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# Conflicted advice about portfolio diversification

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**Abstract**

We investigate the validity of the argument by the financial services industry to "roll over your 'old' 401(k) plan" because 401(k)-type plans have a limited number of investment options, while Individual Retirement Accounts (IRAs) have a virtually unlimited number of options. We empirically analyze the diversification of a large 401(k)-type plan with only five basic investment options. Financial advisers with a conflict of interest may use strategic complexity to encourage rollovers, recommending complex portfolios to impress naïve clients who have a weak understanding of the concept of diversification, while not weighing the cost of the complex portfolios against any added benefits of diversification. 

*JEL classification:* G2; G4

*Keywords:* Portfolio diversification; Financial advice; Conflicts of interest

## 1. Introduction

Because of pension rollovers, Individual Retirement Accounts (IRAs) have become the most important source of pension income in the United States. Thus, pension rollovers play a key role in the U.S. retirement income system. The argument generally made to support the campaign by the financial services industry to "roll over your 'old' 401(k)" is that 401(k)-type plans have a limited number of investment options, while IRAs have a virtually unlimited number of options. This paper investigates the validity of the advice that better diversification is a reason for rolling over to an IRA by empirically analyzing

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the diversification a 401(k)-type plan with only five basic investment options (and five target date funds composed of those options)—the Thrift Savings Plan for federal government workers.

To study the issue of complexity in pension portfolios, we have purposively chosen a pension plan with a small number of investment options. Our argument is not that a plan with a relatively small number of investment options is superior to a plan with say 30 options. Rather, we address the question of whether a plan with only five basic investment options can be adequately diversified. If that is the case, the much larger number of investment options generally found in 401(k) plans indicates that the argument favoring the diversification benefits of the substantially larger number of investment options available in IRAs is generally not valid.

We argue in this paper that conflicted advisers focus on the aspects that are favorable to the case they are making (e.g., "only five funds"), but do not consider whether pension participants need more choice in funds to improve diversification. In addition, they do not weigh the costs associated with their advice to leave a low-cost plan to obtain more investment options. We argue that it is psychologically less costly for advisers to make a true statement that is incomplete than to make a false statement. In addition, some advisers may simply follow the industry standard argument, without considering its merit.

We argue that many participants are susceptible to this argument relating to portfolio complexity because of their naïve understanding of diversification. They think that more investment options are always better, not understanding the characteristics required of new options to improve diversification, and not considering the costs in higher fees of the added options.

This paper thus relates to the more general issue of how many diversified mutual funds are needed to form a diversified investment portfolio. For example, do target date funds need more than a dozen different investments in their portfolios, or would a smaller number be better in part because it would involve less costly funds. It can be argued that an investment that is a small share of the portfolio does not materially affect the risk-return characteristics of the portfolio and should not be included if it is a relatively expensive investment in terms of fees. Some financial advisers and financial products companies may engage in strategic complexity in their portfolios, providing complex portfolios to impress naïve clients.

The remainder of the paper is structured as follows. We first provide background information about roll overs to IRAs and why we focus on the Thrift Savings plan with its five basic options. Focusing on the TSP provides a test of the hypothesis that pension participants should roll over from their 401(k) plans to obtain greater portfolio diversification. We then review the relevant literature concerning portfolio diversification and pension rollovers. Following that, we discuss the investment options available in the Thrift Savings Plan. The main section of the paper follows in which we analyze the effect of adding more investment options and investigate the validity of the advice to roll over from the TSP. Last, we offer our conclusions relating to the quality of the advice from some conflicted financial advisers, the nature of the arguments some conflicted financial advisers make, and the susceptibility of some pension participants with low financial literacy to making decisions based on these arguments.

## 2. Rollovers to IRAs

IRAs are the largest type of pension plan in terms of assets in the United States, having overtaken 401(k) plans and defined benefit plans. Rollovers from 401(k)-type plans are the primary source of funding for IRAs, with relatively few people contributing to IRAs. IRAs had an estimated $7.9 trillion in assets at the end of 2016 and represented 31% of total U.S. retirement market assets, compared with 18% two decades earlier. In 2012, $335 billion was rolled over from employer-provided plans to IRAs (Investment Company Institute, 2016b, 2017).

The Council of Economic Advisers (CEA 2015), which advises the president on economic policy, surveyed the literature on the quality of financial advice provided in the United States. The CEA finds that advice concerning pension rollovers from employer-provided plans to IRAs, and stemming from conflicts of interest, costs U.S. pension participants $17 billion a year in higher fees and lower rates of return. Supporting these conclusions, a study by Munnell, Aubrey and Crawford (2015) finds that IRAs on average receive net rates of return that are about 1 percentage point less than do employer-provided defined contribution plans, such as 401(k) plans and 403(b) plans, in part because of higher fees.

Advisers giving bad advice presumably make an argument to their clients as to why their advice is good advice. Typically, that argument in this context is that pension participants will have more investment options in IRAs than in 401(k)-type plans. That argument has become the industry standard for advice. For example, TIAA (2016) indicates an advantage of rolling over to an IRA is that a pension participant has "a virtually unlimited array of investments." Similarly, Fidelity (2016) indicates an advantage to rolling over is that you have "a wide range of investment options."

According to a survey of persons making pension rollovers, while improved diversification is not the only reason workers give for why they rolled over to an IRA, it is the primary reason for 21% and one of the reasons for 61% of those making rollovers (Investment Company Institute, 2016b).

This argument supporting roll overs seemingly runs counter to the requirements of U.S. pension law. U.S. pension law (ERISA Section 404(c)) requires that 401(k) plans that allow participants the opportunity to make their own investment choices must provide investment options that permit adequate diversification. BrightScope and Investment Company Institute (2014) find that in 2012, 401(k) plans on average provided participants 25 investment options.

## 3. Thrift savings plan

The Thrift Savings Plan (TSP) is the 401(k)-type plan for U.S. federal government employees, members of Congress and the military. In terms of assets, it is the largest pension plan in the United States (Towers Watson, 2014) and the largest defined contribution plan in the world (White, 2011). It has more participants than the social security systems of more than 90 countries (World Bank, 2014). We focus on the TSP because it only offers five basic investment options, plus target date funds. It also charges extremely low fees—three basis points for all its funds, including its international equity fund and its target date funds, which tend to be higher fee funds. The average fee for target date funds outside the TSP is roughly 30 times higher than for the target date funds the TSP provides (Vanguard, 2014).

A survey of TSP participants who made a withdrawal in 2013 finds that an estimated 16,400 participants (about one-third of those making withdrawals) made a withdrawal of all or part of their TSP account because they were advised by their financial adviser to do so (AonHewitt, 2014). Advisers frequently advise TSP participants to roll over from their low-fee account to an IRA that the adviser would manage.

A survey of financial advisers finds that advisers who advise their clients to roll over their TSP accounts commonly use the argument that because the TSP only offers five funds (plus lifecycle or target date funds based on those five funds), the client can obtain greater diversification outside of the TSP (Turner, Klein, and Stein 2016). For example, Ric Edelman, who was three times named the top independent financial adviser in the United States by Barron's, states, "The downside to the Thrift Plan is the fact that you have only five investment choices. None of them are particularly exciting in terms of their performance relative to what's available elsewhere, so we are not terribly thrilled with the choices in the Thrift Plan although we do acknowledge it's really cheap" (Tergesen, 2014).

## 4. Literature review

### *4.1. Quality of advice*

Because of the importance of the 401(k) rollover decision in retirement planning, many people seek financial advice. One survey finds that 61% of the people with rollover IRAs received advice to roll over from a financial adviser (Investment Company Institute, 2016b).

A small but growing literature focuses on the quality of investment advice that some financial advisers provide their clients as being a factor leading to poor investment outcomes. Mullainathan, Noeth, and Schoar (2012) find that people who initially are invested in low-fee, diversified portfolios are frequently advised to invest in higher-fee, less-well-diversified portfolios. Dvorak (2015) compares the 401(k) plan investment choices in the plans of financial advisory firms with the plans of the companies they advise. He finds that the investment options in the advisee firms' plans but not in the adviser firms' plans tend to be high-fee options. Christoffersen, Evans, and Musto (2013) find that brokers tend to sell higher-cost funds that give them higher compensation. An argument counter to these findings is that individual investors tend to underperform the stock market because of bad financial decisions, and that they could do better in avoiding their mistakes if they had the assistance of a financial adviser (Anspach, 2016).

The fundamental explanation for bad advice is the conflict of interest that many advisers have. However, several theories go further to investigate why advisers act on that conflict of interest. Akerlof and Shiller (2015), two Nobel prize laureates, in their book *Phishing for Phools: The Economics of Manipulation & Deception*, argue that many financial advisers take advantage of the behavioral biases of their clients that lead to poor decision making.

A related strand of literature relates to psychological underpinnings of bad advice. Di Tella et al., (2015) analyze instances of self-serving biases, which occur when people take actions that benefit themselves but that harm other people. In such instances, the people taking the action negatively distort their views of the other person (think badly of the other person) to make it psychologically less costly to treat them poorly. Di Tella et al.'s main hypothesis is that "people manage their self image while trying to earn money." This purposeful bias in one's views of another person reduces the psychological cost of taking an action that is favorable to oneself but harmful to the other person.

In our paper, we make a slightly different argument. We argue that some financial advisers exhibit self-serving biases in that they make true but incomplete statements to their clients because it is psychologically less costly to make those statements than it is to make false statements. Because of self-serving biases, the advisers may believe that their advice concerning improved diversification is good advice. As Di Tella et al., (2015) note, "The possibility that beliefs exhibit a self-serving bias has been studied since the development of the theory of cognitive dissonance (e.g., Festinger, 1957; Hastorf and Cantril, 1954)." Chen and Gesche (2016) in an experiment find that some people induced to provide bad advice through use of a cash incentive are likely to continue to provide that advice when the incentive is removed. They argue that the subjects continue to provide bad advice because the subjects have adopted a self-serving bias that justifies the objectively bad advice as actually being good advice.

### *4.2. Diversification*

Relating to the issue of portfolio diversification, Fama (1976) analyzes the effect on the standard deviation of a portfolio of adding an additional stock. He finds a large decline in standard deviation up to 20 stocks, but relatively little further reduction when adding further stocks. Specifically, he finds that about 95% of the reduction in standard deviation in going from a portfolio of one stock to a portfolio with more stocks is achieved with a portfolio of 20 stocks.

According to Betterment (2016), an investment adviser, "Many investors know that they should be diversified, but don't understand what that really means." Lusardi and Mitchell (2011), in a survey of older Americans, find that only half of respondents know that holding stock in a single company is riskier than investing in a mutual fund. Benartzi and Thaler (2001) present experimental evidence suggesting the tendency of investors to engage in naïve diversification, splitting their investments evenly among the available options when a small number of options are provided. This approach is called the 1/n approach. Fisch and Wilkinson-Ryan (2014) present further experimental evidence that unsophisticated investors may be attracted to naïve diversification strategies, which may explain the appeal of the advice that they can have more investment options if they roll over their employer-sponsored defined contribution plan to an IRA. For example, in their experiment, 75% of those participants who invested in a low-fee equity index fund also invested in an identical high-fee fund.

Money Magazine (2015) identifies as a myth some investors believe that investing in a large number of different mutual funds guarantees diversification, writing "Breadth of holdings alone does not guarantee diversification." That myth directly relates to the success of the advice to roll over to have access to a larger number of investment options.

For some pension participants, however, having many options may make investment decisions more difficult. Behavioral economics does not support the idea that having unlimited choice by rolling over to an IRA is a good feature. "The paradox of choice" refers to the negative effects of having too many choices. Several studies document the problems that people have in making decisions when facing a large number of options (Carosa, 2014; Iyengar and Lepper, 2000). Despite the concept from traditional economics that more options are always better, research has documented that for psychological reasons of mental overload, above a certain level, fewer choices are better for many people when the available options allow for a sufficient range of choice. Relating specifically to pension investment options, a study finds that having many investment options in 401(k) plans lowers participation rates (Iyengar, Huberman, and Jiang, 2004). In this paper, we do not explore the issue of what the optimal number of investment options is from the perspective of the participants' ability to make investment decisions, but rather what is the minimal number of investment options needed to provide adequate diversification.

