CHAPTER 71

WAVE-INDUCED OSCILLATIONS OF SMALL MOORED VESSELS

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ABSTRACT

This study deals with the motions of a neutrally buoyant rectangular parallelpiped moored by a linear spring system in standing waves. The results show that a linear theory which considers the response of the body as a single-degree-of-freedom oscillator adequately describes the surge motion for standing waves ranging from shallow-water waves to deep-water waves and for ratios of body length to wave length of from 0.1 to 1 5.

It was found that the response curves for surge motion become more selective with respect to frequency in the vicinity of resonance as the distance of the body from a reflecting surface increases. Therefore, coupling this with viscous effects, which are more important near resonance, it is possible to reduce the effect of resonance considerably for a moored body with a particular natural frequency by choosing the proper mooring location in its standing wave environment. This has possible application in the planning of berthing facilities in marinas.

INTRODUCTION

The problems associated with the mooring of small craft have become increasingly important in recent years with the increase in the number and size of small boat marinas in both coastal and inland areas. Small craft harbors are no longer simply harbors of refuge, some marinas now exist which represent the investment of millions of dollars and provide valuable reclaimed land which can be developed in addition to the usual berthing facilities for pleasure craft. When there are excessive motions and damage to the boats moored in these marinas a great deal of public attention is directed to these projects and also to the necessary corrective measures

The wave environment inside the marina and the attendant motions of the moored small boats are of course very closely related. The small boat and its mooring lines in effect form a system having response characteristics analagous to a mechanical system restrained by linear or non-linear springs. The wave environment within the marina caused by a particular incident wave system can also be thought of as having a similar mechanical analog. Hence, a particular study of the oscillations of small boats in a marina must really include two phases the investigation of wave-induced oscillations of the marina, and the response of the moored small boats to these waves.

The experimental study reported herein is concerned with the latter problem. The nature of the surge motions of a small boat was studied fundamentally in the laboratory by investigating the motions of a simple body moored in a standing wave system. The body is a neutrally-buoyant rectangular parallelpiped which is connected to a fixed support by means of a linear spring so that the motions are limited to those in the direction of the longitudinal axis of the body. An objective of this investigation was to compare the measured motions to those predicted by available theories thereby investigating some of the differences between the motions of large moored ships and small moored boats.

In the case of large ships, usually long period waves are primarily important as far as motions are concerned. However, due to the significantly smaller natural periods of moored small boats, short period wave systems corresponding to the intermediate to deep water wave range may be important. This in turn leads to the fact that relatively large ratios of boat length to wave length may also be of consequence. Another difference between the two cases is the fact that whereas relatively minor motions of a small vessel moored to a fixed dock may be important from the point of view of damage, the same relative motions for a large vessel would be considered unimportant. Therefore, the amplitude of motion for the full range of the response curve is important for small boats. In addition to these two differences, the question arises as to the relative significance of viscous effects upon small boat motions. For large vessels inertial forces are usually considered to be more important than viscous forces, however, for small boats this may not be the case. It was not the intention of the present study to investigate this question, only a detailed investigation of prototype systems would provide a reasonable answer. Nevertheless the problem of dissipative effects is discussed with respect to the experimental results obtained.

The available literature which is reviewed here deals with studies of the mooring of large ships. These studies are both analytical and experimental, the latter primarily dealing with specific problems.

Wilson (1951) begins a series of papers on the motion of large ships moored in standing wave systems. In this paper Wilson presents the concept of the ship and its mooring system as being analogous to a spring-suspended mass. Abramson and Wilson (1955) obtain solutions to the undamped equation of motion developed by Wilson (1951) to determine the amplitude response of a ship moored in a standing wave system as a function of both the wave period and the type of non-linear constraint. The surge motion of the vessel is of primary interest here. Wilson (1958) presents a comprehensive summary of his work on the mooring force problem. In addition to the question of the response of a moored body in the three coordinate directions, the problem of an unmoored ship is treated. Information is also presented on the predicted response characteristics of various ships along with summaries of more general information such as virtual mass coefficients of rectangular bodies in surge and heave. Wilson (1961) applies the approach presented in his previous papers to the case where a large vessel parted its lines while moored in a rectangular basin The non-linear characteristics of the mooring system were taken into account, and the case was treated as a forced undamped oscillation.

Kilner (1960) analyzes the case of a ship restrained in a non-linear fashion after the manner of Wilson (1958) and obtains an approximate solution to the non-linear problem assuming no damping In the experimental phases of his study Kilner moored a model ship at a location which would always correspond to the node of a uninodal seiche which was created by the cyclical pumping of water through a 29-ft. flume. He also showed how a mechanical analogy could be constructed for the case of a vessel moored in a linear fashion at the node of a standing wave.

Russell (1959) has studied the mooring problem experimentally to find a means of reducing the impact between ships and docks. His experiments take two forms the study of the motion of a model tanker moored alongside jetties in a wave system and the study of the lateral motions of a more idealized body which was restricted to motion in one direction by a stiff spring and in the other by a soft spring. These springs were meant to represent flexible fenders and mooring lines.

There have been a number of model and prototype studies of mooring forces and ship motions for particular ships. Investigations such as those of Knapp (1951), O'Brien and Kuchenreuther (1958), O'Brien and Muga (1964), and Wiegel, et al (1959), all add to the fund of general knowledge on the subject of motions of large moored vessels. However, since these studies are of a more specific nature they will not be reviewed here

THEORETICAL CONSIDERATIONS

The theoretical approach used in this study to obtain the equation of motion of a single moored body in surge is essentially the same as that proposed by Wilson (1958) and Kilner (1960). This will be briefly summarized in this section.

The body to be considered is a simple rectangular parallelpiped of length 2L, draft D, and beam B, moored such that only surge motions are possible. A sketch of the body in its wave environment is shown in Fig 1 A standing wave system excites the moored body in a rectangular basin of constant depth d in which the body is moored some distance b from a perfectly reflecting wall. The coordinate x is measured from the center of the body when it is at rest Using small amplitude wave theory the standing wave is described in the usual way by the following expressions for the wave amplitude η and the water particle velocity u in the x-direction

$$\eta = A \cos k(b+x) \cos \sigma t \tag{1}$$

$$u = \frac{Agk}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \sin k(b+x) \sin \sigma t \tag{2}$$

where A is the standing wave amplitude, k is the wave number (2π/wave length, λ), σ is the circular wave frequency (2π/wave period, T), g is the acceleration of gravity, and the other parameters are described in the definition sketch, Fig 1.

The equation of motion of the moored body in surge is

$$M\ddot{x} = \sum \text{(External Forces)}$$

$$M\ddot{x} = F_p + F_i + F_d + F_r \tag{3}$$

where $\ddot{x} = \dfrac{d^2x}{dt^2}$ and M is the mass of the moored body.

The terms on the right-hand side of Eq. 3 are as follows

1. The net pressure force acting on the ends of the body which is a driving force is denoted by F_p.

2. The term F_i is an inertial force which comes about due to the unsteady nature of the problem and the acceleration or deceleration of some mass of fluid in addition to the body. It is the product of the added mass or hydrodynamic mass of the floating body in the x-direction, M'_x, and the relative body acceleration. This term is both a driving force and a restoring force.

3. The viscous drag term F_d is expressed in terms of the relative body velocity and a drag coefficient. It is also both a driving and a restoring force.

4. The restoring force F_r can assume various forms depending on the type of mooring system used Wilson (1951) has suggested that this restoring force be expressed in the form of a power law. (For this study a linear spring system is used.)

