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Even Hamming Distance Labeling of Some Path Related Graphs

E.Esakkiammal [1], K.Thirusangu[2] and S.Seethalakshmi[3]

[1,2] Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai, India.

[3] Department of Mathematics, R.V. Govt. Arts College, Chengalpattu,Chennai,India.

[1]esakkiammal2682@gmail.com.[2]kthirusangu@gmail.com,[3]seetha0687@gmail.com.

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Abstract

Hamming distance is used in error correction while transmitting data over computer networks. In this paper, it is shown that Comb graph P_m^+, Twig graph $TW(P_m)$, Centipede $(m,2)$ graph, and comb product of P_m and P_r graph($P_m \rhd P_r$) are even hamming distance labeled graphs. It is proved that the even hamming distance number of Comb graph and Twig graph are 6 and 8 respectively. Also the even hamming distance number of Centipede graph (m,2) is 6 if m=1 and 8 if $m > 1$ and for the comb product of P_m and P_r graph is 4 if $m = 1$ and 6 if $m > 1$ are obtained. This labeling is applied in Cryptography for sharing secret messages.

Keywords: Even Hamming distance labeling, Comb graph, Twig graph, Centipede graph and comb product of P_m and P_r graph.

1.Introduction

Let G be a graph with vertex set V and edge set E. An alternating sequence of vertices and edges, beginning and ending with vertices is called a path graph[3]. A path graph on m+1 vertices is denoted by $P_m, m \geq 1$. The Corona of path graph P_m is obtained from P_m by attaching a pendent vertex to each vertex of P_m and it is denoted by P_m^+. This graph is also known as comb graph[2]. A Twig TW(P_m)$, n \geq 2$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path[1]. Centipede Graph (m,2) is a graph on 3n vertices obtained by joining two pendant edges that are adjacent in each vertex of a path[6].The comb product between graphs P_m and P_r is a graph obtained by taking one copy of P_m and $|V(P_m)|$ copies of P_r and joining each copy of P_r with each vertex of P_m and this graph is denoted by $P_m \rhd P_r$ [4].

2. Even Hamming Distance Labelling of some Graphs

2.1. Definition: Let G = (V, E) be a graph. A function $f: V \to N \cup \{0\}$ is said to be an Even hamming distance labeling if there exist an induced function $f^* : E \to \{2,4,6, \dots, \text{n}\}$ such that for every $uv \in E, f^*(uv) = hd([f(u)]_2, [f(v)]_2)$ satisfying the following conditions:

(i) For every vertex $v \epsilon V$, the set of all edges incident with v receive distinct even labels.

(ii) For every edge $e = uv$, the adjacent vertices u and v receive distinct labels.

A graph which admits even hamming distance labeling is called even hamming distance graph. The even hamming distance number of a graph G is the least positive integer n such that $2^n - 1 \geq k$, where k = max $\{f(v)/v \in V\}$ and it is denoted by $\eta''_{hd}(\text{G})$.

We have proved the existence of even Hamming distance labeling of cycle related graphs in [5]. Here, we have given the proof for existence of even hamming distance labeling of path related graphs.

We propose the following algorithms only to label the vertices of the graph.

Algorithm 2.1.1: Even hamming Distance Labeling of Comb graph

Input: Vertices of P_m^+ graph, $m \geq 1$

$V \leftarrow \{v_i, v_i'/0 \leq i \leq m\}$

$v_0 \leftarrow 0; v_0' \leftarrow 15;$

for $0 \leq i \leq m$

$$v_i \leftarrow \begin{cases} 3 & if\ i \equiv 1(mod4) \\ 12 & if\ i \equiv 2(mod4) \\ 0 & if\ i \equiv 3(mod4) \\ 15 & if\ i \equiv 0(mod4) \end{cases};$$

end for

for $0 \leq i \leq m$-1

$$v_i' \leftarrow \begin{cases} 60 & if\ i \equiv 1(mod4) \\ 51 & if\ i \equiv 2(mod4) \\ 63 & if\ i \equiv 3(mod4) \\ 48 & if\ i \equiv 0(mod4) \end{cases};$$

end for

$$v_m' \leftarrow \begin{cases} 12 & if\ m \equiv 1(mod4) \\ 0 & if\ m \equiv 2(mod4) \\ 15 & if\ m \equiv 3(mod4) \\ 3 & if\ m \equiv 0(mod4) \end{cases};$$

end procedure

output: The labeled vertices of P_m^+ graph.