Another aspect of too much choice, in the context of IRAs, is the tradeoff between quantity and quality of choice. A large number of choices that are not preselected by a financial expert with a fiduciary obligation, as is the case with IRAs, will include more options that are of poor quality, are poorly diversified, have high fees and poor rates of return (Goldreich and Halaburda, 2011).

A substantial literature demonstrates that the cognitive costs of greater choice, beyond a certain number of choices, can lead to worse savings and retirement investment choices (Ashraf, Karlan, and Yin, 2006; Choi et al., 2007, 2010; Duarte and Hastings, 2009; Hastings and Tajeda-Ashton, 2008; Madrian and Shea, 2001).

## 5. Investment options in the TSP

The Thrift Savings Plan for federal government employees, members of the military, and members of Congress uses passively managed index funds. The TSP offers a choice of 10 funds, five of which are lifecycle or target date funds, based on the participant's expected date of retirement. In the empirical analysis, we focus on individual portfolios constructed from the five basic funds. The five basic funds are: (1) the Government Securities Investment Fund (G Fund, which is based on medium-term and long-term government bond rates); (2) the Fixed Income Index Investment Fund (F Fund, which tracks the Barclays Capital U.S. Aggregate Bond Index), which includes Treasury Securities, Government-agency bonds, mortgage-backed bonds, corporate bonds and a small amount of foreign bonds traded in the United States; (3) the Common Stock Index Investment Fund (C Fund, which tracks the Standard & Poor's 500 Index); (4) the Small Capitalization Index Fund (S Fund, which tracks the Dow Jones U.S. Completion Total Stock Market Index, it represents all U.S. equities other than those in the Standard & Poor's 500 index); and (5) the International Stock Index Investment Fund (I Fund, which tracks the Morgan Stanley Capital International EAFE, which is the Europe, Australasia, and Far East Index; Thrift Savings Plan, 2015), which includes securities from more than 20 developed countries.

## 6. Empirical analysis

The TSP stock funds do not cover emerging markets, Canada, and international small capitalization stocks. They also do not include real estate, commodities, and international bond funds. Copeland (2013) finds that, in aggregate, IRA participants invest 13.8% of their assets in the category "other," which refers to investments not in stocks, bonds, or target date funds. This finding suggests that IRA participants do hold a wider range of investments, because the TSP does not have any investment that would be in that category. The TSP also does not offer actively managed funds.

The spanning test introduced by Huberman and Kandel (1987) is a well-known method for investigating whether a set of funds is adequately diversified. It uses a likelihood ratio test to examine whether additional risky assets can span the minimum–variance frontier. Seminal work by Tang et al. (2010), and Elton et al. (2006) also adopt this method for examining the adequacy and efficiency of 401(k) investment options. Kan and Zhou (2012) impose a comprehensive test for a mean-variance spanning method. However, Bessler and Wolff (2015) argue that the spanning method is limited to in-sample tests which may exaggerate the benefits of additional assets.

We adopt the Bessler and Wolff (2015) method to test whether the optimal portfolio constructed by the basic five TSP funds is fully diversified. First, we investigate whether the TSP participants can benefit from greater diversification when extra investment options are available in addition to the five existing options. Second, we investigate the closely related question of whether rolling over from the TSP with its five basic investment options results in better diversification.

### *6.1. Asset allocation strategies*

Suppose an investor can allocate her wealth among $N$ risky funds. We use four different asset allocation strategies–the mean-variance approach, the minimum-variance portfolio; the $1/N$ naïve diversification rule, and the risk parity asset allocation strategy. We impose borrowing constraints on each of the asset allocation strategies; hence, a short position is prohibited.

In the ***mean-variance*** approach (Markowitz, 1952), the investor faces a trade-off between risk and return. The investor maximizes the mean-variance utility function

$$\max_{\omega} U = \mu'\omega - \frac{\gamma}{2}\omega'\Omega\omega \quad (1)$$

where $\gamma$ measures risk aversion. Portfolio weights on risky assets are indicated by the $N-$ vector $\omega$, with $\omega = (\omega_1 \cdots \omega_N)'$. The expected excess return (relative to the risk-free rate) is denoted by the $N-$ vector $\mu$ with $\mu = (\mu_1 \cdots \mu_N)'$, and the $N \times N$ covariance matrix $\Omega$ is given by $\Omega = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_N^2 \end{pmatrix}$. The fraction of wealth not invested in risky assets, $1 - i'\omega$, is invested in the risk-free asset. If borrowing is allowed, the optimal investment in risky assets is

$$\omega_{mv} = \frac{1}{\gamma}\Omega^{-1}\mu \tag{2}$$

If borrowing is not allowed ($\omega_i > 0$), we need to solve the quadratic optimization problem numerically.

For the purpose of in-sample analysis, there is no need to analyze different asset allocation strategies because the Markowitz mean-variance strategy dominates all other strategies under the assumption that the in-sample analysis includes a perfect forecast of all asset returns. However, this assumption does not reflect reality, as this approach is limited to the condition that future performance of the return series is known in advance and input parameters are estimated without error. Therefore, we also introduce alternative asset allocation strategies that rely less on the estimation performance of input variables and analyze the out-of-sample benefit of having additional funds.

The expected asset returns are notoriously difficult to estimate from historical data (Merton, 1980). Hence, it can be beneficial to exclude return estimates and to focus solely on risk estimates. Therefore, we also introduce alternative strategies that require only risk estimates and a naïve strategy that requires no estimates.

The minimum-variance portfolio is obtained by solving the following optimization problem

$$\min_{\omega} U = \omega'\Omega\omega \tag{3}$$

subject to $\omega' l = 1$. Without the borrowing constraint, the global minimum variance portfolio is given by

$$\omega_{gmv} = \frac{\Omega^{-1}l}{l'\Omega^{-1}l}$$

The $1/N$ *naïve portfolio* denoting $\omega_{na} = 1/N$ can be attractive to some private investors (Benartzi and Thaler, 2001)). DeMiguel, Garlappi, and Uppal (2009) show that the $1/N$ naive strategy might outperform the mean-variance portfolios when the input mean and covariance parameters are estimated with errors because the $1/N$ naïve portfolio is independent of the input data, hence is estimation error neutral.

The ***risk parity*** strategy is commonly implemented by index providers, defined benefit pension funds and long-term investors (see Anderson, Bianchi, and Goldberg 2012). The idea of the risk parity strategy is to adjust the risk allocation of each fund so that each fund has the same risk level. The risk parity portfolio, denoted by $\omega_{rp}$, is given by

$$\omega_{rp}(i) = \frac{\frac{1}{\sigma_i}}{\sum_{i=1}^{N}\frac{1}{\sigma_i}}$$

where $i = 1, \ldots, N$ and $\sigma i$ is the volatility of asset $i$.

### 6.2. Estimation methods

Mean-variance portfolios are highly sensitive to estimates of means, volatilities, and correlations. Mis-specified inputs make mean-variance portfolios problematic. In this paper, we introduce three robust estimators, including factor models and the Bayesian shrinkage method, aiming at enhancing the quality of input estimators.

Define an N-variate random vector for each time period $r_t = (r_{1,t} \cdots r_{N,t})'$. We first introduce the least robust method to estimate $\mu$ and $\Omega$, namely using their historical sample analogues:

$$\mu_{sm} = \frac{1}{T}\sum_{t=1}^{T} r_t \qquad \Omega_{sm} = \frac{1}{T-N-2}\sum_{t=1}^{T}(r_t - \mu_{sm})(r_t - \mu_{sm})'.$$

where $T$ refers to the sample observations. Historical averages are very noisy estimates of the mean return. Hence, we also adopt two indexed-model based estimation approaches and a Bayesian Shrinkage method recommended by Ledoit and Wolf (2003) to estimate the input parameters.

The ***capital asset pricing model (CAPM)***, a well-known linear one-factor model that analyzes equilibrium expected returns of risky assets, is used for our analysis. The model assumes the rate of excess return of asset $i$ is given by

$$r_{i,t} = \alpha_i + \beta_i r_{M,t} + e_{i,t} \tag{4}$$

where $r_{M,t}$ is the excess market return at time $t$. The difference between the fair return and the actually expected rate of excess return on asset $i$ is captured by the constant term, $\alpha_i$. The constant coefficient term $\beta_i$ measures the contribution of asset $i$ to the variance of the market portfolio. The error term of the regression is denoted by $e_{i,t}$. Therefore, the CAPM estimates for $\mu$ and $\Omega$ are given by

$$\mu_{CAPM} = \alpha_{CAPM} + \beta_{CAPM}\,\mu_M \tag{5}$$

$$\Omega_{CAPM} = \sigma_M^2\,\beta_{CAPM}\beta_{CAPM}' + \psi_{CAPM} \tag{6}$$

where

$\alpha_{CAPM} = (\alpha_1 \cdots \alpha_N)'$ and $\beta_{CAPM} = (\beta_1 \cdots \beta_N)'$ are coefficient vectors. $\psi_{CAPM}$ is the $N \times N$ diagonal matrix with entries $(\sigma^2(e_1) \cdots \sigma^2(e_N))$. $\mu_M$ and $\sigma_M^2$ are the mean and variance of the market excess return series.

We also use the ***Fama-French three-factor model*** to estimate expected asset returns:

$$r_{it} = \alpha_i + \beta_{iM} r_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it} \tag{7}$$

where

*SMB* = Small Minus Big is the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of large stocks.

*HML* = High Minus Low is the return of a portfolio of stocks with a high book-to-market ratio in excess of the return of portfolio of stocks with a low book-to-market ratio.

Under the Fama-French multifactor model, the expected excess return and covariance matrix of the assets are

$$\mu_{FF3} = \alpha_{FF3} + \beta_{FF3}\,\mu_f$$

$$\Omega_{FF3} = \beta_{FF3}\Omega_f\beta_{FF3}{}' + \psi_{FF3}$$

where $\alpha_{FF3} = (\alpha_1 \cdots \alpha_N)'$ and $\beta_{FF3} = (\beta_M, \beta_{SMB}, \beta_{HML})$ is a $N \times 3$ matrix with $\beta_f = (\beta_{1F} \cdots \beta_{NF})'$ for $f = M, SMB$ and $HML$, respectively. $\mu_f = (\mu_M, \mu_{SMB}, \mu_{HML})'$ and $\Omega_f = E[(F_t - \mu_f)(F_t - \mu_f)']$ are the mean and covariance matrix of the three factors, with $F_t = (r_{Mt}, SMB_t, HML_t)'$. $\psi_{FF3}$ is the $N \times N$ diagonal matrix with entities equal to the variance of the regression residual for each asset $i$.

For both the CAPM and Fama-French models, we use least-squares estimates $\hat{\alpha}_i$, $\hat{\beta}_i$ for the time-series regression for each asset $i$. We use moment methods to estimate the remaining parameters.

Investors can also improve the estimates of inputs by using robust statistical estimators. The ***Bayesian shrinkage*** approach by Stein (1956) and James and Stein (1961) is one of the most well received robust estimators. The shrinking estimators take care of outliers and extreme values that may jeopardize estimation performance. We follow Bessler, Opfer, and Wolff (2017) to formulate the Bayesian shrinkage methods. The idea behind the Bayesian shrinkage method is to shrink the sample mean $\mu_{sm}$ towards the expected return of the minimum variance portfolio $\mu_{min}$ with:

$$\mu_{min} = \omega'_{gmv}\mu_{sm} = \frac{\Omega^{-1}l}{l'\Omega^{-1}l}\,\mu_{sm}$$

The return estimates based on the Bayesian shrinkage approach take the form

$$\mu_{BS} = (1 - v)\mu_{sm} + v\mu_{min}l$$

with $v = \dfrac{N+2}{(N+2) + T(\mu_{sm} - \mu_{min}l)'\Omega^{-1}(\mu_{sm} - \mu_{min}l)}$, where $N$ is the number of assets and $T$ is the sample size. We set Bayesian shrinkage estimates of the covariance matrix to be the same as sample moment methods. With $\Omega_{BS} = \Omega_{sm}$.