After substituting the appropriate terms which define the various forces into Eq. 3, the equation of motion of the body in surge can be written in a general form as

$$M\dot{x} = M\dot{U} + M'_x (\dot{U} - \ddot{x}) + \frac{\rho}{2} C_{D_x} BD (U - x) | U - \dot{x} | - Cx^n \tag{4}$$

where $\dot{U}$ is the water particle acceleration averaged over the length and the draft of the body and U is the areal average of water particle velocity obtained in a similar way. After defining the virtual mass coefficient, C_M as

$$C_M = 1 + \frac{M'_x}{M}$$ Eq. 4 reduces to·

$$C_M M(\dot{x} - \dot{U}) + \frac{\rho}{2} C_{D_x} BD(\dot{x} - U) | x - U | + Cx^n = 0 \tag{5}$$

For the purpose of this study both the drag term and the restoring force in Eq 5 have been linearized The final reduced form of the equation of motion is·

$$x + \beta_x \dot{x} + \omega^2 x = \dot{U} + \beta_x U \tag{6}$$

where· $$\beta_x = \frac{C_{d_x}}{4\, C_M L}$$

$$\omega^2 = \frac{C}{C_M M}$$

and the coefficient of drag redefined as C_{d_x} incorporates the effect of the relative velocity. If it is assumed that C_{d_x} does not vary significantly for a particular system then the solution to Eq. 6, after some simplifications, is given by

$$\frac{x}{A} = \frac{\delta}{A\sigma^2} \frac{\sqrt{\left[1 - \frac{\sigma^2}{\omega^2} - \frac{\beta_x^{\,2}}{\omega^2}\right]^2 + \frac{\beta_x^{\,2}}{\sigma^2}}}{\frac{\omega^2}{\sigma^2}\left[1 - \frac{\sigma^2}{\omega^2}\right]^2 + \frac{\beta_x^{\,2}}{\omega^2}} \cos(\sigma t - \varphi) \tag{7a}$$

$$\text{and } \tan\varphi = \frac{\beta_x/\sigma}{1 - \frac{\sigma^2}{\omega^2} - \frac{\beta_x^{\,2}}{\omega^2}} \tag{7b}$$

$$\text{where } \delta \equiv \frac{Ag}{kLD}\left[\frac{\sinh kd - \sinh k(d-D)}{\cosh kd}\right] \sin kL \sin kb \tag{7c}$$

It can be shown that as the wave length becomes large relative to the depth and the dimensions of the system, for the case of zero damping, the maximum relative displacement of the body, X/A, approaches

$$\frac{X}{A} = \frac{b}{d} \frac{\sigma^2/\omega^2}{1 - (\sigma^2/\omega^2)} \tag{8}$$

However, when such simplifications cannot be made, the variation of the maximum displacement will be governed by the ratio of body length to wave length and the ratio of distance of the body from a reflecting surface and wave length in accordance with the trigonometric terms of Eqs. 7a and 7c.

A set of theoretical response curves obtained from Eq. 7a and plotted as X/A vs τ/T (ratio of the maximum surge amplitude to the maximum wave amplitude as a function of the ratio of the natural period of the body to the wave period) for constant damping ratios β_x/ω, are presented in Fig. 2 for the particular system dimensions and body characteristics indicated in the figure. These curves are given in this section only to show the general shape of the response curves and the effect of the linearized friction on the body response. The zero values of X/A are caused by the trigonometric functions of Eqs. 7a and 7c. In a physical sense this occurs when there is no net driving force acting on the body. As the dissipation function β_x/ω increases, the relative resonant period of the body, τ/T, becomes smaller. Since τ is the undamped natural period, this means that the resonant period of the body is increased with increased damping, the same as its mechanical analog, the single-degree-of-freedom oscillator. Near resonance, the response of the system decreases as the damping decreases; however, for the case shown, for small values of τ/T the response actually increases with increasing damping. This is because in this region the viscous effects become important as a driving force in the system. For very large values of β_x/ω the response at resonance actually increases with increasing damping. The general characteristics of the response curve will be discussed in more detail later.

EXPERIMENTAL EQUIPMENT

This experimental study was conducted in a wave basin 21 inches deep with a working region approximately 30 ft. long by 12 ft. wide. The standing wave system in this basin was produced by a pendulum-type wave generator, 12 ft. wide, located at one end of the basin. By a simple adjustment of the support members of the plate of the generator, it could be operated either as a piston or a flap-type wave generator. The plate is driven by two arms connected to independent Scotch yokes which are in turn driven through a pulley system by a variable speed motor. A maximum amplitude of ± 6 in. can be obtained with this arrangement and the amplitude of the wave machine adjusted and measured to within ± 0.0005 in. Some of the details of the wave basin and generator system can be seen in the photograph, Fig. 3.

The wave period, which can be varied from 0.34 sec. to 3.8 sec., is measured by a pulse counting technique. The pulse is generated by interrupting a light beam which is directed at a photocell by a disc with 360 evenly spaced holes arranged in a circle near its outer edge. This disc can be seen in Fig. 3. These pulses are counted on an electronic counter and the wave period so obtained is a 10-second average. Using this technique the wave period could be maintained throughout an experiment within ± 0.05 percent of the desired value.

The moored body was a rectangular parallelpiped having a length of 24 in., a beam of 6 in. and a height of 8 in. The model is supported from overhead by a structure independent of the basin, which can be located anywhere on the centerline of the basin. Thus, the distance b in Eq. 7c can be varied over a wide range.

The body is connected to the support structure by means of two aluminum leaf springs (0.09 in. thick and 2 in. wide) mounted 2 in. apart which represent a linear mooring system. These springs can be clamped anywhere along their length so that the natural frequency of the body can be varied over a wide range. Also, the draft of the body can be changed independent of the springs by raising or lowering the spring support plate, and ballast can be added or removed to make the body neutrally buoyant The motions of the body in surge are measured by means of a linear variable differential transformer, the core of which is attached to the body and the coil to the fixed overhead support structure. Some of these features can be seen in the photograph Fig. 4.

Wave heights were measured at three locations along the wall of the basin opposite the wave machine by means of resistance wave gages. The gages consist of two stainless steel wires 0.01 in. in diameter, approximately 3-1/2 in. long spaced 1/8-in. apart. They are insulated from each other and stretched taut in a 1/8-in. diameter stainless steel frame which becomes an integral part of a point gage used for calibration. These resistance wave gages were used in conjunction with a Sanborn direct-writing recording system to obtain time histories of the wave amplitude.

For additional details the interested reader is referred to Raichlen (1965).

PRESENTATION AND DISCUSSION OF RESULTS

The variation of the virtual mass coefficient C_M and the damping coefficient β_x with system parameters was evaluated by investigating the nature of the damped free oscillations of the body in surge. The solution to Eq. 6 reduced to the case of free oscillations is

$$x = Xe^{-\frac{\beta_x}{2}t} \sin\left[\sqrt{\omega^2 - \left[\frac{\beta_x}{2}\right]^2}\, t + \varphi\right] \tag{9}$$

From Eq. 9 and the definition of ω it is seen that the virtual mass coefficient C_M can be determined by measuring the spring constant of the mooring system and the natural frequency of the body in air and in water. The difference between the masses so obtained is the hydrodynamic or added mass of the body and the virtual mass coefficient follows directly from its definition

The virtual mass coefficient C_M determined in this way is presented in Fig. 5 plotted as a function of the ratio of the draft to beam, D/B. The data shown are for a number of experiments with different depths of water, body weights, and natural frequencies. In all cases the data were obtained before waves generated by the motion of the body were reflected from the basin walls and returned. There appears to be scatter in these data which may be due in part to experimental error. For instance, if both the spring constant and the natural frequency of the body were in error in opposite directions by 2 percent this could lead to an error in the analytically determined mass of the body of approximately 6 percent. Assuming that the evaluation of the mass in water and air to be an error in the same amount, an error of approximately 10 percent could occur in the virtual mass coefficient C_M.