Theorem: **2.1.2**. The corona of path (comb) graph P_m^+ is an even hamming distance labeled graph and the even hamming distance number is $\eta_{hd}''(P_m^+) = 6$.

Proof: Let us consider the comb graph P_m^+ with vertex set $V = \{\{v_0, v_1, v_2, \dots, v_m\} \cup \{v_0', v_1', v_2', \dots, v_m'\}\}$ and edge set $E = \{\{v_i v_{i+1} / 0 \leq i \leq m-1\} \cup \{v_i v_i'/0 \leq i \leq m\}\}$. Define a function f: V → N ∪ {0} such that $f(u) \neq f(v)$ for any two adjacent vertices u and v as given in the above algorithm 2.1.1. hence the adjacent vertices receive distinct labels. The edge labels are obtained as follows:

$f^*(v_0 v_0')$ = $hd([f(v_0)]_2, [f(v_0')]_2)$ = hd$([0]_2, [15]_2)$ = hd(00000, 01111) =4.

$f^*(v_0 v_1)$ = $hd([f(v_0)]_2, [f(v_1)]_2)$= = hd$([0]_2, [3]_2$ = hd(00000, 00011) = 2.

For $1 \leq \mathrm{i} \leq \mathrm{m}-1$

Case (i): if i $\equiv 1(\mathrm{mod}\ 4)$or $m \equiv 1(mod\ 4)$

$f^*(\mathrm{v_i v_{i+1}})$ = $\mathrm{hd}([\mathrm{f(v_i)}]_2, [\mathrm{f(v_{i+1})}]_2) =$ $\mathrm{hd}([3]_2, [12]_2)$ = 4.

$f^*(v_i v_i')$ =$hd([f(v_i)]_2, [f(v_i')]_2) =$ $\mathrm{hd}([3]_2, [60]_2)$ = 6.

$f^*(v_m v_m')$ =$hd([f(v_m)]_2, [f(v_m')]_2)$= $\mathrm{hd}([3]_2, [12]_2)$ = 4.

Case (ii): if i $\equiv 2(\mathrm{mod}4)$; or $m \equiv 2(mod\ 4)$

$f^*(\mathrm{v_i v_{i+1}})$ = $\mathrm{hd}([\mathrm{f(v_i)}]_2, [\mathrm{f(v_{i+1})}]_2) =$ $\mathrm{hd}([12]_2, [0]_2)$ = 2.

$f^*(v_i v_i')$ =$hd([f(v_i)]_2, [f(v_i')]_2) =$ $\mathrm{hd}(12_2, [51]_2)$ = 6.

$f^*(v_m v_m')$ =$hd([f(v_m)]_2, [f(v_m')]_2)$= $\mathrm{hd}([12]_2, [0]_2)$ = 2.

Case (iii): if i $\equiv 3(\mathrm{mod}\ 4)$ or $m \equiv 2(mod\ 4)$

$f^*(\mathrm{v_i v_{i+1}})$ = $\mathrm{hd}([\mathrm{f(v_i)}]_2, [\mathrm{f(v_{i+1})}]_2) =$ $\mathrm{hd}([0]_2, [15]_2)$ = 4.

$f^*(v_i v_i')$ =$hd([f(v_i)]_2, [f(v_i')]_2) =$ $\mathrm{hd}([0]_2, [63]_2)$ = 6.

$f^*(v_m v_m')$ =$hd([f(v_m)]_2, [f(v_m')]_2)$= $\mathrm{hd}([0]_2, [15]_2)$ = 4.