### 6.3. Performance measures

To measure the out-of-sample performance of optimal portfolios under various estimation methods, we compute the portfolio's net return $\omega'\mu$, and volatility $\sqrt{\omega'\Omega\omega}$ as well as the net Sharpe ratio $\dfrac{\omega'\mu}{\sqrt{\omega'\Omega\omega}}$ in excess of transaction cost for each asset allocation strategy.

### *6.4. Data*

TSP funds track the performance of various stock and bond indices, so we use those indices to run the diversification analysis. We collect monthly returns from January 1993 to April 2015. We use the Barclays Capital U.S. Aggregate Bond Index for the F Fund; Standard and Poor's (S&P) 500 Stock Index for the C Fund; the Dow Jones U.S. Completion Total Stock Market Index for the S Fund; and the MSCI EAFE Stock Index for the I Fund. Because the G Fund has different risk and return characteristics from publicly available U.S. government bonds, we use monthly-rate-of-return data provided to us by the Thrift Savings Board. We use three-month U.S Treasury discount bond yields as a proxy for the risk-free rate.

We consider four additional investment options. First, we include a real estate fund. We use the FTSE NAREIT U.S. Real Estate Index Series (REIT). Previous studies, such as Burns and Epley (1982), Ennis and Burik (1991), and Giliberto (1993), use REIT data to show that investing in real estate funds improves diversification for U.S investors. Second, we add an emerging market fund. Li, Sarkar, and Wang (2003) find substantial international diversification benefit for U.S equity investors. The data we use are from the MSCI Emerging Markets Index. Third, we consider the commodity market. Daskalaki and Skiadopoulos (2011) show that only nonmean-variance investors can benefit from commodity investment, and this result only holds in sample. We use the S&P Goldman Sachs Commodity Index to calculate the return from investing in the commodity market. Fourth, we add an international bonds fund. We use the Citi Non-USD Non-GBP world government bond index as a proxy.

Table 1 shows summary statistics for the data. The upper panel presents the sample moments of the five TSP funds. The G Fund is almost risk free while providing an average annual return of 4.525%. The 10-year government bonds are roughly comparable with the G fund in terms of average return and volatility. Other TSP funds and additional funds all have higher annualized returns compared with the G fund and also higher levels of risk. The average return of REITs during the sample period is 9.926%, which is comparable with the expected stock index return (C Fund). The Sharpe ratios of all the rest of the additional funds are lower than the Sharpe ratios of most of the TSP funds, suggesting that the additional funds are not attractive as a stand-alone investment. The Jarque-Bera statistic of most funds is significant at the 5%-level besides the fixed income indices, rejecting the assumption of normal distribution of returns for all funds except bonds.

Even if the additional funds do not appear to be attractive in terms of stand-alone investments, they may still improve the risk-return profile if the correlations with the TSP funds are low or negative. To gain insights in terms of potential diversification benefit, we present the pair-wise correlation matrix in Table 2. We find low but significantly positive correlation between the international bond index and most of the TSP funds. There is also a low but significant correlation between the real estate index and the F fund. The emerging market index is highly correlated with most of the TSP funds. Based on our correlation analysis, an international bond index fund might be able to bring additional diversification benefit to the TSP portfolio.

Table 1 Descriptive statistics of asset returns (January 1993 to April 2015)

| | Mean (%) | Standard deviation (%) | Skewness | Kurtosis | Sharpe | VaR (99%) | JB (*p*-value %) | Observations |
|---|---|---|---|---|---|---|---|---|
| TSP G fund | 4.525 | 0.504 | −0.190 | 1.940 | 0.000 | 0.643 | 0.084 | 268 |
| TSP F fund | 5.686 | 3.595 | −0.238 | 3.966 | 0.821 | 2.881 | 0.154 | 268 |
| TSP C fund | 8.160 | 14.579 | −0.711 | 4.300 | 0.372 | 8.966 | 0.000 | 268 |
| TSP S fund | 10.090 | 18.567 | −0.634 | 4.634 | 0.396 | 13.752 | 0.000 | 268 |
| TSP I fund | 5.684 | 16.462 | −0.659 | 4.351 | 0.179 | 9.816 | 0.000 | 268 |
| 10-year bond | 4.508 | 0.449 | −0.057 | 2.167 | 0.000 | 0.640 | 1.925 | 268 |
| Real estate | 9.926 | 19.174 | −1.649 | 13.110 | 0.375 | 11.851 | 0.000 | 268 |
| Emerging | 8.116 | 23.118 | −0.710 | 5.071 | 0.233 | 14.854 | 0.000 | 268 |
| Commodity | 6.201 | 21.224 | −0.269 | 4.603 | 0.163 | 14.943 | 0.000 | 268 |
| Intern. bond | 5.203 | 8.300 | 0.223 | 3.897 | 0.297 | 6.510 | 0.368 | 268 |
| 3-month T-bill | 2.734 | 0.633 | 0.025 | 1.387 | 0.000 | 0.520 | 0.000 | 268 |

This table provides sample moments, Sharpe ratios, Value-at-risk, and Jarque-Bera statistics of the five TSP-fund indices, the 10-year government bond, the four additional fund indices and the risk-free rate used in the empirical analysis. The evaluation period covers 268 months from January 1993 to April 2005. Mean and standard deviation represent annualized time-series mean and annualized standard deviation of monthly returns. Skewness and Kurtosis denote the third and the fourth moment of the return distribution. Sharpe represents the annualized Sharpe ratios of the respective asset classes. We treat G fund index and 10-year government bond as riskless assets; therefore, their Sharpe ratios are zeros. VaR (99%) shows the nonparametric 99% value-at-risk of the monthly returns during the sample period. JB (*p*-value) is the *p*-value of the Jarque-Bera statistics for testing normality of sample returns.

VaR = value at risk; JB = Jarque-Bera; TSP = thrift savings plan; Intern. = international.

### *6.5. Analysis*

We start the empirical analysis by examining the in-sample benefit of adding extra funds to the TSP portfolio. Table 3a and 3b reports the optimal weight on each risky asset class for the mean-variance portfolio. As a benchmark, we set the risk aversion level at $\gamma = 5$. Both

Table 2 Correlation matrix of asset returns (January 1993 to April 2015)

| Correlation | TSP funds | | | | | Additional funds | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | G fund | F fund | C fund | S fund | I fund | 10 year bond | Real estate | Emerging | Commodity | Intern. bond |
| G fund | 1.000 | 0.197** | −0.017 | −0.048 | −0.031 | 0.969** | −0.027 | −0.066 | −0.009 | 0.108* |
| F fund | 0.197** | 1.000 | 0.039 | −0.017 | 0.033 | 0.163** | 0.176** | 0.002 | 0.005 | 0.210** |
| C fund | −0.017 | 0.039 | 1.000 | 0.851** | 0.801** | −0.012 | 0.556** | 0.718** | 0.255** | 0.002 |
| S fund | −0.048 | −0.017 | 0.851** | 1.000 | 0.753** | −0.045 | 0.581** | 0.737** | 0.308** | 0.010 |
| I fund | −0.031 | 0.033 | 0.801** | 0.753** | 1.000 | −0.019 | 0.527** | 0.784** | 0.380** | 0.154* |
| 10 year bond | 0.969** | 0.163** | −0.012 | −0.045 | −0.019 | 1.000 | −0.034 | −0.061 | 0.021 | 0.064 |
| Real estate | 0.001 | 0.182** | 0.563** | 0.585** | 0.532** | −0.005 | 1.000 | 0.476** | 0.178** | 0.078 |
| Emerging | −0.066 | 0.002 | 0.718** | 0.737** | 0.783** | −0.061 | 0.472** | 1.000 | 0.361** | 0.028 |
| Commodity | −0.009 | 0.005 | 0.255** | 0.308** | 0.380** | 0.021 | 0.163** | 0.361** | 1.000 | 0.074 |
| Intern. bond | 0.108* | 0.210** | 0.002 | 0.010 | 0.154* | 0.064 | 0.072 | 0.028 | 0.074 | 1.000 |

The table displays the correlation matrix for the five TSP-fund indices, the 10-year government bond and the four additional fund indices used in the empirical analysis over the period from January 1993 to April 2015.

* and ** represent the correlation values significantly different from zero at the 5% and 1% level, respectively.

Intern. = international.

Table 3a In-sample optimal portfolio without constraint (in percentage)

| Method | Sample moments | | | | | | | CAPM | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP |
| F fund | 92.75 | 92.64 | 92.84 | 91.51 | 83.81 | 82.67 | 82.67 | 79.99 | 75.47 | 77.49 | 77.97 | 71.36 | 64.92 | 64.92 |
| C fund | 6.14 | 6.11 | 6.13 | 7.01 | 7.57 | 8.31 | 8.31 | 8.81 | 8.09 | 8.46 | 8.57 | 7.83 | 6.87 | 6.87 |
| S fund | 11.92 | 11.82 | 11.71 | 11.22 | 11.53 | 10.50 | 10.50 | 7.43 | 6.77 | 7.12 | 7.23 | 6.60 | 5.73 | 5.73 |
| I fund | −10.82 | −10.90 | −11.21 | −11.98 | −12.36 | −14.05 | −14.05 | 3.77 | 3.39 | 3.60 | 3.66 | 3.34 | 2.85 | 2.85 |
| Real | | 0.32 | | | | 0.32 | 0.32 | | 6.28 | | | | 5.28 | 5.28 |
| Emer. | | | 0.53 | | | 0.71 | 0.71 | | | 3.33 | | | 2.66 | 2.66 |
| Com. | | | | 2.24 | | 2.08 | 2.08 | | | | 2.57 | | 2.06 | 2.06 |
| Int. B | | | | | 9.46 | 9.46 | 9.46 | | | | | 10.87 | 9.63 | 9.63 |
| Method | Fama-French 3 factors | | | | | | | Bayesian shrinkage | | | | | | |
| | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP |
| F fund | 80.05 | 75.43 | 77.62 | 77.99 | 71.31 | 64.94 | 64.94 | 92.63 | 92.52 | 92.80 | 91.47 | 83.76 | 82.85 | 82.85 |
| C fund | 8.78 | 8.11 | 8.42 | 8.55 | 7.82 | 6.91 | 6.91 | 6.35 | 6.23 | 6.18 | 7.07 | 7.63 | 8.12 | 8.12 |
| S fund | 7.41 | 6.76 | 7.09 | 7.21 | 6.58 | 5.72 | 5.72 | 12.38 | 12.07 | 11.83 | 11.34 | 11.67 | 10.08 | 10.08 |
| I fund | 3.76 | 3.38 | 3.58 | 3.65 | 3.33 | 2.84 | 2.84 | −11.35 | −11.25 | −11.37 | −12.12 | −12.53 | −13.32 | −13.32 |
| Real | | 6.33 | | | | 5.28 | 5.28 | | 0.44 | | | | 0.10 | 0.10 |
| Emer. | | | 3.28 | | | 2.69 | 2.69 | | | 0.55 | | | 0.64 | 0.64 |
| Com. | | | | 2.60 | | 2.03 | 2.03 | | | | 2.24 | | 2.06 | 2.06 |
| Int. B | | | | | 10.96 | 9.60 | 9.60 | | | | | 9.47 | 9.47 | 9.47 |

This table reports the portfolio weights (%) of risky assets for the in-sample optimized mean-variance portfolio. Without borrowing or short constraint, portfolio weights can be beyond 1 or be negative. Input parameters, mean and covariance matrix, are estimated using four different methods. The optimal portfolio is net of administration cost that is set at 3 *bp* for the TSP participants and 60 *bp* for the non-TSP participants. Risk aversion level is set at $\gamma = 5$.

TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int.B. = international bond.

Table 3b In-sample optimal portfolio with constraint (in percentage)

| Method | Sample moments | | | | | | | CAPM | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP |
| F fund | 71.62 | 67.03 | 71.62 | 71.62 | 71.62 | 67.02 | 67.02 | 45.91 | 27.65 | 38.03 | 43.09 | 45.91 | 18.53 | 18.53 |
| C fund | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 25.57 | 23.80 | 24.93 | 25.38 | 25.57 | 23.09 | 23.09 |
| S fund | 28.37 | 21.97 | 28.38 | 28.38 | 28.38 | 21.97 | 21.97 | 26.97 | 25.54 | 26.48 | 26.84 | 26.97 | 25.02 | 25.02 |
| I fund | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.56 | 0.05 | 1.02 | 1.40 | 1.56 | 0.00 | 0.00 |
| Real | | 11.01 | | | | 11.01 | 11.01 | | 22.97 | | | | 22.39 | 22.39 |
| Emer. | | | 0.00 | | | 0.00 | 0.00 | | | 9.54 | | | 8.64 | 8.64 |
| Com. | | | | 0.00 | | 0.00 | 0.00 | | | | 3.29 | | 2.33 | 2.33 |
| Int. B | | | | | 0.00 | 0.00 | 0.00 | | | | | 0.00 | 0.00 | 0.00 |
| Method | Fama-French 3 factors | | | | | | | Bayesian shrinkage | | | | | | |
| | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP | TSP | +Real | +Emer | +Com | +Int. B | +All | Non-TSP |
| F fund | 46.13 | 27.49 | 38.50 | 43.19 | 46.13 | 18.43 | 18.43 | 73.71 | 70.14 | 74.38 | 74.40 | 74.41 | 72.34 | 72.34 |
| C fund | 25.46 | 23.86 | 24.76 | 25.29 | 25.46 | 23.10 | 23.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| S fund | 26.91 | 25.52 | 26.38 | 26.78 | 26.91 | 24.95 | 24.95 | 26.29 | 20.32 | 25.62 | 25.60 | 25.60 | 19.15 | 19.15 |
| I fund | 1.51 | 0.00 | 0.97 | 1.35 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Real | | 23.13 | | | | 22.62 | 22.62 | | 9.54 | | | | 8.50 | 8.50 |
| Emer. | | | 9.39 | | | 8.65 | 8.65 | | | 0.00 | | | 0.00 | 0.00 |
| Com. | | | | 3.40 | | 2.25 | 2.25 | | | | 0.00 | | 0.00 | 0.00 |
| Int. B | | | | | 0.00 | 0.00 | 0.00 | | | | | 0.00 | 0.00 | 0.00 |

This table reports the portfolio weights (%) of risky assets for the in-sample optimized mean-variance portfolio with constraint on borrowing or short positions. Input parameters are estimated using four different methods. The optimal portfolio is net of administration cost that is set at 3 *bp* for the TSP participants and 60 *bp* for the non-TSP participants. As a benchmark, we set risk aversion level at $\gamma = 5$.

TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int.B. = international bond.

tables contain four panels representing four different estimates of parameter inputs. The first column of each panel reports the portfolio weight of the four TSP risky funds. Table 3a does not impose a borrowing constraint, while Table 3b does. For both cases, the F-fund dominates the optimal portfolio because of its high Sharpe ratio (see Table 1) compared with the other funds. We do not include the G-fund in our diversification analysis since it is risk free. The next four columns report the optimal weight after introducing one more instrument to the TSP funds, while keeping the low administration fee of 3 bps. The sixth column of each panel gives the optimal portfolio assuming all additional funds are included the TSP plan. The same market instruments are included in the last column while based on the non-TSP members cost which is 20 times more expensive. The cost of investments does not influence the mean-variance optimal portfolio; hence the last two columns are always the same.

The optimal result demonstrates the attractiveness of the F-fund under various estimation methods. The CAPM and Fama-French models make the real estate fund a valuable additional instrument with more than 20% portfolio weight for the constraint case. The international bond is also an attractive investment vehicle with around a 10% portfolio weight if borrowing is allowed. The results also demonstrate how sensitive the portfolio weights are to the input estimates and the consideration of the borrowing constraint.

The in-sample performance is presented in Table 4. In general, TSP risky funds are sufficiently diversified, with limited room for further diversification. Table 4 shows that adding more investment options to the TSP benchmark portfolio may only bring a small while significant improvement to the optimal mean-variance portfolio under CAPM or the Fama-French approach when borrowing is permitted. However, for non-TSP investors, the effect of adding more funds is not appealing because of the high administration cost. In other words, investors can only benefit from rolling over from the TSP while accessing alternative instruments if a substantial result can be achieved to cover the high administration cost. However, with the borrowing constraint, the possibility of having substantially high returns is fairly low.

The in-sample test assumes a perfect forecast of all asset returns. This assumption does not reflect reality as it is limited to the condition that future performance of the return series is known in advance. Therefore, we also analyze the out-of-sample benefit of having additional funds. We use the rolling estimation window method used by DeMiguel, Garlappi, and Uppal (2009) to compare the performance of various asset allocation strategies. It is not necessary to consider other asset allocation strategies in the in-sample analysis since the Markowitz (1952) mean-variance strategy dominates any alternative strategies if investors only care about portfolio risk and return. The alternative asset allocation strategies we consider include the global minimum-variance strategy, risk parity strategy, and 1/*N* naïve strategy.

Table 5a through 5d provide the average portfolio weight of each risky asset as well as the respective standard deviation for the various optimization strategies and for the different estimation methods. The optimal portfolio for non-TSP participants is displayed in the last column of each panel in Table 5, in which all the eight risky assets are included. The standard deviation of the optimal mean-variance portfolio (Table 5a and 5b) is much larger than other asset allocation strategies, indicating a strong fluctuation of portfolio shares over time, and

Table 4 In sample performance of optimal mean-variance portfolio

| Estimation method | Sample moments | | | CAPM | | | Fama-French 3 factor | | | Bayesian shrinkage | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe |
| Mean-variance asset allocation strategy without constraint | | | | | | | | | | | | |
| TSP | 20.91 | 19.08 | 0.95 | 23.18 | 20.24 | 1.01 | 23.16 | 20.23 | 1.01 | 17.22 | 17.04 | 0.85 |
| +Real estate | 20.92 | 19.09 | 0.95 | 26.03 | 21.60 | 1.08* | 26.06 | 21.61 | 1.08* | 17.10 | 16.97 | 0.85 |
| +Emerging | 20.92 | 19.09 | 0.95 | 24.27 | 20.77 | 1.04 | 24.22 | 20.74 | 1.04 | 17.10 | 16.97 | 0.85 |
| +Commodity | 21.16 | 19.21 | 0.96 | 23.72 | 20.50 | 1.02 | 23.71 | 20.50 | 1.02 | 17.26 | 17.06 | 0.85 |
| +Intern. bond | 21.68 | 19.48 | 0.97 | 24.95 | 21.09 | 1.05 | 24.96 | 21.10 | 1.05 | 17.58 | 17.25 | 0.86 |
| +All | 21.93 | 19.61 | 0.98* | 29.31 | 23.07 | 1.15** | 29.32 | 23.07 | 1.15** | 17.39 | 17.14 | 0.86 |
| Non-TSP | 21.36 | 19.61 | 0.95 | 28.74 | 23.07 | 1.13** | 28.75 | 23.07 | 1.13** | 16.82 | 17.14 | 0.82 |
| Mean-variance asset allocation strategy with constraint | | | | | | | | | | | | |
| TSP | 6.91 | 5.83 | 0.72 | 7.48 | 6.47 | 0.73 | 7.47 | 6.46 | 0.73 | 6.76 | 5.86 | 0.69 |
| +Real estate | 7.09 | 6.20 | 0.70 | 8.34 | 7.51 | 0.75 | 8.35 | 7.52 | 0.75 | 6.86 | 6.12 | 0.68 |
| +Emerging | 6.91 | 5.83 | 0.72 | 7.67 | 6.68 | 0.74 | 7.66 | 6.66 | 0.74 | 6.70 | 5.76 | 0.69 |
| +Commodity | 6.91 | 5.83 | 0.72 | 7.48 | 6.45 | 0.74 | 7.48 | 6.44 | 0.74 | 6.70 | 5.76 | 0.69 |
| +Intern. bond | 6.91 | 5.83 | 0.72 | 7.48 | 6.47 | 0.73 | 7.47 | 6.46 | 0.73 | 6.70 | 5.76 | 0.69 |
| +All | 7.09 | 6.20 | 0.70 | 8.50 | 7.64 | 0.76 | 8.51 | 7.64 | 0.76* | 6.69 | 5.83 | 0.68 |
| Non-TSP | 6.52 | 6.20 | 0.61 | 7.93 | 7.64 | 0.68 | 7.94 | 7.64 | 0.68 | 6.76 | 5.86 | 0.69 |

This table displays the in-sample optimal mean-variance portfolio performance net of administering cost for the full sample from January 1993 to April 2015. The upper panel presents the results without borrowing constraint while the lower panel restricts short positions. The first row of each panel is the benchmark TSP 5-fund portfolio and the next five rows show the TSP portfolios complemented with additional funds. The administrative expense for the TSP plan participants is 0.03% per year, which is applied to the first six rows. The last row shows the performance for the non-TSP portfolio with annual cost 20 times more expensive than the TSP participants. Mean denotes the annualized monthly (in percentage) returns. Standard deviation represents the associated annualized standard deviation of portfolio returns. Sharpe is the annualized Sharpe ratio. The mean and the covariance-matrix are estimated using four different methods.

* and ** indicate the significant higher values of Sharpe ratio in comparison to the value of the benchmark TSP portfolio at the 10% and 5% level, respectively.

TSP = thrift savings plan; Intern. = international.

Table 5a Out-of-sample optimal portfolio weights (in percentage) of mean-variance portfolio without constraint

| Method | Sample moments | | | | | | CAPM | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Mean-variance asset allocation strategy without borrowing constraint | | | | | | | | | | | | |
| F fund | 99.36 | 93.59 | 102.34 | 94.77 | 91.04 | 82.68 | 89.98 | 82.53 | 86.41 | 85.01 | 77.81 | 67.36 |
| | (7.59) | (12.14) | (11.71) | (7.07) | (8.04) | (9.74) | (12.30) | (14.34) | (10.78) | (10.29) | (7.72) | (7.92) |
| C fund | −13.06 | −11.91 | −10.36 | −6.64 | −10.08 | −2.38 | 3.01 | 2.31 | 2.88 | 2.83 | 2.83 | 2.01 |
| | (24.23) | (22.90) | (23.45) | (20.35) | (22.48) | (17.31) | (7.32) | (6.75) | (7.05) | (6.85) | (6.20) | (5.36) |
| S fund | 23.50 | 22.55 | 18.81 | 20.00 | 22.00 | 14.91 | 5.60 | 4.80 | 5.18 | 5.12 | 4.84 | 3.67 |
| | (15.58) | (18.09) | (13.28) | (13.72) | (13.80) | (13.44) | (2.41) | (2.28) | (2.40) | (2.37) | (2.15) | (2.01) |
| I fund | −9.80 | −9.55 | −25.08 | −14.40 | −11.40 | −24.72 | 1.41 | 1.03 | 1.15 | 1.20 | 1.24 | 0.65 |
| | (11.50) | (10.72) | (16.48) | (10.19) | (13.08) | (21.24) | (4.05) | (3.73) | (3.83) | (3.81) | (3.46) | (2.95) |
| Real | | 5.32 | | | | 6.82 | | 9.33 | | | | 7.66 |
| | | (11.63) | | | | (12.07) | | (6.08) | | | | (5.99) |
| Emer. | | | 14.30 | | | 9.11 | | | 4.38 | | | 3.16 |
| | | | (19.32) | | | (18.36) | | | (3.42) | | | (2.45) |
| Com. | | | | 6.27 | | 5.28 | | | | 5.84 | | 4.31 |
| | | | | (3.21) | | (3.02) | | | | (3.23) | | (2.27) |
| Int. B | | | | | 8.44 | 8.29 | | | | | 13.28 | 11.19 |
| | | | | | (9.87) | (13.13) | | | | | (5.97) | (5.08) |