Three curves are shown in Fig. 5 experimental curves describing the upper and lower bounds of the experimental data and a dashed curve, which is an average of these data. The upper and lower curves at most show a deviation of ± 5 percent from the average curve, indicating that the observed scatter could be due to the experimental errors mentioned. Nevertheless for this body there is a definite trend in the variation of C_M with draft, i.e., an increase of approximately 15 percent as the draft increases from 25 percent of the width to nearly the full body width. The values of the virtual mass coefficient presented by Wilson (1958) for a rectangular parallelpiped having a beam to length ratio of 0.25 are shown by the arrows. These are reported by Wilson as values proposed by Wendel (1956) and Browne et al (1929-1930) for the upper and lower values respectively, and it is seen that they correspond essentially to average values of the virtual mass coefficients determined in these experiments.

The damping coefficient β_x can be obtained from the decay of the measured damped free oscillations in accordance with Eq. 9. The values of the damping ratio, β_x/ω, determined in this way and corrected for the effect of structural and air damping are presented in Fig. 6. These data are plotted as a function of the product of the circular natural frequency and the draft, ωD. It may be considered better to use a type of Reynolds number as the abscissa, however, it was felt that this would tend to indicate a more general relationship than the data warranted.

A part of the energy dissipation and the resultant increase in the damping ratio β_x/ω with increasing ωD can probably be attributed to wave generation by the body. In effect the height of the waves generated and the resultant energy which goes into wave making, increases with the product of depth of immersion of the generating plate (D) and the frequency of oscillation of the generator (ω). Therefore, it appears that for this case this type of energy dissipation is more important than viscous dissipation.

In Figs. 7 and 8 the general characteristics of the wave systems used in studying the forced response of the body to standing waves is presented. These curves are for two cases· body natural periods of 1.305 sec. and 0.664 sec. In each of these figures the wave length, wave number, ratio of depth to wave length, and ratio of body length to wave length for these experiments are plotted as functions of the ratio of the natural period of the body to the wave period.

Three response curves were obtained for each of two natural periods of the body, $\tau = 1.305$ sec. and $\tau = 0.664$ sec , for various distances of the center of the body from the backwall of the basin, b. Figs. 9, 10, and 11 show theoretical response curves and corresponding experimental data for values of the distance b of 1.95 ft., 4 0 ft , and 6.0 ft. respectively and for a natural period of the body of 1.305 sec.

The weight of the body was adjusted so that for the depth of immersion chosen ($D \simeq 0.375$ ft.) the body was neutrally buoyant.

The different experiments shown in each figure correspond to different stroke settings of the wave machine and both piston and flap type wave machine operation. Therefore, in general it can be said that away from resonance the system operated as a linear system since there is relatively good agreement among data which correspond to essentially the same wave period but different wave heights.

In comparing the theoretical response curves shown in Figs. 9, 10, and 11 for the inviscid case to the experimental data it is seen that away from resonance the theoretical curves agree relatively well with the experimental data. However, in the vicinity of resonance generally the agreement between theory and experiment is poor. It is possible that in part this disagreement can be attributed to the waves which are generated by the oscillating body. These waves after reflecting from the boundaries of the basin return to the body and modify the net pressure force on the body This in turn alters its steady state motion and the theoretical approach which neglects the wave-making ability of the body would no longer be applicable. In addition to this, of course, non-linear effects which are not included in the theory become most important near resonance and can contribute to the disagreement between theory and experiment. The effect of resonance of the basin itself may be important near body-resonance ($\tau/T = 1$). The values of τ/T which correspond to the harmonies of the basin are indicated by the arrows at the abscissa of the figures with n denoting the number of the harmonic.

A theoretical curve corresponding to a damping ratio, β_x/ω, of 0.1 is shown in Fig. 9. The effect of damping on the response curve for this case appears small except in the immediate vicinity of resonance. However, in this region, as mentioned previously, due to finite amplitude effects the applicability of this small amplitude theory would itself be in question. The value of β_x/ω obtained for this configuration from the free damped oscillation of the moored body was 0.11. It should be mentioned that due to wave generation by the oscillating body itself, one should not expect the damping effect in free oscillation to be the same as that in forced oscillation. Nevertheless, it can be said with reference to the experimental data and the undamped theoretical response curve shown in Fig. 9 that in this case dissipative effects are small.

It is interesting to view Figs. 9, 10 and 11 in an overall sense to observe the effect of location of a moored body in a standing wave system. As described previously, since the net driving force acting on the ends of the body periodically goes to zero, the response curve also goes to zero periodically for each body location. For a given range of wave periods, as the distance of the body from the back wall increases, the number of zeroes in the response curve increases. This in effect forces the response curve near resonance to become more peaked, or in other words, for the body response to become more selective in a periodwise sense.

Theoretical and experimental response curves were also obtained for essentially the same conditions as those shown in Figs 9, 10 and 11, except that the natural period of the body was reduced by nearly a factor of two to $\tau = 0.664$ sec by increasing the spring constant. These results are presented in Figs. 12, 13 and 14 for distances from the center of the body to the reflecting surface of 2 ft, 4 ft, and 6 ft respectively.

The scatter of the data in Figs 12, 13 and 14 is less than for the corresponding data obtained with the moored body having a larger natural period ($\tau = 1.305$ sec.). This may be attributed to the smaller effect of wave generation in the case of a body with a smaller natural period. Since the natural period of the body was decreased by increasing the spring constant of the system, the restoring force was correspondingly increased and the body motions and the height of the waves generated by the body were reduced compared to the system with the larger natural period.

As before it can be seen that the number of zeroes of the response curve increases as the distance from the reflecting surface increases, and this in turn causes the response curve to become more selective near resonance For instance, for a response X/A equal to unity, when the center of the body is located 2 ft from the backwall, the band width of the major peak ranges from $\tau/T = 0.91$ to $\tau/T = 1.015$, or a band width $\Delta(\tau/T) = 0.11$ When the distance increases to 6 ft the width of the peak at unit response is $\Delta(\tau/T) = 0.04$ Since the major effect of damping is to reduce the body response at resonance, it is possible that, due to combined effects of energy dissipation and the narrowing of the response curve near resonance, an attenuated response always occurs. In other words, due to a high body natural frequency and a large distance from a reflecting surface, the response of the body in surge may be unimportant as far as gross movements are concerned.

This change of the response curve as a function of the location of the moored body can raise some interesting speculation with regard to the operation of marinas. For instance, it may be feasible for a harbor excited by a predominant wave period to locate boats of various sizes in different areas of the marina depending upon the natural frequency of the boat-mooring system. This would take advantage of the demonstrated fact that for a particular moored body if properly located the response defined as X/A, with dissipative effects included, could always be less than unity. Of course, the preferred location would also be influenced by the response characteristics of the basin and the expected three-dimensional amplitude distribution within the basin

It is of interest to note the range of the ratio of body length to wave length represented by the experimental data presented in the two sets of response curves, i.e., Figs. 9, 10 and 11 for $\tau = 1.305$ sec. and Figs. 12, 13, and 14 for $\tau = 0.664$ sec. For the moored body having a natural period of $\tau = 1.305$ sec. the ratio of the body length to wave length varied from 0.1 to 0.8. This ratio varied from 0.25 to 1.5 for the moored body with the smaller natural period ($\tau = 0.664$ sec.) Even for the latter case where the wave length is comparable to the body length, agreement with the small amplitude theory is fairly good. Therefore, it can be stated that the theory presented by Wilson (1958) applies reasonably well even when the wave length becomes small compared to the length of the body. Also, in addition to this, the theoretical approach appears applicable over the full range of wave length to depth ratios, i.e., from shallow water waves through the intermediate region to deep water waves.