Case (iv): if i $\equiv 0(\mathrm{mod}4)$;

$f^*(\mathrm{v_i v_{i+1}})$ = $\mathrm{hd}([\mathrm{f(v_i)}]_2, [\mathrm{f(v_{i+1})}]_2) =$ $\mathrm{hd}([15]_2, [3]_2)$ = 2 .

$f^*(v_i v_i')$ =$hd([f(v_i)]_2, [f(v_i')]_2) =$ $\mathrm{hd}([15]_2, [48]_2)$ = 6.

$f^*(v_m v_m')$ =$hd([f(v_m)]_2, [f(v_m')]_2)$= $\mathrm{hd}([15]_2, [3]_2)$ = 2.

From all the above cases, all the adjacent edges receive distinct Even labels. Hence the Path graph $\mathrm{P_m}$ admits even hamming distance labeling and the even hamming distance number is $\eta_{hd}''(P_m^+) = 6$.

Algorithm 2.1.3. Even hamming Distance Labeling of Twig graph $TW(P_m)$

Input: Vertices of $TW(P_m)$ graph , $m \geq 2$

$V \leftarrow \{v_{0\,,}\, v_m\ , v_i'\ , v_i''/1 \leq i \leq m-1\}$

$v_0\ \leftarrow 0;$

for $i = 1\ to\ m$ do

$$v_i\ \leftarrow \begin{cases} 3 & if\ \ i\ \equiv 1(mod4) \\ 252 & if\ \ i\ \equiv 2(mod4) \\ 60 & if\ \ i\ \equiv 3(mod4) \\ 195 & if\ \ i\ \equiv 0(mod4) \end{cases}$$

end for

for $i = 1\ to\ m-1$ do

$$v_i' \leftarrow \begin{cases} 12 & if\ \ i\ \equiv 1(mod4) \\ 12 & if\ \ i\ \equiv 2(mod4) \\ 0 & if\ \ i\ \equiv 3(mod4) \\ 0 & if\ \ i\ \equiv 0(mod4) \end{cases} \qquad v_i'' \leftarrow \begin{cases} 60 & if\ \ i\ \equiv 1(mod4) \\ 0 & if\ \ i\ \equiv 2(mod4) \\ 3 & if\ \ i\ \equiv 3(mod4) \\ 12 & if\ \ i\ \equiv 0(mod4) \end{cases}$$

end for

end procedure

output: The labeled vertices of twig $TW(P_m)$graph.

Theorem 2.1.4. The Twig graph $TW(P_m)$ is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(TW(P_m)) = 8$, for any $m \geq 2$.

Proof: Let us consider the twig graph $TW(P_m)$ with vertex set $V = \{v_{0\,,}\, v_m\,, v_i'\,, v_i''/1 \leq i \leq m-1\}$ and edge set $E = \{\{v_i v_{i+1}\ /\ 0 \leq i \leq m-1\} \cup \{v_i v_i'\ /\ 1 \leq i \leq m-1\} \cup \{v_i v_i''\ /\ 1 \leq i \leq m-1\}\}$. Define a function $f: V \to N \cup \{0\}$ such that $f(u) \neq f(v)$ for any two adjacent vertices u and v as given in the above algorithm 2.1.3. Hence the adjacent vertices receive distinct labels. The edge labels are obtained as follows:

$f^*(v_0 v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = = hd([0]_2, [3]_2)$=2.

For $1 \leq i \leq m-1$,

Case (i): if i $\equiv 1(\text{mod } 4)$or m $\equiv 1(\text{mod } 4)$

$f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([3]_2, [252]_2) = 8$.

$f^*(v_i v_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([3]_2, [12]_2) = 4$.

$f^*(v_i v_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([3]_2, [60]_2) = 6$.

$f^*(v_{m-1} v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([195]_2, [3]_2) = 2$.

Case (ii): if i $\equiv 2(\text{mod}4)$

$f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([252]_2, [60]_2) = 2$.

$f^*(v_i v_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd(252_2, [12]_2) = 4$.

$f^*(v_i v_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([252]_2, [0]_2) = 6$.

$f^*(v_{m-1} v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([3]_2, [252]_2) = 8$.