*(continued on next page)*

This table reports the average portfolio weights (%) of risky assets in the out-of-sample optimized mean-variance portfolio without borrowing constraint. The input variables are estimated using sample moment method and CAPM method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parenthesis denote the associated standard deviations of the optimal weights over the testing period. Remark: the optimal portfolio weight is not affected by the administration cost.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5a (Continued)

| Method | Fama-French 3 factor | | | | | | Bayesian shrinkage | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Mean-variance asset allocation strategy without borrowing constraint | | | | | | | | | | | | |
| F fund | 90.50 | 83.01 | 87.25 | 85.50 | 78.51 | 68.09 | 100.00 | 93.55 | 104.26 | 95.15 | 90.71 | 81.99 |
| | (12.06) | (14.52) | (10.55) | (10.22) | (7.35) | (8.33) | (8.49) | (13.47) | (15.34) | (7.65) | (8.78) | (12.18) |
| C fund | 2.75 | 2.15 | 2.52 | 2.62 | 2.54 | 1.77 | −14.05 | −12.92 | −12.92 | −7.40 | −10.38 | −4.02 |
| | (7.29) | (6.76) | (7.13) | (6.83) | (6.18) | (5.48) | (27.83) | (26.58) | (29.44) | (23.19) | (25.69) | (22.35) |
| S fund | 5.50 | 4.72 | 5.00 | 4.99 | 4.67 | 3.47 | 25.11 | 24.01 | 20.61 | 21.30 | 23.08 | 16.09 |
| | (2.32) | (2.19) | (2.37) | (2.33) | (2.10) | (2.06) | (16.72) | (18.81) | (14.35) | (14.59) | (14.81) | (13.67) |
| I fund | 1.25 | 0.91 | 0.93 | 1.04 | 1.06 | 0.48 | −11.07 | −11.07 | −29.07 | −15.64 | −12.76 | −29.35 |
| | (3.97) | (3.65) | (3.78) | (3.76) | (3.39) | (2.94) | (13.92) | (12.97) | (21.18) | (12.16) | (15.58) | (26.75) |
| Real | | 9.22 | | | | 7.47 | | 6.43 | | | | 8.37 |
| | | (6.13) | | | | (6.08) | | (12.63) | | | | (13.17) |
| Emer. | | | 4.30 | | | 3.14 | | | 17.13 | | | 11.53 |
| | | | (3.44) | | | (2.42) | | | (25.04) | | | (23.73) |
| Com. | | | | 5.86 | | 4.32 | | | | 6.59 | | 5.84 |
| | | | | (3.19) | | (2.24) | | | | (3.68) | | (3.72) |
| Int. B | | | | | 13.21 | 11.26 | | | | | 9.36 | 9.55 |
| | | | | | (6.15) | (5.28) | | | | | (10.21) | (14.69) |

This table continues Table 5a reporting the average portfolio weights (%) of risky assets in the out-of-sample optimized mean-variance portfolio without borrowing constraint. The input variables are estimated using the Fama-French 3-factor model and the Bayesian shrinkage method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parentheses denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5b Out-of-sample optimal portfolio weights of mean-variance portfolio with constraint

| Method | Sample moments | | | | | | CAPM | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Mean-variance asset allocation strategy with borrowing and no-short constraint | | | | | | | | | | | | |
| F fund | 79.26 | 58.31 | 72.64 | 65.38 | 60.29 | 27.54 | 67.93 | 41.12 | 52.96 | 49.37 | 49.35 | 11.47 |
| | (12.23) | (26.08) | (8.27) | (10.31) | (24.63) | (24.95) | (21.13) | (33.21) | (21.05) | (18.73) | (20.45) | (18.08) |
| C fund | 3.44 | 2.11 | 3.44 | 3.89 | 3.44 | 2.13 | 11.12 | 8.84 | 10.52 | 10.41 | 10.91 | 6.62 |
| | (7.13) | (4.75) | (7.13) | (7.37) | (7.14) | (4.75) | (10.41) | (8.58) | (10.30) | (10.22) | (10.42) | (8.22) |
| S fund | 17.01 | 10.37 | 10.00 | 11.38 | 16.30 | 4.22 | 17.98 | 15.94 | 16.82 | 16.73 | 17.14 | 12.13 |
| | (12.91) | (14.60) | (11.89) | (10.99) | (12.76) | (9.71) | (11.49) | (11.07) | (11.12) | (11.05) | (11.48) | (10.56) |
| I fund | 0.29 | 0.26 | 0.00 | 0.17 | 0.28 | 0.00 | 2.98 | 2.25 | 2.44 | 2.50 | 2.77 | 0.88 |
| | (2.05) | (1.80) | (0.00) | (1.20) | (1.96) | (0.00) | (5.24) | (4.38) | (4.51) | (4.70) | (5.05) | (2.02) |
| Real | | 28.95 | | | | 27.03 | | 31.85 | | | | 27.08 |
| | | (29.89) | | | | (31.32) | | (24.37) | | | | (24.18) |
| Emer. | | | 13.92 | | | 7.18 | | | 17.26 | | | 14.36 |
| | | | (14.55) | | | (9.46) | | | (12.88) | | | (11.42) |
| Com. | | | | 19.18 | | 17.09 | | | | 20.99 | | 17.64 |
| | | | | (13.35) | | (11.88) | | | | (13.02) | | (11.62) |
| Int. B | | | | | 19.69 | 14.82 | | | | | 19.83 | 9.82 |
| | | | | | (25.89) | (20.96) | | | | | (24.40) | (14.89) |

*(continued on next page)*

This table shows the average portfolio weights (%) of risky assets in the out-of-sample optimized mean-variance portfolio with borrowing and short-position constraint. The inputs are based on the sample moments and the CAPM approach. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parentheses denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5b (Continued)

| Method | Fama-French 3 factor | | | | | | Bayesian shrinkage | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Mean-variance asset allocation strategy with borrowing and no-short constraint | | | | | | | | | | | | |
| F fund | 68.94 | 42.26 | 54.77 | 50.31 | 50.85 | 12.53 | 80.89 | 62.16 | 74.95 | 68.78 | 63.57 | 33.75 |
| | (20.17) | (33.34) | (19.51) | (17.95) | (19.35) | (18.40) | (11.47) | (23.37) | (8.18) | (9.20) | (23.74) | (23.39) |
| C fund | 10.54 | 8.41 | 9.84 | 9.95 | 10.36 | 6.38 | 3.25 | 1.99 | 3.23 | 3.64 | 3.21 | 2.02 |
| | (10.17) | (8.42) | (10.09) | (10.09) | (10.19) | (8.13) | (6.70) | (4.47) | (6.65) | (6.80) | (6.62) | (4.38) |
| S fund | 17.77 | 15.69 | 16.22 | 16.39 | 16.83 | 11.83 | 15.58 | 9.76 | 8.80 | 10.19 | 14.53 | 3.85 |
| | (11.33) | (10.78) | (10.94) | (10.92) | (11.35) | (10.35) | (12.03) | (13.48) | (10.61) | (9.88) | (11.60) | (8.73) |
| I fund | 2.75 | 2.06 | 2.18 | 2.32 | 2.56 | 0.78 | 0.28 | 0.24 | 0.00 | 0.17 | 0.28 | 0.00 |
| | (5.01) | (4.16) | (4.24) | (4.53) | (4.83) | (1.97) | (1.88) | (1.69) | (0.00) | (1.20) | (1.77) | (0.00) |
| Real | | 31.58 | | | | 26.59 | | 25.85 | | | | 23.67 |
| | | (24.67) | | | | (24.52) | | (26.70) | | | | (27.37) |
| Emer. | | | 16.99 | | | 14.23 | | | 13.02 | | | 6.58 |
| | | | (12.80) | | | (11.34) | | | (13.63) | | | (8.66) |
| Com. | | | | 21.03 | | 17.52 | | | | 17.21 | | 15.46 |
| | | | | (12.95) | | (11.64) | | | | (12.12) | | (10.66) |
| Int. B | | | | | 19.41 | 10.14 | | | | | 18.40 | 14.68 |
| | | | | | (24.44) | (15.41) | | | | | (24.31) | (20.69) |

This table shows the average portfolio weights (%) of risky assets in the out-of-sample optimized mean-variance portfolio with borrowing and short-position constraint. The inputs are based on the Fama-French 3-factor model and the Bayesian shrinkage method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parentheses denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5c Out-of-sample optimal portfolio weights of global minimum variance portfolio

| Method | Sample moment | | | | | | CAPM | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int.B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Minimum-variance asset allocation strategy | | | | | | | | | | | | |
| F fund | 93.23 | 92.43 | 93.27 | 91.90 | 89.05 | 86.46 | 87.53 | 84.44 | 85.96 | 85.78 | 76.18 | 71.83 |
| | (1.82) | (2.44) | (1.62) | (1.93) | (3.64) | (3.76) | (0.95) | (3.03) | (1.13) | (1.44) | (1.44) | (3.68) |
| C fund | 7.09 | 7.31 | 6.20 | 8.60 | 7.52 | 7.86 | 4.94 | 4.66 | 4.79 | 4.78 | 4.23 | 3.82 |
| | (6.27) | (6.31) | (6.12) | (6.96) | (6.88) | (7.01) | (0.22) | (0.19) | (0.20) | (0.21) | (0.23) | (0.18) |
| S fund | 1.05 | 3.89 | 1.40 | 0.30 | 0.89 | 4.06 | 3.08 | 2.86 | 2.96 | 2.97 | 2.62 | 2.29 |
| | (2.01) | (5.61) | (3.05) | (1.69) | (1.90) | (6.74) | (0.27) | (0.25) | (0.25) | (0.22) | (0.15) | (0.12) |
| I fund | −1.37 | −1.16 | −0.24 | −2.99 | −1.82 | −0.67 | 4.45 | 4.14 | 4.29 | 4.32 | 3.85 | 3.43 |
| | (8.33) | (7.99) | (6.27) | (9.31) | (9.09) | (7.01) | (0.98) | (0.85) | (0.96) | (1.01) | (0.97) | (0.86) |
| Real | | −2.47 | | | | −2.51 | | 3.90 | | | | 3.20 |
| | | (5.06) | | | | (5.11) | | (2.42) | | | | (2.22) |
| Emer. | | | −0.63 | | | −1.99 | | | 2.00 | | | 1.58 |
| | | | (2.63) | | | (3.49) | | | (0.18) | | | (0.18) |
| Com. | | | | 2.19 | | 2.09 | | | | 2.15 | | 1.66 |
| | | | | (0.76) | | (0.68) | | | | (0.42) | | (0.40) |
| Int. B | | | | | 4.35 | 4.70 | | | | | 13.13 | 12.19 |
| | | | | | (5.56) | (5.99) | | | | | (0.59) | (0.52) |