CONCLUSIONS

The following major conclusions may be drawn from this study

1. In general it can be stated that the theory proposed by Wilson (1958) adequately describes the surge motion of this simple body moored in a linear fashion in a standing wave system. The wave length to depth ratios covered by these experiments varied from approximately the shallow water wave limit to the deep water wave limit. The ratio of body length to wave length varied from 0.1 to 1.5.

2. The coefficient of virtual mass of the body (rectangular parallelpiped of aspect ratio 4·1) determined from simple free oscillations was found to correlate best with the ratio of draft to beam (D/B) For a variation of D/B from 0.25 to 0.95 the average coefficient of virtual mass varied from approximately 1 1 to 1 25.

3. The response of the moored body becomes more selective with respect to wave period as the distance of the body from a reflecting surface increases.

4. It is possible to reduce the effect of resonance considerably, simply by choosing the proper body location in its standing wave environment for a particular natural frequency.

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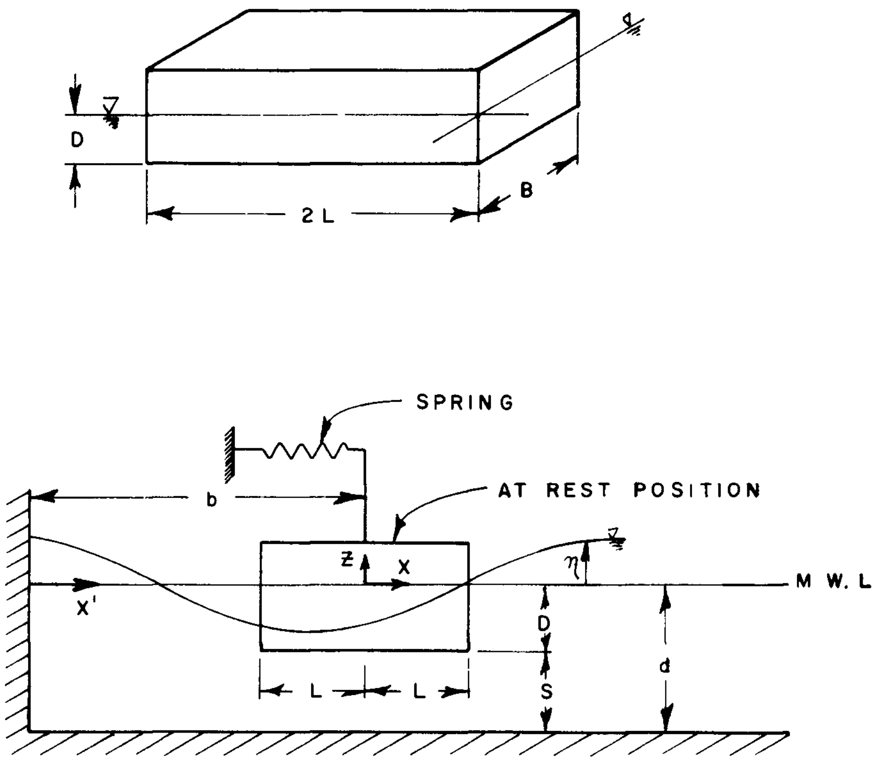

Fig. 1. Definition sketch of single moored body.

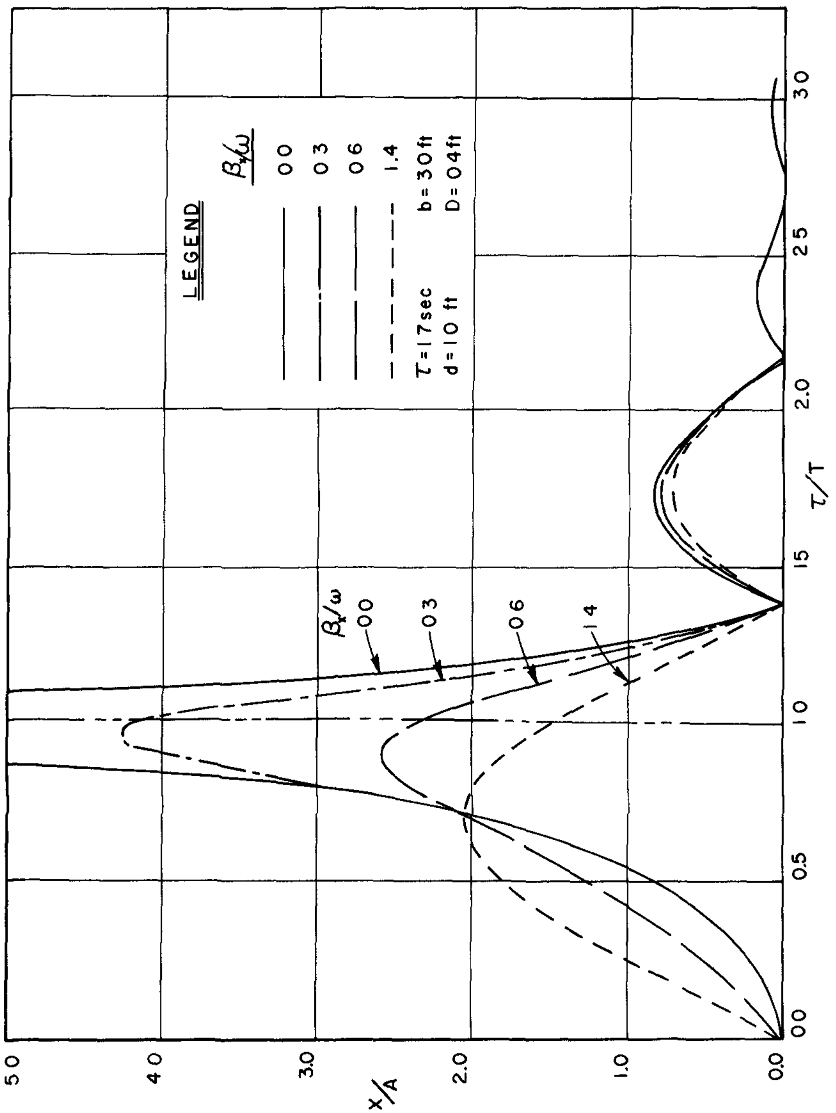

Fig. 2. Theoretical response curve single moored body.

Fig. 3. Overall view of wave basin and wave generator.

Fig. 4. View of model and model support structure.