Case (iii): if i $\equiv 3(\text{mod } 4)$

$f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([60]_2, [195]_2) = 8$.

$f^*(v_i v_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([60]_2, [0]_2) = 4$.

$f^*(v_i v_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([60]_2, [3]_2) = 6$.

$f^*(v_{m-1} v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([252]_2, [60]_2) = 2$.

Case (iv): if i $\equiv 0(\text{mod}4)$

$f^*(\mathrm{v_i v_{i+1}}) = \mathrm{hd}([\mathrm{f(v_i)}]_2, [\mathrm{f(v_{i+1})}]_2) = \mathrm{hd}([195]_2, [3]_2) = 2$.

$f^*(v_i v_i') = hd([f(v_i)]_2, [f(v_i')]_2) = \mathrm{hd}([195]_2, [0]_2) = 4$.

$f^*(v_i v_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = \mathrm{hd}([195]_2, [12]_2) = 6$.

$f^*(\mathrm{v_{m-1} v_m}) = \mathrm{hd}([\mathrm{f(v_{m-1})}]_2, [\mathrm{f(v_m)}]_2) = \mathrm{hd}([60]_2, [195]_2) = 8$.

From all the above cases, all the adjacent edges receive distinct even labels. Hence the twig graph $TW(P_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(TW(P_m)) = 8$, for any $m \geq 2$.

Algorithm 2.1.5. Even Hamming Distance labeling of Centipede graph$(\boldsymbol{m}, \mathbf{2})$.

Input: Vertices of Centipede graph $(m, 2)$ graph, $m \geq 1$

$V \leftarrow \{v_i, v_i', v_i''/0 \leq i \leq m\}$

$v_0 \leftarrow 0; v_0' \leftarrow 15; v_0'' \leftarrow 63;$

$$v_m' \leftarrow \begin{cases} 12 & if \;\; m \equiv 1(mod4) \\ 0 & if \;\; m \equiv 2(mod4) \\ 15 & if \;\; m \equiv 3(mod4) \\ 3 & if \;\; m \equiv 0(mod4) \end{cases}; \qquad v_m'' \leftarrow \begin{cases} 60 & if \;\; m \equiv 1(mod4) \\ 51 & if \;\; m \equiv 2(mod4) \\ 63 & if \;\; m \equiv 3(mod4) \\ 48 & if \;\; m \equiv 0(mod4) \end{cases}$$

for $i = 1 \; to \; m - 1$ do

$$v_i' \leftarrow \begin{cases} 60 & if \;\; i \equiv 1(mod4) \\ 51 & if \;\; i \equiv 2(mod4) \\ 63 & if \;\; i \equiv 3(mod4) \\ 48 & if \;\; i \equiv 0(mod4) \end{cases}; v_i'' \leftarrow \begin{cases} 252 & if \;\; i \equiv 1(mod4) \\ 243 & if \;\; i \equiv 2(mod4) \\ 255 & if \;\; i \equiv 3(mod4) \\ 240 & if \;\; i \equiv 0(mod4) \end{cases}$$

end for

for $i = 1 \; to \; m$ do

$$v_i \leftarrow \begin{cases} 3 & if \;\; i \equiv 1(mod4) \\ 12 & if \;\; i \equiv 2(mod4) \\ 0 & if \;\; i \equiv 3(mod4) \\ 15 & if \;\; i \equiv 0(mod4) \end{cases};$$

end for

end procedure

Output: The labeled vertices of Centipede (m,2) graph.