*(continued on next page)*

This table reports the average portfolio weights (%) of risky assets in the out-of-sample global minimum variance portfolio. The input variables are based on the sample moment method and the CAPM method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parenthesis denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5c (Continued)

| Method | Fama-French 3 factor | | | | | | Bayesian shrinkage | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Minimum-variance asset allocation strategy | | | | | | | | | | | | |
| F fund | 88.04 | 85.01 | 86.70 | 86.30 | 76.83 | 72.63 | 92.63 | 92.36 | 92.49 | 91.58 | 88.79 | 86.78 |
| | (1.47) | (3.69) | (1.75) | (2.11) | (2.00) | (4.62) | (1.69) | (1.72) | (1.68) | (1.68) | (3.96) | (4.59) |
| C fund | 4.77 | 4.52 | 4.59 | 4.64 | 3.99 | 3.63 | 9.32 | 9.53 | 7.77 | 10.56 | 9.70 | 9.15 |
| | (0.33) | (0.32) | (0.32) | (0.37) | (0.43) | (0.42) | (9.34) | (9.32) | (8.60) | (9.94) | (10.03) | (9.39) |
| S fund | 2.88 | 2.66 | 2.74 | 2.77 | 2.36 | 2.04 | −1.52 | 1.49 | −0.45 | −2.17 | −1.70 | 2.53 |
| | (0.17) | (0.15) | (0.18) | (0.20) | (0.29) | (0.32) | (1.27) | (3.73) | (2.00) | (1.32) | (1.43) | (5.60) |
| I fund | 4.31 | 4.03 | 4.15 | 4.19 | 3.69 | 3.30 | −0.42 | −0.10 | 2.16 | −1.71 | −0.77 | 1.97 |
| | (1.10) | (1.00) | (1.07) | (1.13) | (1.10) | (0.99) | (9.47) | (9.12) | (5.99) | (10.29) | (10.27) | (6.44) |
| Real | | 3.78 | | | | 3.07 | | −3.28 | | | | −3.53 |
| | | (2.42) | | | | (2.31) | | (4.37) | | | | (4.42) |
| Emer. | | | 1.82 | | | 1.44 | | | −1.96 | | | −3.14 |
| | | | (0.31) | | | (0.33) | | | (3.96) | | | (4.91) |
| Com. | | | | 2.11 | | 1.63 | | | | 1.74 | | 1.74 |
| | | | | (0.52) | | (0.50) | | | | (0.76) | | (0.75) |
| Int. B | | | | | 13.13 | 12.27 | | | | | 3.99 | 4.51 |
| | | | | | (0.62) | (0.56) | | | | | (5.58) | (5.68) |

This table continues Table 5c reporting the average portfolio weights (%) of risky assets in the out-of-sample global minimum-variance portfolio. The input variables are estimated using the Fama-French 3-factor model and the Bayesian shrinkage method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parenthesis denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5d Out-of-sample optimal portfolio weights of risk parity portfolio

| Method | Sample moment | | | | | | CAPM | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Risk parity asset allocation strategy | | | | | | | | | | | | |
| F fund | 60.96 | 54.29 | 55.81 | 55.49 | 48.46 | 38.81 | 60.93 | 54.27 | 55.78 | 55.46 | 48.50 | 38.82 |
| | (1.18) | (2.79) | (1.21) | (1.54) | (1.09) | (1.99) | (1.01) | (2.57) | (1.03) | (1.35) | (0.89) | (1.76) |
| C fund | 14.40 | 12.82 | 13.18 | 13.11 | 11.45 | 9.16 | 14.42 | 12.84 | 13.21 | 13.13 | 11.48 | 9.19 |
| | (0.25) | (0.49) | (0.21) | (0.24) | (0.19) | (0.33) | (0.25) | (0.52) | (0.21) | (0.26) | (0.20) | (0.34) |
| S fund | 11.20 | 9.98 | 10.25 | 10.20 | 8.90 | 7.13 | 11.21 | 9.99 | 10.26 | 10.20 | 8.92 | 7.14 |
| | (0.54) | (0.74) | (0.49) | (0.53) | (0.45) | (0.51) | (0.59) | (0.78) | (0.53) | (0.57) | (0.47) | (0.53) |
| I fund | 13.45 | 11.93 | 12.31 | 12.23 | 10.69 | 8.53 | 13.44 | 11.93 | 12.31 | 12.23 | 10.70 | 8.54 |
| | (1.36) | (0.85) | (1.23) | (1.15) | (1.06) | (0.61) | (1.27) | (0.77) | (1.15) | (1.07) | (1.00) | (0.58) |
| Real | | 10.98 | | | | 7.85 | | 10.97 | | | | 7.85 |
| | | (3.06) | | | | (2.19) | | (2.99) | | | | (2.16) |
| Emer. | | | 8.44 | | | 5.87 | | | 8.45 | | | 5.87 |
| | | | (0.26) | | | (0.20) | | | (0.24) | | | (0.22) |
| Com. | | | | 8.98 | | 6.26 | | | | 8.99 | | 6.28 |
| | | | | (0.86) | | (0.45) | | | | (0.81) | | (0.43) |
| Int. B | | | | | 20.50 | 16.40 | | | | | 20.40 | 16.32 |
| | | | | | (0.41) | (0.45) | | | | | (0.34) | (0.50) |

(continued on next page)

This table shows the average portfolio weights (%) of risky assets in the out-of-sample risk parity portfolio. The input variables are estimated using the sample moment and the CAPM method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parentheses denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

Table 5d (Continued)

| Method | Fama-French 3 factor | | | | | | Bayesian shrinkage | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TSP | +Real | +Emer | +Com | +Int. B | +All | TSP | +Real | +Emer | +Com | +Int. B | +All |
| Risk parity asset allocation strategy | | | | | | | | | | | | |
| F fund | 60.93 | 54.27 | 55.78 | 55.46 | 48.50 | 38.82 | 58.68 | 52.00 | 53.46 | 53.15 | 46.26 | 36.73 |
| | (1.01) | (2.57) | (1.03) | (1.35) | (0.89) | (1.76) | (1.52) | (3.05) | (1.56) | (1.90) | (1.42) | (2.25) |
| C fund | 14.42 | 12.84 | 13.21 | 13.13 | 11.48 | 9.19 | 15.22 | 13.49 | 13.87 | 13.79 | 12.00 | 9.52 |
| | (0.25) | (0.52) | (0.21) | (0.26) | (0.20) | (0.34) | (0.27) | (0.47) | (0.22) | (0.23) | (0.20) | (0.34) |
| S fund | 11.21 | 9.99 | 10.26 | 10.20 | 8.92 | 7.14 | 11.82 | 10.48 | 10.77 | 10.71 | 9.32 | 7.40 |
| | (0.59) | (0.79) | (0.53) | (0.57) | (0.47) | (0.53) | (0.50) | (0.70) | (0.45) | (0.49) | (0.42) | (0.50) |
| I fund | 13.44 | 11.94 | 12.31 | 12.23 | 10.70 | 8.54 | 14.28 | 12.61 | 13.01 | 12.92 | 11.25 | 8.90 |
| | (1.27) | (0.77) | (1.15) | (1.07) | (1.00) | (0.58) | (1.56) | (0.97) | (1.39) | (1.29) | (1.18) | (0.67) |
| Real | | 10.97 | | | | 7.85 | | 11.42 | | | | 8.06 |
| | | (2.99) | | | | (2.16) | | (3.12) | | | | (2.19) |
| Emer. | | | 8.45 | | | 5.87 | | | 8.88 | | | 6.09 |
| | | | (0.24) | | | (0.22) | | | (0.33) | | | (0.18) |
| Com. | | | | 8.99 | | 6.28 | | | | 9.43 | | 6.49 |
| | | | | (0.81) | | (0.43) | | | | (1.00) | | (0.52) |
| Int. B | | | | | 20.40 | 16.32 | | | | | 21.19 | 16.81 |
| | | | | | (0.34) | (0.50) | | | | | (0.54) | (0.39) |

This table continues Table 5d reporting the average portfolio weights (%) of risky assets in the out-of-sample risk parity portfolio. The input variables are estimated using the Fama-French 3-factor model and the Bayesian shrinkage method. The estimation window length is 120 months and the testing window has the length of 168 months. Numbers in parentheses denote the associated standard deviations of the optimal weights over the testing period.

CAPM = capital asset pricing model; TSP = thrift savings plan; Real = real estate; Emer = emerging market; Com = commodity; Int. B = international bond.

also implicitly showing the noise of estimating expected returns. The results demonstrate the attractiveness of the F-fund because it dominates all asset allocation strategies for all estimation methods. Real estate fund is also attractive under the mean-variance portfolio, but is less attractive for the minimum-variance strategy, indicating the high riskiness of the REITs. The international bond index fund is also attractive, with almost 20% of average portfolio weight, especially under the minimum variance and risk parity strategies. Both the minimum variance strategy and risk parity strategy are stable over time with much lower standard deviations. This result is because for both strategies, asset volatility is the only input parameter, and the volatility term is less volatile compared with the first moment.

In Table 6a, we present the out-of-sample performance of the optimal mean-variance portfolio under various estimation methods and for different portfolio combinations. Without the borrowing constraint, the optimal mean-variance portfolio can generate substantially high returns but also substantially high volatility, which leads to a lower Sharpe ratio compared with the constraint case. Under the mean-variance strategy, adding extra funds in most of the cases can enhance the portfolio returns by approximately 2.5%, while the risk of the portfolio increases by more than about 5%. If an investor only cares about portfolio return regardless of the underlying risk, having additional investment vehicles can indeed bring much higher expected returns even with high investment fees. However, the marginal gain from the risk-return trade-off is negligible. Therefore, unless there is a dramatic increase in the portfolio returns, non-TSP investors can benefit from investing in a larger class of assets even with much higher investment fees. However, with the borrowing constraint, a dramatic increase in portfolio returns is less likely to happen. Hence, the chance to benefit from rolling-over from TSP plan is also small.

Table 6b reports the out-of-sample performance under the minimum variance strategy, risk parity strategy and 1/N naïve strategy. In many cases, additional investment options increase the portfolio returns by less than 0.5% and can also reduce the portfolio volatility by 2–3%. The result demonstrates that there is still limited but significant room for further diversification. However, the marginal increase of portfolio returns is too small to cover the high investment expense for the non-TSP investors. As a result, almost none of the Sharpe ratios of the non-TSP scenarios beat the TSP benchmark scenario.

As a robustness check, in Table 7 we present the out-of-sample analysis in three subperiods. Only during the subperiod 2001–2008 (Table 7b), we observe significant improvement by adding extra funds under various asset allocation strategies. The out of sample performance using CAPM and Fama-French estimation approaches is more stable than other estimation methods. The mean-variance strategy (with constraint) persistently presents a significant improvement of the Sharpe ratio for TSP portfolios complemented with additional funds, especially using factor-model based estimation approaches across subsamples. Therefore, Table 7 shows that rolling over from the TSP plan may give investors an opportunity to enhance their portfolio performance in terms of diversification benefit even with much higher administration fees. However, such benefit highly relies on the asset allocation strategy, market timing, estimation approach of input variables as well as investors' risk preference. In other words, this result indicates that a larger number of investment options