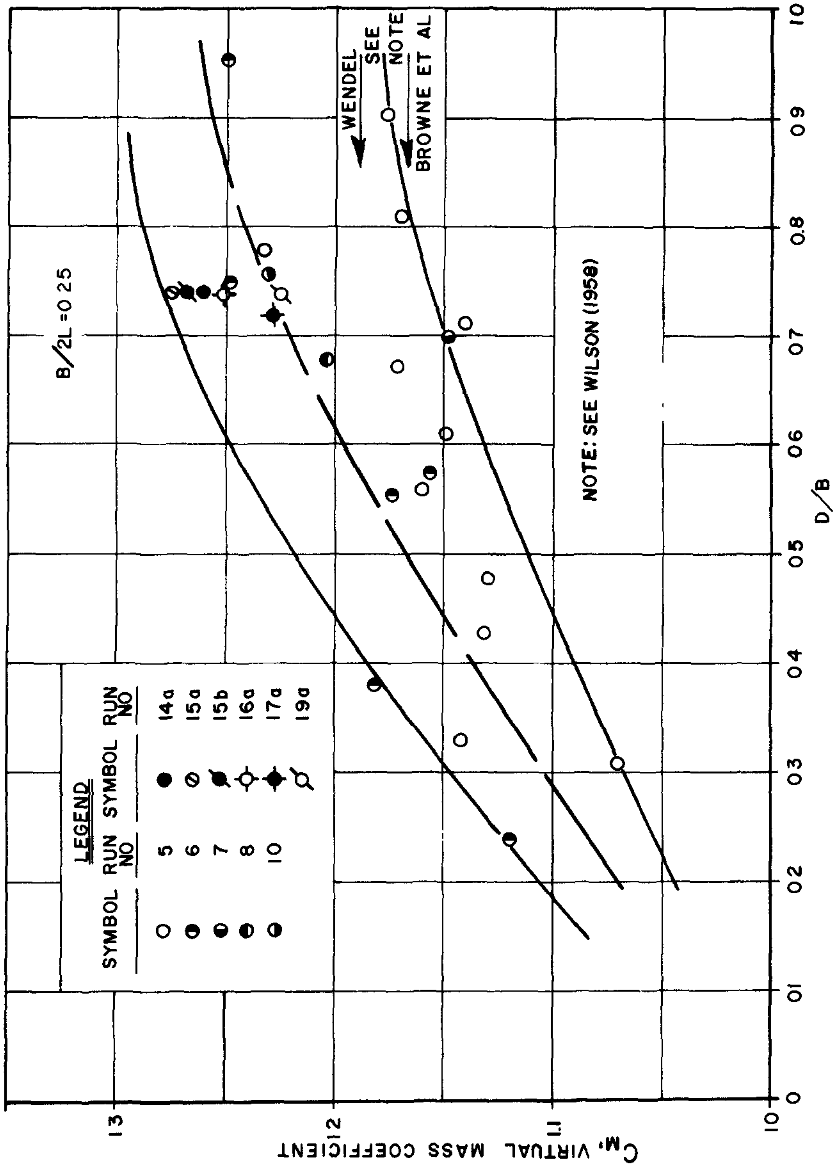

Fig. 5. Variation of virtual mass coefficient, C_M, with ratio of draft of body to beam of body, D/B.

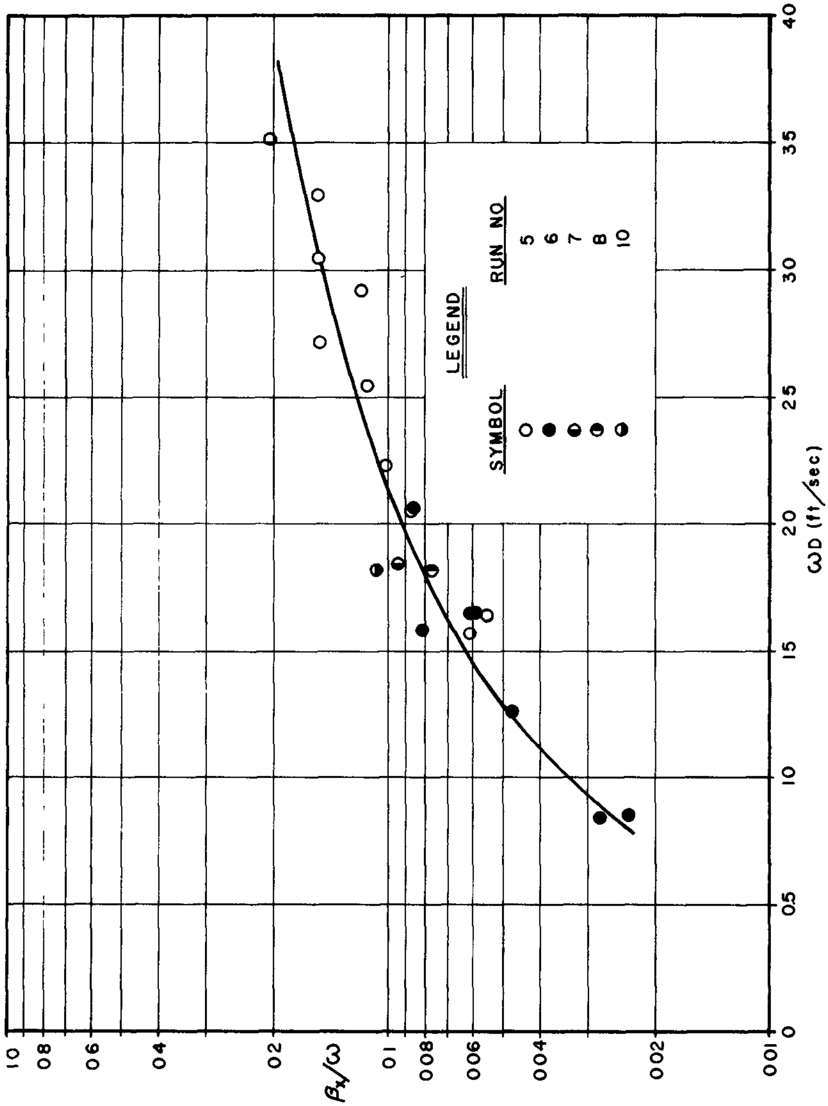

Fig. 6. Variation of damping ratio β_x/ω with product of circular frequency and draft, ωD.