Theorem: **2.1.6**. The centipede graph $(m, 2)$ is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(\mathrm{m}, 2) = \begin{cases} 6, & if \; m = 1 \\ 8, & if \; m > 1 \end{cases}$

Proof: Let us consider the centipede graph $(m, 2)$ with vertex set $V = \{\{v_{0,} v_{1,} v_{2,} \dots, v_m\} \cup \{v_{0,}' v_{1,}' v_{2,}' \dots, v_m'\} \cup \{v_{0,}'' v_{1,}'' v_{2,}'' \dots, v_m''\}\}$ and edge set $E =$

$\{\{v_iv_{i+1} / 0 \le i \le m-1\} \cup \{v_iv_i'/0 \le i \le m\} \cup \{v_iv_i''/0 \le i \le m\}\}$. Define a function $f: V \to N \cup \{0\}$ such that $f(u) \ne f(v)$ for any two adjacent vertices u and v as given in the above algorithm 2.1.5. hence the adjacent vertices receive distinct labels. The edge labels are obtained as follows:

$f^*(v_0v_0') = hd([f(v_0)]_2, [f(v_0')]_2) = hd([0]_2, [15]_2) = 4$.

$f^*(v_0v_0'') = hd([f(v_0)]_2, [f(v_0'')]_2 = hd([0]_2, [63]_2) = 6$.

For $1 \le i \le m-1$,

Case (i): if i $\equiv 1(\text{mod } 4)$ or $m \equiv 1(\text{mod } 4)$

$f^*(v_iv_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([3]_2, [60]_2) = 6$.

$f^*(v_iv_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([3]_2, [252]_2) = 8$.

$f^*(v_mv_m') = hd([f(v_m)]_2, [f(v_m')]_2) = hd([3]_2, [12]_2) = 4$.

$f^*(v_mv_m'') = hd([f(v_m)]_2, [f(v_m'')]_2) = hd([3]_2, [60]_2) = 6$.

Case (ii): if i $\equiv 2(\text{mod}4)$ or $m \equiv 2(\text{mod } 4)$

$f^*(v_iv_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([12]_2, [51]_2) = 6$.

$f^*(v_iv_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([12]_2, [243]_2) = 8$.

$f^*(v_mv_m') = hd([f(v_m)]_2, [f(v_m')]_2) = hd([12]_2, [0]_2) = 2$.

$f^*(v_mv_m'') = hd([f(v_m)]_2, [f(v_m'')]_2) = hd([12]_2, [51]_2) = 6$.

Case (iii): if i $\equiv 3(\text{mod } 4)$ or $m \equiv 3(\text{mod } 4)$

$f^*(v_iv_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([0]_2, [63]_2) = 6$.

$f^*(v_iv_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([0]_2, [255]_2) = 8$.

$f^*(v_mv_m') = hd([f(v_m)]_2, [f(v_m')]_2) = hd([0]_2, [15]_2) = 4$.

$f^*(v_mv_m'') = hd([f(v_m)]_2, [f(v_m'')]_2) = hd([0]_2, [63]_2) = 6$.

Case (iv): if i $\equiv 0(\text{mod}4)$ or $m \equiv 0(\text{mod } 4)$

$f^*(v_iv_i') = hd([f(v_i)]_2, [f(v_i')]_2) = hd([15]_2, [48]_2) = 6$.

$f^*(v_iv_i'') = hd([f(v_i)]_2, [f(v_i'')]_2) = hd([15]_2, [240]_2) = 8$.

$f^*(v_mv_m') = hd([f(v_m)]_2, [f(v_m')]_2) = hd([15]_2, [3]_2) = 2$.

$f^*(v_mv_m'') = hd([f(v_m)]_2, [f(v_m'')]_2) = hd([15]_2, [48]_2) = 6$.

From all the above cases, all adjacent edges receive distinc event labels. Hence it is proved that the centipede graph (m,2) admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(m,2) = \begin{cases} 6, & if\ m = 1 \\ 8, & if\ m > 1 \end{cases}$.