Table 6a Out-of-sample portfolio benefits of additional funds using mean-variance asset allocation strategies

| Method | Sample moments | | | CAPM | | | Fama-French 3 factor | | | Bayesian shrinkage | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe |
| Mean-variance asset allocation strategy without constraint | | | | | | | | | | | | |
| TSP | 15.80 | 21.08 | 0.62 | 18.81 | 19.02 | 0.89 | 18.23 | 18.90 | 0.87 | 12.05 | 29.95 | 0.32 |
| +Real estate | $18.76^{+}$ | 22.80 | $0.70^{+}$ | $22.13^{+}$ | 21.19 | $0.97^{+}$ | $21.39^{+}$ | 21.02 | $0.94^{+}$ | $14.71^{+}$ | 30.99 | $0.40^{+}$ |
| +Emerging | $16.40^{+}$ | 23.46 | 0.58 | 17.72 | 20.31 | 0.79 | 16.99 | 20.11 | 0.77 | $13.08^{+}$ | 30.95 | $0.34^{+}$ |
| +Commodity | $16.06^{+}$ | 22.33 | 0.60 | $18.86^{+}$ | 20.60 | 0.85 | $18.36^{+}$ | 20.46 | 0.83 | 11.90 | 30.74 | 0.30 |
| +Intern. bond | 15.70 | 21.72 | 0.60 | $20.45^{+}$ | 20.55 | $0.92^{+}$ | $19.78^{+}$ | 20.40 | $0.90^{+}$ | 11.06 | 30.42 | 0.27 |
| +All | $20.54^{+}$ | 26.68 | $0.71^{+}$ | $22.35^{+}$ | 24.79 | $0.92^{+}$ | $21.55^{+}$ | 24.47 | $0.90^{+}$ | $15.32^{+}$ | 33.04 | $0.39^{+}$ |
| Non-TSP | $19.97^{+}$ | 26.68 | $0.69^{+}$ | $21.78^{+}$ | 24.79 | 0.89 | $20.98^{+}$ | 24.47 | $0.88^{+}$ | $13.83^{+}$ | 33.06 | $0.34^{+}$ |
| Mean-variance asset allocation strategy with constraint | | | | | | | | | | | | |
| TSP | 6.73 | 5.03 | 0.77 | 6.94 | 5.31 | 0.77 | 6.92 | 5.29 | 0.76 | 6.61 | 5.19 | 1.23 |
| +Real estate | $8.11^{+}$ | 6.92 | 0.76 | $8.53^{+}$ | 7.14 | $0.79^{+}$ | $8.50^{+}$ | 7.10 | $0.79^{+}$ | $7.75^{+}$ | 6.82 | 1.10 |
| +Emerging | $7.59^{+}$ | 6.37 | 0.73 | $8.19^{+}$ | 7.00 | 0.74 | $8.14^{+}$ | 6.96 | 0.74 | $7.37^{+}$ | 6.38 | 1.13 |
| +Commodity | $8.06^{+}$ | 6.98 | 0.73 | $8.51^{+}$ | 7.45 | $0.75^{+}$ | $8.48^{+}$ | 7.44 | 0.75 | $7.71^{+}$ | 6.84 | 1.10 |
| +Intern. bond | $7.09^{+}$ | 5.58 | 0.74 | $7.27^{+}$ | 5.67 | 0.77 | $7.24^{+}$ | 5.65 | $0.77^{+}$ | $6.89^{+}$ | 5.62 | 1.19 |
| +All | $9.98^{+}$ | 8.99 | $0.78^{+}$ | $10.79^{+}$ | 9.25 | $0.84^{+}$ | $10.74^{+}$ | 9.25 | $0.84^{+}$ | $9.30^{+}$ | 8.66 | 1.04 |
| Non-TSP | $9.41^{+}$ | 8.99 | 0.71 | $10.22^{+}$ | 9.25 | $0.78^{+}$ | $10.17^{+}$ | 9.25 | $0.77^{+}$ | $8.73^{+}$ | 8.66 | 0.66 |

This table displays the out-of-sample portfolio performance following mean-variance asset allocation strategies with and without borrowing constraint for the TSP 5-fund portfolio and portfolios complemented with additional funds during the period from January 1993 to April 2015. The results are net of administration cost which is 0.03% for the TSP participants and 0.6% for the non-TSP investors. Parameters are estimated using four different methods. The estimation window length is 120 months and the testing window has the length of 168 months. Improvements in comparison to the TSP benchmark portfolio are highlighted with $^{+}$.

CAPM = capital asset pricing model; TSP = thrift savings plan; Intern. = international.

Table 6b Out-of-sample portfolio benefits of additional funds using minimum-variance, risk parity, and *1/N* naïve asset allocation strategies

| Method | Sample moments | | | CAPM | | | Fama-French 3 factor | | | Bayesian shrinkage | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe | Mean | Standard deviation | Sharpe |
| Minimum-variance asset allocation strategy | | | | | | | | | | | | |
| TSP | 5.03 | 3.41 | 1.04 | 5.06 | 3.38 | 1.06 | 5.02 | 3.40 | 1.05 | 5.11 | 3.78 | 0.97 |
| +Real estate | 5.24$^{+}$ | 3.29$^{+}$ | 1.16$^{+}$ | 5.30$^{+}$ | 3.31$^{+}$ | 1.15$^{+}$ | 5.26$^{+}$ | 3.33$^{+}$ | 1.13$^{+}$ | 5.30$^{+}$ | 3.69$^{+}$ | 1.06$^{+}$ |
| +Emerging | 5.19$^{+}$ | 3.39$^{+}$ | 1.10$^{+}$ | 5.22$^{+}$ | 3.35$^{+}$ | 1.12$^{+}$ | 5.18$^{+}$ | 3.37$^{+}$ | 1.10$^{+}$ | 5.26$^{+}$ | 3.75$^{+}$ | 1.02$^{+}$ |
| +Commodity | 5.05$^{+}$ | 3.37$^{+}$ | 1.06$^{+}$ | 5.13$^{+}$ | 3.35$^{+}$ | 1.09$^{+}$ | 5.12$^{+}$ | 3.36$^{+}$ | 1.08$^{+}$ | 5.14$^{+}$ | 3.76$^{+}$ | 0.98$^{+}$ |
| +Intern. bond | 4.82 | 3.35$^{+}$ | 1.00 | 4.93 | 3.16$^{+}$ | 1.10$^{+}$ | 4.89 | 3.17$^{+}$ | 1.08$^{+}$ | 4.91 | 3.74$^{+}$ | 0.93 |
| +All | 5.22$^{+}$ | 3.17$^{+}$ | 1.20$^{+}$ | 5.32$^{+}$ | 3.05$^{+}$ | 1.25$^{+}$ | 5.29$^{+}$ | 3.08$^{+}$ | 1.23$^{+}$ | 5.26$^{+}$ | 3.56$^{+}$ | 1.10$^{+}$ |
| Non-TSP | 4.65 | 3.17$^{+}$ | 1.02 | 4.75 | 3.05$^{+}$ | 1.07$^{+}$ | 4.72 | 3.08$^{+}$ | 1.05 | 4.69 | 3.56$^{+}$ | 0.93 |
| Risk parity asset allocation strategy | | | | | | | | | | | | |
| TSP | 6.36 | 6.52 | 0.71 | 6.37 | 4.46 | 1.07 | 6.37 | 4.54 | 1.05 | 6.49 | 7.06 | 0.68 |
| +Real estate | 6.84$^{+}$ | 7.12 | 0.70 | 6.85$^{+}$ | 4.51 | 1.15$^{+}$ | 6.85$^{+}$ | 4.60 | 1.13 | 6.99$^{+}$ | 7.67 | 0.67 |
| +Emerging | 6.92$^{+}$ | 7.66 | 0.68 | 6.92$^{+}$ | 4.60 | 1.15$^{+}$ | 6.92$^{+}$ | 4.72 | 1.13 | 7.09$^{+}$ | 8.22 | 0.66 |
| +Commodity | 6.52$^{+}$ | 6.73 | 0.71 | 6.53$^{+}$ | 4.58 | 1.07 | 6.53$^{+}$ | 4.66 | 1.06 | 6.66$^{+}$ | 7.28 | 0.68 |
| +Intern. bond | 5.94 | 5.75$^{+}$ | 0.76$^{+}$ | 5.94 | 3.99$^{+}$ | 1.10$^{+}$ | 5.94 | 4.07$^{+}$ | 1.08 | 6.03 | 6.20$^{+}$ | 0.72$^{+}$ |
| +All | 6.83$^{+}$ | 7.11 | 0.74$^{+}$ | 6.83$^{+}$ | 4.17$^{+}$ | 1.25$^{+}$ | 6.83$^{+}$ | 4.29$^{+}$ | 1.23 | 6.95$^{+}$ | 7.61 | 0.71$^{+}$ |
| Non-TSP | 6.26 | 7.11 | 0.65 | 6.26$^{+}$ | 4.17$^{+}$ | 1.11$^{+}$ | 6.26 | 4.29$^{+}$ | 1.10 | 6.38 | 7.61 | 0.63 |
| *1*/N naive asset allocation strategy | | | | | | | | | | | | |
| TSP | 8.19 | 12.16 | 0.53 | 8.19 | 7.75 | 0.84 | 8.19 | 7.91 | 0.82 | 8.19 | 12.30 | 0.52 |
| +Real estate | 8.71$^{+}$ | 12.48 | 0.53 | 8.71$^{+}$ | 7.59$^{+}$ | 0.89$^{+}$ | 8.71$^{+}$ | 7.76$^{+}$ | 0.87 | 8.71$^{+}$ | 12.65 | 0.53$^{+}$ |
| +Emerging | 9.14$^{+}$ | 13.98 | 0.53 | 9.14$^{+}$ | 7.99 | 0.93$^{+}$ | 9.14$^{+}$ | 8.20 | 0.91 | 9.14$^{+}$ | 14.12 | 0.53$^{+}$ |
| +Commodity | 8.02 | 11.80$^{+}$ | 0.54$^{+}$ | 8.02 | 7.81 | 0.83 | 8.02 | 7.93 | 0.82 | 8.02 | 11.98$^{+}$ | 0.53$^{+}$ |
| +Intern. bond | 7.41 | 10.04$^{+}$ | 0.57$^{+}$ | 7.41 | 6.47$^{+}$ | 0.89$^{+}$ | 7.41 | 6.62$^{+}$ | 0.87$^{+}$ | 7.41 | 10.20$^{+}$ | 0.56$^{+}$ |
| +All | 8.51$^{+}$ | 11.66$^{+}$ | 0.60$^{+}$ | 8.51$^{+}$ | 6.64$^{+}$ | 1.04$^{+}$ | 8.51$^{+}$ | 6.82$^{+}$ | 1.03$^{+}$ | 8.51$^{+}$ | 11.87$^{+}$ | 0.59$^{+}$ |
| Non-TSP | 7.94 | 11.66$^{+}$ | 0.55$^{+}$ | 7.94 | 6.64$^{+}$ | 0.95$^{+}$ | 7.94 | 6.82$^{+}$ | 0.95$^{+}$ | 7.94 | 11.87$^{+}$ | 0.54$^{+}$ |

This table displays the out of sample portfolio performance following various alternative asset allocation strategies including the minimum variance, the risk parity and the naïve strategies during the period from January 1993 to April 2015. The results are net of administration cost which is for the TSP participants and for the non-TSP investors. Parameters are estimated using four different methods. The estimation window length is 120 months and the testing window has the length of 168 months. Improvement in comparison to the TSP benchmark portfolio are highlighted with $^{+}$.

CAPM = capital asset pricing model; TSP = thrift savings plan; Intern. = international.

Table 7a Out-of-sample analysis for sub-period from January 1993 to January 2001

| | Mean-variance asset allocation strategy | | | | Minimum-variance asset allocation strategy | | | |
|---|---|---|---|---|---|---|---|---|
| | Sample Moments | CAPM | Fama-French | Bayesian Shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 1.195 | 1.360 | 1.329 | 1.010 | −0.164 | 1.039 | 0.945 | −0.305 |
| +Real estate | 1.262+ | 1.447+ | 1.415+ | 1.053+ | −0.118+ | 1.197+ | 1.038+ | −0.292+ |
| +Emerging | 1.196+ | 1.366+ | 1.334+ | 1.008 | −0.499 | 0.890 | 0.827 | −0.709 |
| +Commodity | 1.224+ | 1.388+ | 1.359+ | 1.021+ | 0.005+ | 0.959 | 0.865 | −0.103+ |
| +Intern. bond | 1.226+ | 1.390+ | 1.355+ | 1.024+ | −0.135 | 0.847 | 0.808 | −0.251+ |
| +All | 1.323+ | 1.506+ | 1.471+ | 1.090+ | −0.364 | 0.816 | 0.718 | −0.684 |
| Non-TSP | 1.270+ | 1.445+ | 1.411+ | 1.041+ | −0.568 | 0.613 | 0.523 | −0.870 |
| | **Risk parity asset allocation strategy** | | | | **1/N naive asset allocation strategy** | | | |
| | Sample moments | CAPM | Fama-French | Bayesian shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 0.704 | 1.071 | 1.001 | 0.617 | 0.598 | 0.925 | 0.868 | 0.549 |
| +Real estate | 0.748+ | 1.190+ | 1.075+ | 0.638+ | 0.641+ | 1.049+ | 0.945+ | 0.565+ |
| +Emerging | 0.422 | 0.699 | 0.630 | 0.370 | 0.217 | 0.353 | 0.319 | 0.221 |
| +Commodity | 0.601 | 0.933 | 0.874 | 0.535 | 0.475 | 0.758 | 0.694 | 0.457 |
| +Intern. bond | 0.489 | 0.733 | 0.709 | 0.426 | 0.427 | 0.668 | 0.646 | 0.395 |
| +All | 0.300 | 0.538 | 0.443 | 0.249 | 0.143 | 0.285 | 0.201 | 0.143 |
| Non-TSP | 0.200 | 0.372 | 0.289 | 0.159 | 0.072 | 0.167 | 0.090 | 0.076 |

This table displays the out-of-sample portfolio Sharpe ratios for different asset allocation strategies and different parameter estimation methods for the TSP 4-risky-fund portfolio and portfolios complemented with additional funds during the sub period from January 1993 to January 2001. The results are net of administration cost. The length of the estimation window is 30 months. Improvements in comparison to the TSP benchmark portfolio are highlighted with +.