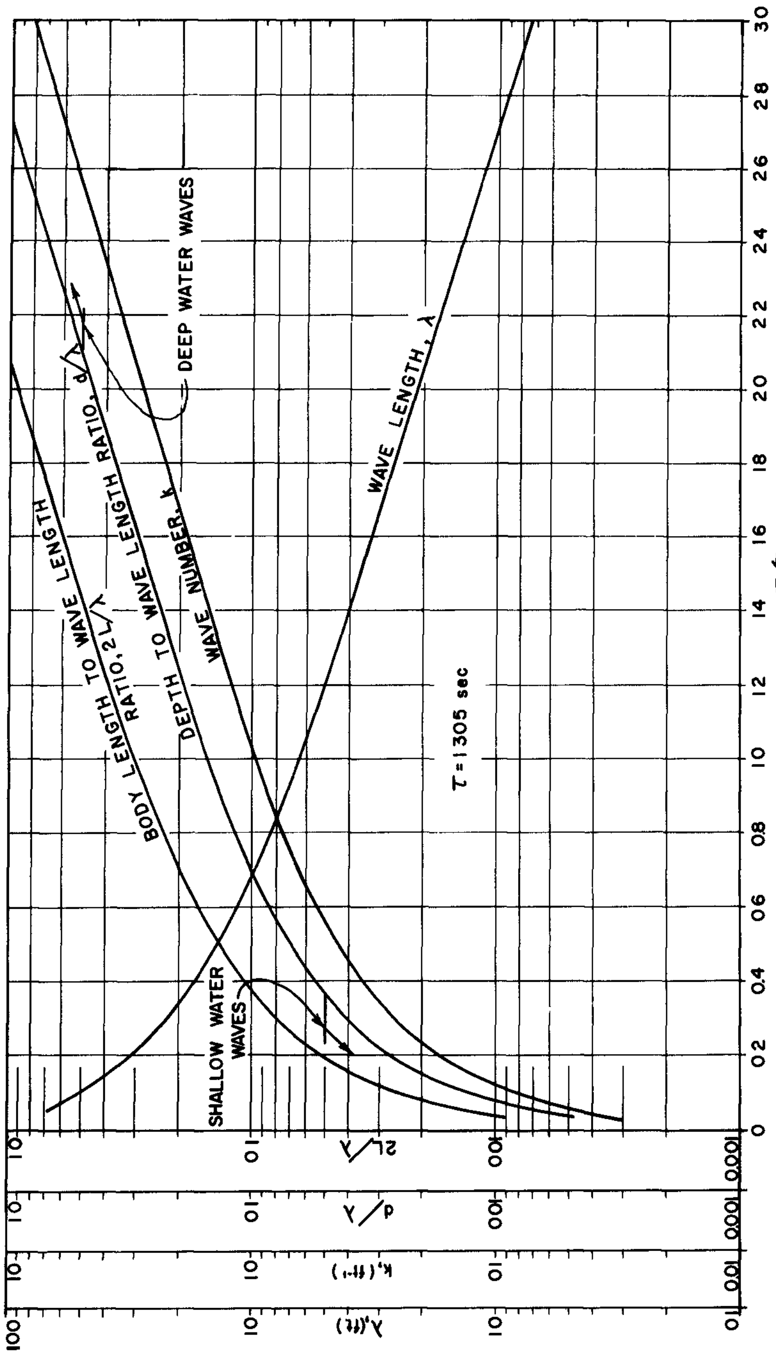

Fig. 7. General characteristics of wave system ($\tau = 1.305$ sec.).

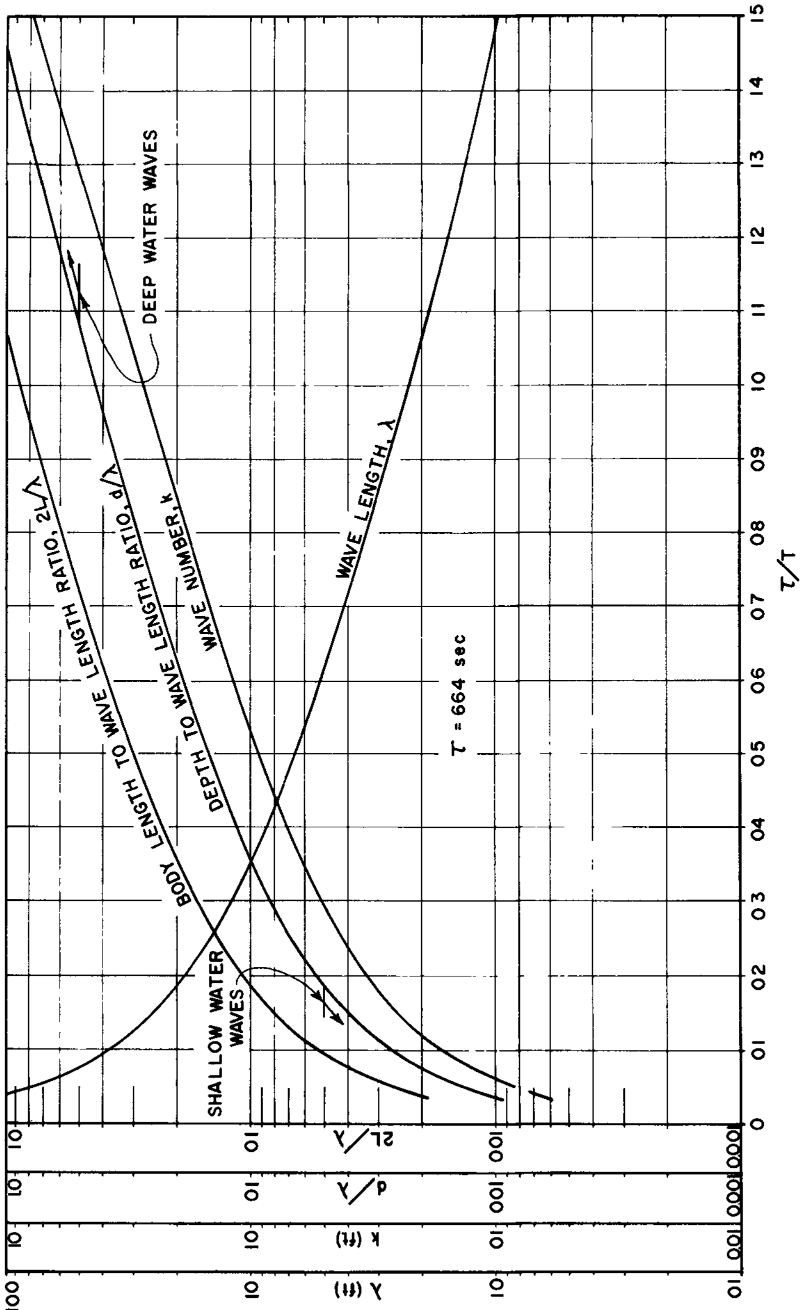

Fig. 8. General characteristics of wave system ($\tau = 0.664$ sec.).

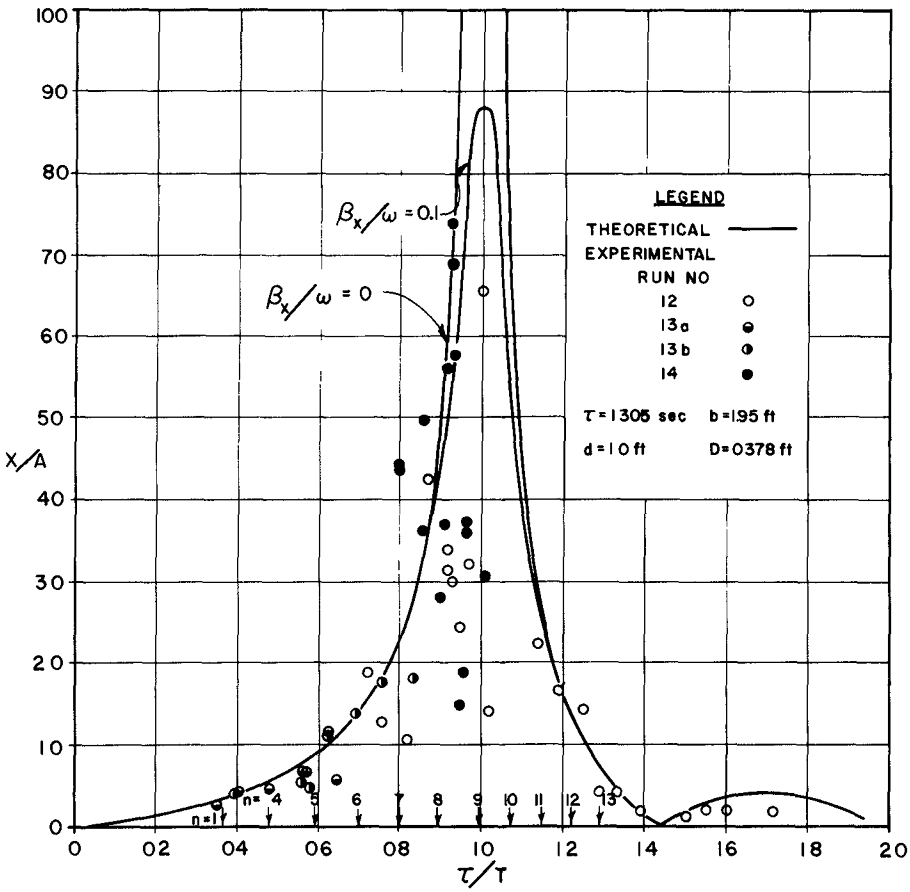

Fig. 9. Response curve of single moored body
(τ = 1.305 sec., b = 1.95 ft).