Algorithm 2.1.7. Even Hamming Distance labeling of Comb Product of P_m and P_r :

Input: Vertices of $(P_m \rhd P_r)$ graph, $m, r \ge 1$

$V \leftarrow \left\{v_i, u_i^{(j)}/0 \leq i \leq m,\ 2 \leq j \leq r+1, where\ \ v_i = u_i^{(1)}\right\}$

$v_0 \ = u_0^{(1)} \leftarrow 0; u_0^{(2)} \leftarrow 15;$

for $i = 1\ to\ m$ do

$$v_i \ = \ u_i^{(1)} \leftarrow \begin{cases} 3 & if\ \ i\ \equiv 1(mod4) \\ 60 & if\ i\ \equiv 2(mod4) \\ 12 & if\ i\ \equiv 3(mod4) \\ 51 & if\ \ i\ \equiv 0(mod4) \end{cases} \qquad ;u_i^{(2)} \leftarrow \begin{cases} 12 & if\ \ i\ \equiv 1(mod4) \\ 0 & if\ \ i\ \equiv 2(mod4) \\ 3 & if\ \ i\ \equiv 3(mod4) \\ 0 & if\ \ i\ \equiv 0(mod4) \end{cases}$$

end for

for i = 0 to m do

for j = 3 to r+1 do

if i ≡ $1(mod2)$ do

$$u_i^{(j)} \leftarrow \begin{cases} 3 & if\ \ j\ \equiv 1(mod4) \\ 12 & if\ \ j \equiv 2(mod4) \\ 0 & if\ \ j\ \equiv 3(mod4) \\ 15 & if\ \ j\ \equiv 0(mod4) \end{cases}$$

else

$$u_i^{(j)} \leftarrow \begin{cases} 0 & if\ \ j\ \equiv 1(mod4) \\ 15 & if\ \ j \equiv 2(mod4) \\ 3 & if\ \ j\ \equiv 3(mod4) \\ 12 & if\ \ j\ \equiv 0(mod4) \end{cases}$$

end if

end for

end for

end procedure

output: The labeled vertices of Comb Product $(P_m \rhd P_r)$graph.

Theorem 2.1.8. The comb product of P_m and P_r graph $(P_m \rhd P_r)$is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(P_m \rhd P_r) = \begin{cases} 4, & if\ m = 1 \\ 6, & if\ m > 1 \end{cases}$.

Proof: Let us consider the comb product of P_m and P_r graph $(P_m \rhd P_r)$ with vertex set $V = \left\{v_i, u_i^{(j)}/0 \leq i \leq m,\ 2 \leq j \leq r, where\ \ v_i = u_i^{(1)}\right\}$ and edge set $E = \left\{\{v_i v_{i+1}\ /\ 0 \leq i \leq m-1\} \cup \left\{u_i^{(j)} u_{i+1}^{(j+1)}/0 \leq i \leq m, 1 \leq j \leq r-1\right\}\right\}$. Define a function $f\colon \mathrm{V} \to \mathrm{N} \cup \{0\}$ such that $f(u) \neq f(v)$ for any two adjacent vertices u and v as given in the above algorithm 2.1.7. Hence the adjacent vertices receive distinct labels. The edge labels are obtained as follows: $f^*(u_0^{(1)}u_0^{(2)}) = hd(\left[f\left(u_0^{(1)}\right)\right]_2, \left[f\left(u_0^{(2)}\right)\right]_2) = hd([0]_2, [15]_2) = 4$.

$f^*(u_0^{(2)}u_0^{(3)}) = hd(\left[f\left(u_0^{(2)}\right)\right]_2, \left[f\left(u_0^{(3)}\right)\right]_2) = hd([15]_2, [3]_2) = 2$.

$f^*(v_0v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = hd([0]_2, [3]_2) = 2$.

For $1 \leq i \leq m-1$

Case (i): if $i \equiv 1(mod4)$ or $m \equiv 1(mod4)$

$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([3]_2, [60]_2) = 6$.

$f^*(u_i^{(1)}u_i^{(2)}) = hd(\left[f\left(u_i^{(1)}\right)\right]_2, \left[f\left(u_i^{(2)}\right)\right]_2) = hd([3]_2, [12]_2) = 4$.

$f^*(u_i^{(2)}u_i^{(3)}) = hd(\left[f\left(u_i^{(2)}\right)\right]_2, \left[f\left(u_i^{(3)}\right)\right]_2) = hd([12]_2, [0]_2) = 2$.

$f^*(v_{m-1}v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([51]_2, [3]_2) = 2$.