CAPM = capital asset pricing model; TSP = thrift savings plan; Intern. = international.

Table 7b Out-of-sample analysis for sub-period from February 2001 to February 2008

| | Mean-variance asset allocation strategy | | | | Minimum-variance asset allocation strategy | | | |
|---|---|---|---|---|---|---|---|---|
| | Sample moments | CAPM | Fama-French | Bayesian shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 1.423 | 1.539 | 1.537 | 1.239 | 0.592 | 0.837 | 0.866 | 0.496 |
| +Real estate | 1.497$^{+}$ | 1.712$^{+}$ | 1.718$^{+}$ | 1.283$^{+}$ | 0.375 | 0.898$^{+}$ | 0.913$^{+}$ | 0.331 |
| +Emerging | 1.434$^{+}$ | 1.570$^{+}$ | 1.558$^{+}$ | 1.253$^{+}$ | 0.343 | 1.026$^{+}$ | 1.019$^{+}$ | 0.191 |
| +Commodity | 1.534$^{+}$ | 1.605$^{+}$ | 1.600$^{+}$ | 1.312$^{+}$ | 0.711$^{+}$ | 0.960$^{+}$ | 0.973$^{+}$ | 0.593$^{+}$ |
| +Intern. bond | 1.559$^{+}$ | 1.678$^{+}$ | 1.676$^{+}$ | 1.330$^{+}$ | 0.825$^{+}$ | 1.032$^{+}$ | 1.062$^{+}$ | 0.679 |
| +All | 1.760$^{+}$ | 1.905$^{+}$ | 1.897$^{+}$ | 1.473$^{+}$ | 0.405 | 1.350$^{+}$ | 1.304$^{+}$ | 0.254 |
| Non-TSP | 1.494$^{+}$ | 1.856$^{+}$ | 1.848$^{+}$ | 1.155 | 0.182 | 1.148$^{+}$ | 1.101$^{+}$ | 0.050 |
| | **Risk parity asset allocation strategy** | | | | **1/N naive asset allocation strategy** | | | |
| | Sample moments | CAPM | Fama-French | Bayesian shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 0.710 | 0.959 | 0.948 | 0.672 | 0.578 | 0.874 | 0.841 | 0.563 |
| +Real estate | 0.728$^{+}$ | 1.032$^{+}$ | 1.028$^{+}$ | 0.689$^{+}$ | 0.612$^{+}$ | 0.932$^{+}$ | 0.914$^{+}$ | 0.588$^{+}$ |
| +Emerging | 0.901$^{+}$ | 1.411$^{+}$ | 1.371$^{+}$ | 0.839$^{+}$ | 0.896$^{+}$ | 1.520$^{+}$ | 1.446$^{+}$ | 0.850$^{+}$ |
| +Commodity | 1.032$^{+}$ | 1.284$^{+}$ | 1.272$^{+}$ | 0.955$^{+}$ | 1.070$^{+}$ | 1.329$^{+}$ | 1.311$^{+}$ | 1.004$^{+}$ |
| +Intern. bond | 0.856$^{+}$ | 1.152$^{+}$ | 1.153$^{+}$ | 0.805$^{+}$ | 0.660$^{+}$ | 1.001$^{+}$ | 0.985$^{+}$ | 0.648$^{+}$ |
| +All | 1.177$^{+}$ | 1.786$^{+}$ | 1.766$^{+}$ | 1.079$^{+}$ | 1.219$^{+}$ | 1.831$^{+}$ | 1.797$^{+}$ | 1.122$^{+}$ |
| Non-TSP | 1.075$^{+}$ | 1.622$^{+}$ | 1.605$^{+}$ | 0.990$^{+}$ | 1.147$^{+}$ | 1.721$^{+}$ | 1.689$^{+}$ | 1.060$^{+}$ |

This table continues Table 5a, showing the out-of-sample portfolio Sharpe ratios for different asset allocation strategies and different parameter estimation methods during the sub period from February 2001 to February 2008. Improvements in comparison to the TSP benchmark portfolio are highlighted with $^{+}$.

CAPM = capital asset pricing model; TSP = thrift savings plan; Intern. = international.

Table 7c Out-of-sample analysis for sub-period from March 2008 to April 2015

| | Mean-variance asset allocation strategy | | | | Minimum-variance asset allocation strategy | | | |
|---|---|---|---|---|---|---|---|---|
| | Sample moments | CAPM | Fama-French | Bayesian shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 1.356 | 1.544 | 1.520 | 1.253 | 1.758 | 1.392 | 1.379 | 1.330 |
| +Real estate | 1.344 | 1.654+ | 1.612+ | 1.228 | 1.892+ | 1.440+ | 1.422+ | 1.451+ |
| +Emerging | 1.362+ | 1.572+ | 1.548+ | 1.258+ | 2.119+ | 1.374 | 1.353 | 1.545+ |
| +Commodity | 1.371+ | 1.600+ | 1.575+ | 1.264+ | 1.576 | 1.285 | 1.209 | 1.106 |
| +Intern. bond | 1.371+ | 1.553 | 1.527+ | 1.265+ | 1.563 | 1.186 | 1.187 | 1.153 |
| +All | 1.378+ | 1.738+ | 1.694+ | 1.249 | 2.502+ | 1.112 | 1.034 | 1.726+ |
| Non-TSP | 1.702+ | 1.672+ | 1.629+ | 1.017 | 2.197+ | 0.881 | 0.802 | 1.494+ |
| | **Risk parity asset allocation strategy** | | | | **1/N naive asset allocation strategy** | | | |
| | Sample moments | CAPM | Fama-French | Bayesian shrinkage | Sample moments | CAPM | Fama-French | Bayesian shrinkage |
| TSP | 1.166 | 1.621 | 1.584 | 1.074 | 0.853 | 1.306 | 1.255 | 0.840 |
| +Real estate | 1.081 | 1.682+ | 1.626+ | 1.006 | 0.839 | 1.376+ | 1.320+ | 0.821 |
| +Emerging | 0.911 | 1.443 | 1.381 | 0.859 | 0.636 | 1.059 | 0.993 | 0.641 |
| +Commodity | 0.816 | 1.281 | 1.224 | 0.763 | 0.499 | 0.852 | 0.786 | 0.518 |
| +Intern. bond | 1.063 | 1.403 | 1.350 | 0.974 | 0.819 | 1.228 | 1.182 | 0.810 |
| +All | 0.648 | 1.148 | 1.073 | 0.617 | 0.443 | 0.834 | 0.773 | 0.459 |
| Non-TSP | 0.556 | 0.990 | 0.924 | 0.532 | 0.391 | 0.739 | 0.685 | 0.408 |

This table continues Table 5a and 5b, reporting the out-of-sample portfolio performance net of administration cost during the sub period from March 2008 to April 2015. for different asset allocation strategies and different parameter estimation methods. Improvements in comparison to the TSP benchmark portfolio are highlighted with $^{+}$.

CAPM = capital asset pricing model; TSP = thrift savings plan; Intern. = international.

does not necessarily result in a better performance if the investor fails to obtain correct input parameters or does not follow the optimal asset allocation strategy.

In summary, three factors affect our conclusions—portfolio volatility, gross rates of the portfolio return, and fees. The empirical analysis shows the TSP 5-fund plan is fully diversified in term of reducing volatility. However, one can still make a small improvement in net investment returns by including additional instruments if the newly added funds charge low investment fees. This additional benefit fades away when the investment fee is roughly 60 basis points. Thus, although rolling over from the TSP plan allows an investor to have more investment opportunities, the marginal cost of investing cancels out the marginal benefit. If the advice to roll over to an IRA also involves an ongoing investment management fee of 60 basis points or more, or if the investment fees in total are roughly 65 basis points or higher, the advice will lead to reduced net returns.

## 7. Policy results

The main result concerns the quality of advice that pension participants are receiving. The advice to roll over from the TSP, or from 401(k) or 403(b) plans with more investment options, for better diversification is generally not valid. Thus, we document that many participants in the TSP, and in defined contribution plans with more investment options, are receiving bad advice that is costly. The advice to roll over can result in present-value losses of thousands of dollars (Turner, Klein, and Stein 2016).

It should be noted that some pension participants, particularly those in defined contribution plans provided by small employers, are in plans with relatively high fees, and they can reduce their fees by rolling over to a low-fee IRA provider. In addition, for some participants, financial advisers may add value by keeping them from engaging in panic selling when there is a market downturn. In addition, in some circumstances, participants may benefit from a partial roll over, in particular when the disbursement options are limited within the pension plan, which has been the case for the TSP.

More generally, our results indicate that for participants in large 401(k) plans, which typically have lower fees than small plans or IRAs, the advice to roll over for better diversification is based on a true statement that it may be possible to obtain better diversification, but ignores the costs. Because the improvement in diversification is generally relatively small, the increase in costs from rolling over to an IRA outweighs the improvement in diversification. Thus, bad advice is supported by bad analysis. In particular, the analysis focuses on only one aspect of the situation, in this case portfolio diversification, without adequately considering costs.

The second main result is that pension plans can be well diversified with a relatively small number of funds. For example, with its five basic investment options, the TSP is well diversified. Adding an additional four investment options results in slightly better Sharpe ratios using some investment strategies. Thus, this result suggests that defined contribution plans and other funds of funds, such as target date plans, can provide their participants the opportunity to have well diversified funds while still retaining the simple choice menu of a small number of funds that are themselves well diversified and that are selected to work well together in a portfolio. This result has relevance for litigation as to the adequacy of the investment options offered by pension plans, where some plaintiffs have charged imprudent management based on a small number of investment options being offered (Fisch and Wilkinson-Ryan, 2016).

In addition, the results suggest that some financial advisers and some providers of financial products may use strategic complexity to impress naïve investors, recommending or providing complex investment portfolios when simpler portfolios may be superior, once fees are taken into account.

## 8. Conclusions

Mounting evidence documents that some financial advisers with conflicts of interest provide advice that is costly to their clients. Nevertheless, these advisers presumably have arguments that they use to persuade their clients to follow their advice. We conclude that bad advice is sometimes supported by bad analysis. This paper analyzes one such argument. In doing so, it investigates the hypothesis that some advisers with a conflict of interest, in communicating with their clients, focus on the benefits of their advice without weighing the marginal benefits against the marginal costs. We argue that it is psychologically less costly to make a true statement that is incomplete than to make a false statement. Many advisers, however, may simply be following the industry standard advice.

We present evidence against the industry standard advice concerning 401(k) plan rollovers to have more investment options. A plan with as few as five well diversified investment options can provide adequate diversification, while leaving an employer-provided plan and rolling over to an IRA for more options will generally result in higher fees. That is not always the case because small plans tend to have relatively high fees, and sophisticated investors can find low-fee investment options outside of employer-provided pension plans. However, the typical 401(k) plan has far more than five investment options, so our results suggest that the industry standard advice to roll over for greater diversification is generally not valid.

Our results suggest that some advisers and financial service providers may engage in strategic complexity in the portfolios they provide to impress naïve clients as to their complex diversification.

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