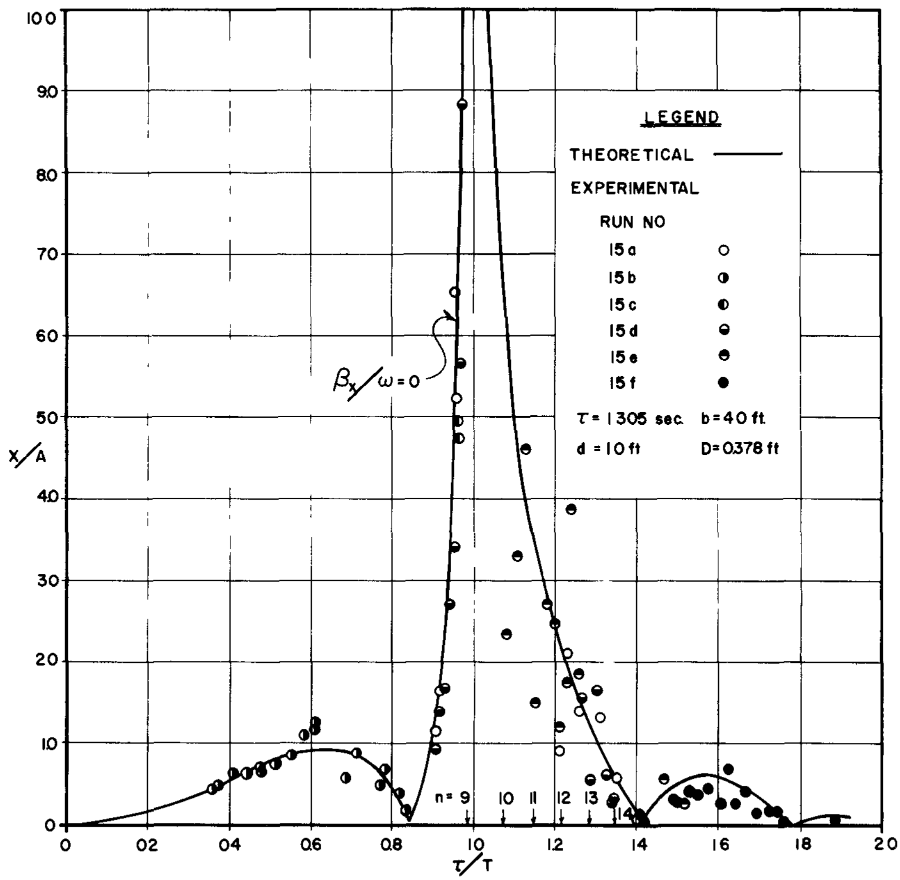

Fig. 10. Response curve of single moored body (τ = 1.305 sec., b = 4.0 ft).

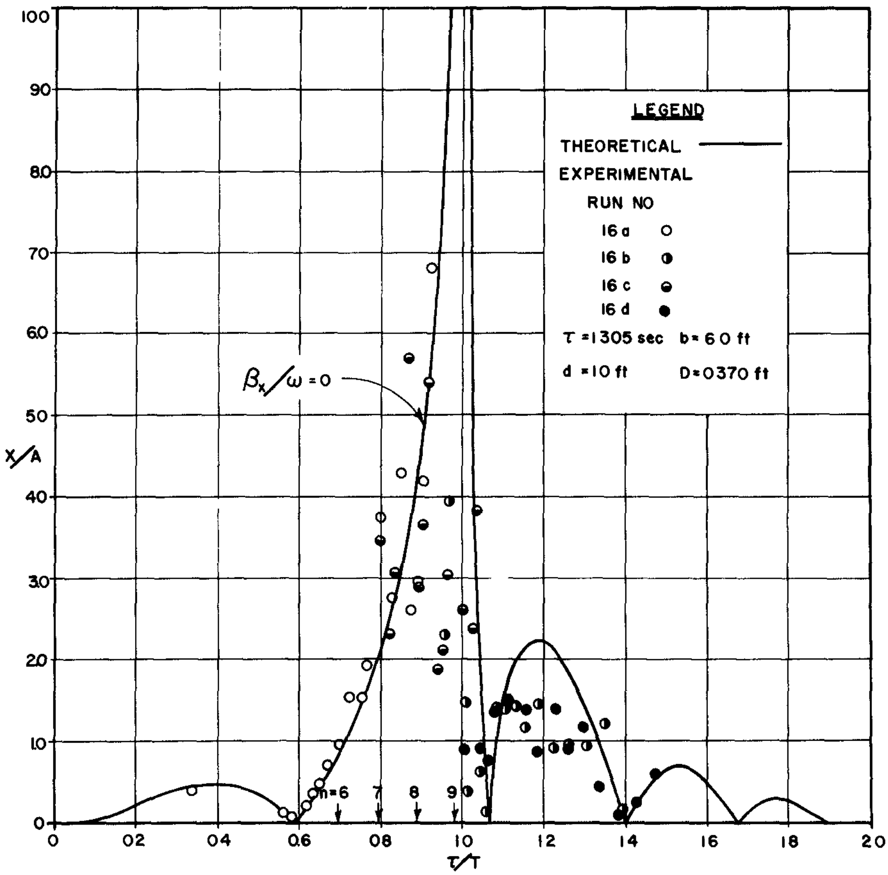

Fig. 11. Response curve of single moored body (τ = 1.305 sec., b = 6.0 ft).