Case (ii): if $i \equiv 2(mod4)$ or $m \equiv 2(mod4)$

$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([60]_2, [12]_2) = 2$.

$f^*(u_i^{(1)}u_i^{(2)}) = hd(\left[f\left(u_i^{(1)}\right)\right]_2, \left[f\left(u_i^{(2)}\right)\right]_2) = hd([60]_2, [0]_2) = 4$.

$f^*(u_i^{(2)}u_i^{(3)}) = hd(\left[f\left(u_i^{(2)}\right)\right]_2, \left[f\left(u_i^{(3)}\right)\right]_2) = hd([0]_2, [3]_2) = 2$.

$f^*(v_{m-1}v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd(3_2, [60]_2) = 6$.

Case (iii): if $i \equiv 3(mod4)$ or $m \equiv 3(mod4)$

$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([12]_2, [51]_2) = 6$.

$f^*(u_i^{(1)}u_i^{(2)}) = hd(\left[f\left(u_i^{(1)}\right)\right]_2, \left[f\left(u_i^{(2)}\right)\right]_2) = hd([12]_2, [3]_2) = 4$.

$f^*(u_i^{(2)}u_i^{(3)}) = hd(\left[f\left(u_i^{(2)}\right)\right]_2, \left[f\left(u_i^{(3)}\right)\right]_2) = hd([3]_2, [0]_2) = 2$.

$f^*(v_{m-1}v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([60]_2, [12]_2) = 2$.

Case (iv): if $i \equiv 0(mod4)$ or $m \equiv 0(mod4)$

$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([51]_2, [3]_2) = 2$.

$f^*(u_i^{(1)}u_i^{(2)}) = hd(\left[f\left(u_i^{(1)}\right)\right]_2, \left[f\left(u_i^{(2)}\right)\right]_2) = hd(51_2, [0]_2) = 4$.

$f^*(u_i^{(2)}u_i^{(3)}) = hd(\left[f\left(u_i^{(2)}\right)\right]_2, \left[f\left(u_i^{(3)}\right)\right]_2) = hd([0]_2, [3]_2) = 2$.

$f^*(v_{m-1}v_m) = hd([f(v_{m-1})]_2, [f(v_m)]_2) = hd([12]_2, [51]_2) = 6$.

For $0 \leq i \leq m$; $3 \leq j \leq r$

if $j \equiv 1(mod4)$; $f^*(u_i^{(j)}u_i^{(j+1)}) = hd(\left[f\left(u_i^{(j)}\right)\right]_2, \left[f\left(u_i^{(j+1)}\right)\right]_2) = hd([0]_2, [15]_2) = 4$.

if $j \equiv 2(mod4)$; $f^*(u_i^{(j)}u_i^{(j+1)}) = hd(\left[f\left(u_i^{(j)}\right)\right]_2, \left[f\left(u_i^{(j+1)}\right)\right]_2) = hd([15]_2, [3]_2) = 2$.

if $j \equiv 3(mod4)$; $f^*(u_i^{(j)}u_i^{(j+1)}) = hd(\left[f\left(u_i^{(j)}\right)\right]_2, \left[f\left(u_i^{(j+1)}\right)\right]_2) = hd([3]_2, [12]_2) = 4$.

if $j \equiv 0(mod4)$; $f^*(u_i^{(j)}u_i^{(j+1)}) = hd(\left[f\left(u_i^{(j)}\right)\right]_2, \left[f\left(u_i^{(j+1)}\right)\right]_2) = hd([12]_2, [0]_2) = 2$

From all the above cases, all adjacent edges receive distinct even labels. Hence it is proved that the comb product of P_m and P_r graph $(P_m \rhd P_r)$ admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(P_m \rhd P_r) = \begin{cases} 4, & if\ m = 1 \\ 6, & if\ m > 1 \end{cases}$.

3. Conclusion

In this paper, the even hamming distance number of Comb graph P_m^+, Twig graph $TW(P_m)$, Centipede graph$(m, 2)$, and comb product of P_m and P_r graph$(P_m \rhd P_r)$ were obtained.

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