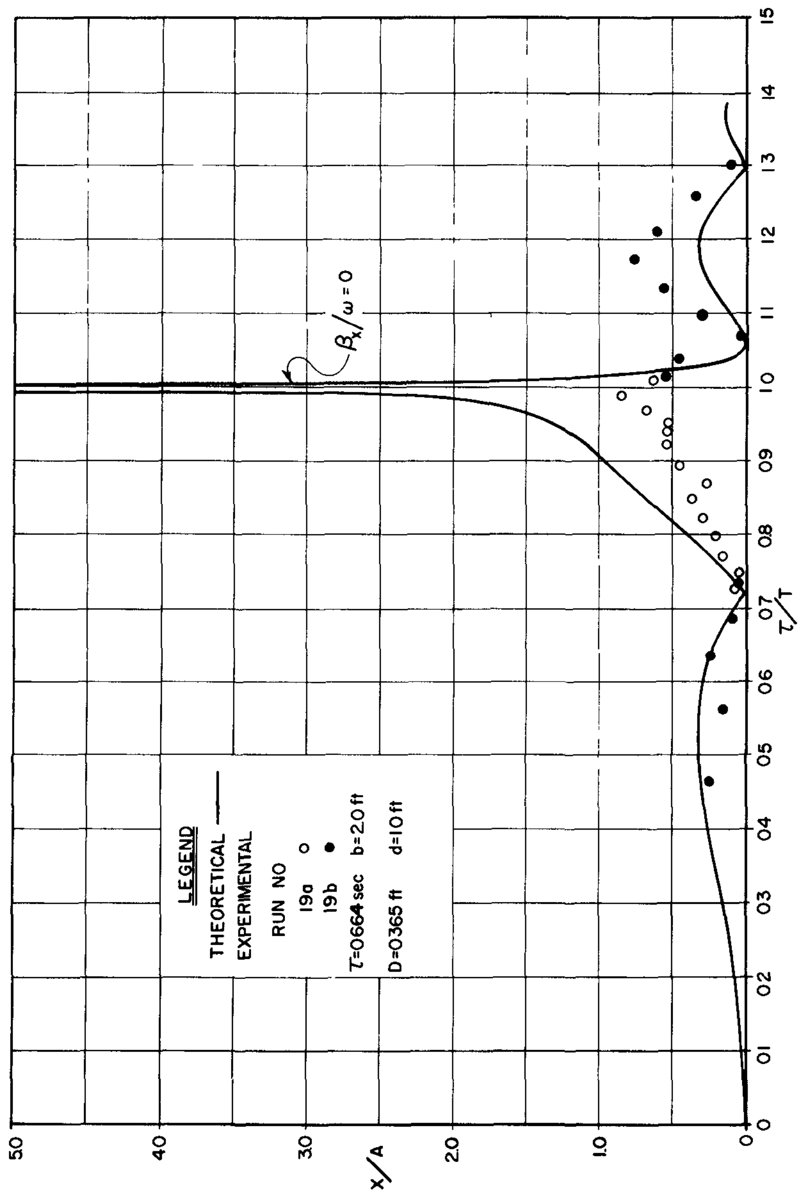

Fig. 12. Response curve of single moored body (τ = 0.664 sec., b = 2.0 ft).

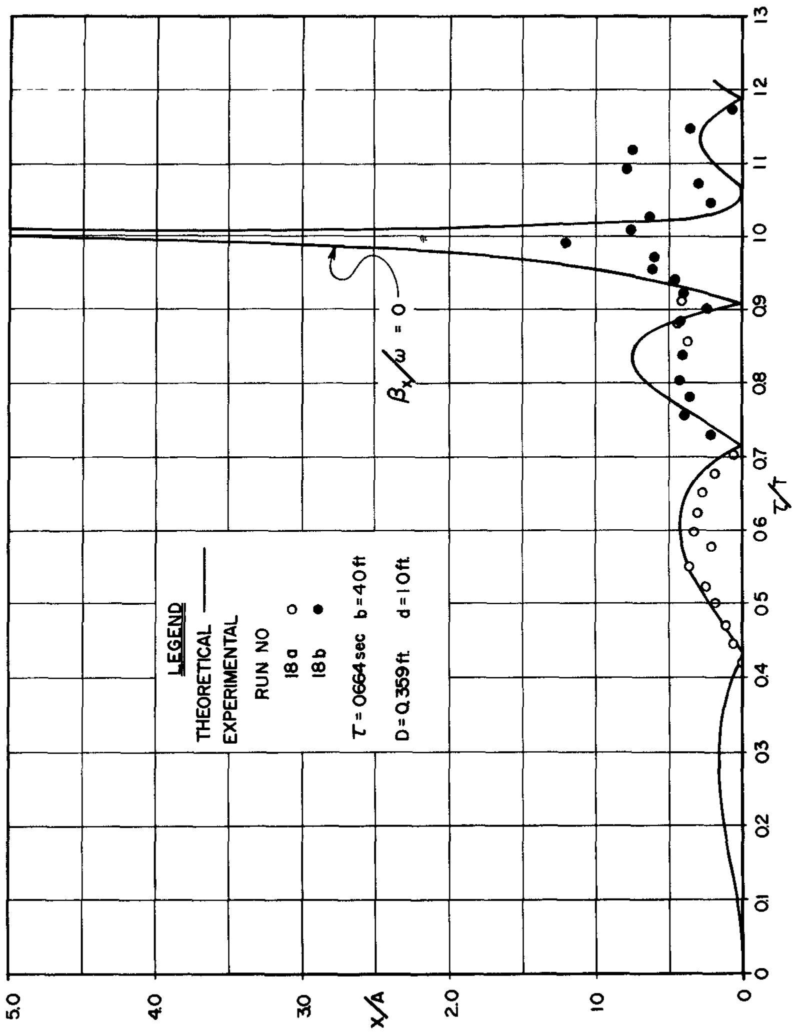

Fig. 13. Response curve of singled moored body (τ = 0.664 sec., b = 4.0 ft).

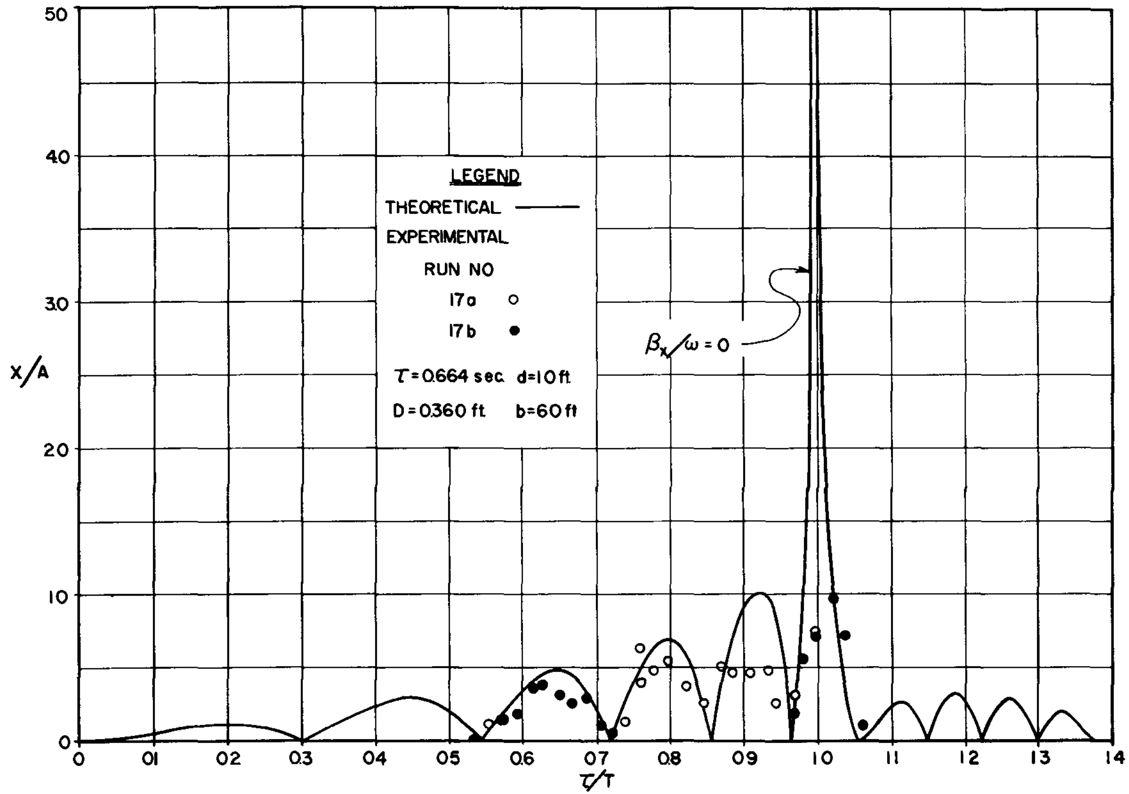

Fig. 14. Response curve of single moored body
(τ = 0.664 sec., b = 6.0 ft).