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Non-Locality in Classical Electrodynamics

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ABSTRACT

Classical electrodynamics—if developed consistently, as in Dirac's classical theory of the electron—is causally non-local. I distinguish two distinct causal locality principles and argue, using Dirac's theory as my main case study, that neither can be reduced to a non-causal principle of local determinism.

1 Introduction

Classical electrodynamics is generally understood to be the paradigm of a local and causal physical theory. After all, electromagnetic fields, which mediate all electromagnetic influences, propagate at a finite speed. One of my aims in this paper is to set the record straight: perhaps surprisingly, non-locality is already a feature of purely classical electromagnetic systems, independently of any quantum mechanical considerations. On its standard interpretation, P. A. M. Dirac's classical theory of the electron (Dirac [1938]) allows for forces to act where they are not, and for superluminal causal propagation.

The general philosophical question in which I am interested is: what is it for a theory to be local or non-local? I will argue that there are several logically distinct locality conditions that are invoked in the context of classical physics and that these conditions are often not distinguished carefully enough. Two of these conditions, I want to argue, are irreducibly causal.

Informally, locality principles are often introduced in causal terms. Newtonian gravitational theory, for example, is said to be non-local, because it allows for action-at-a-distance; while the theory of special relativity is sometimes (if perhaps incorrectly) said to imply the locality condition that there can be no superluminal causal propagation. According to a widespread view, however, the notion of causation is inherently suspect and whatever genuine content such *prima facie* causal principles have should be explicated in non-causal terms. For example, Bertrand Russell famously argued that in the advanced sciences the notion of functional dependency has replaced that of causation. Any more substantive notion of causality, he claimed, 'is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm' ([1918], p. 180).

Russell's wariness of a rich notion of causation is echoed by a currently influential view of scientific theories that identifies theories (in the physical sciences) with a mathematical formalism (or a class of mathematical models) and an interpretation whose *only* job it is to fix the ontology of the theory. Absent from this account is the idea that part of the job of an interpretation may be to stipulate causal structures consistent with the mathematical models with which the theory provides us. Thus, *prima facie* causal locality conditions—such as the prohibition against action-at-a-distance—would, on this view, have to be spelled out purely in terms of the mathematical formalism (plus its associated ontology).

Gordon Belot's recent discussion of non-locality in electromagnetism (Belot [1998]) can be understood as contributing to this broadly neo-Humean project. Belot there proposes a non-causal principle of determinism which he takes to be equivalent to the condition that effects propagate at a finite speed. But I will argue that Belot's condition is logically independent from the causal principle it is meant to explicate. In particular, I will argue that Dirac's classical theory of the electron comes out as local according to Belot's condition, even though the theory is causally non-local.

Although my main focus will be on Belot's principle and its relation to causal locality conditions, I believe that my conclusion generalizes: no non-causal explication of the two causal locality principles—that of a finite velocity of all causal propagation and that of action-by-contact—seems possible. As additional evidence for this claim I will discuss a locality criterion proposed by John Earman ([1987]) that is meant to capture the content of the action-by-contact principle. Earman's criterion, as I will argue, also fails as a strictly non-causal explication.

In the next section (section 2), I will present certain salient features of Dirac's classical theory of the electron and I will show that (under its standard interpretation) the theory violates both the action-by-contact condition and the finite causal propagation condition. In section 3 I introduce Belot's locality condition and discuss his own application of that condition to interpretations of classical electrodynamics. In sections 4 and 5 I show that the three locality principles that I have distinguished—the two causal principles and Belot's condition—are all logically independent from one another. Section 6 argues that Earman's condition fails to provide a non-causal explication of the action-by-contact condition and addresses a Russellian objection to my account. In the last section I provide a brief summary.

2 Dirac's classical theory of the electron

Microscopic classical electrodynamics is concerned with the interaction between electric charges and electromagnetic fields. In the theory familiar from physics textbooks, such as Jackson ([1999]), electromagnetic phenomena are treated as being governed by two sets of laws—the microscopic Maxwell equations, on the one hand, and the Lorentz force law together with Newton's second law (or its relativistic generalization), on the other. The Maxwell equations allow us to calculate the electromagnetic fields associated with a given charge and current configuration, while the Lorentz law allows us to calculate the motion of a charge in an external field. The Maxwell–Lorentz approach to electrodynamics results in a local and forward-causal theory. It is this theory that people seem to have in mind when they refer to electromagnetism, as Belot does, as 'the paradigm of all that a classical (i.e. non-quantum) theory should be' ([1998], p. 531). The only trouble with this theory is that it is inconsistent. According to the Maxwell equations, accelerated charges radiate off energy, but the Lorentz law ignores any effects on the motion of a charge due to its own radiation. If one assumes energy-momentum conservation (in its standard formulation), then the Maxwell equations imply that the acceleration of a charge cannot be what the Lorentz law predicts. The reason why the Lorentz force law ignores a charge's own radiation field is that this field—as predicted by the Maxwell equations—is infinite at the location of the charge, if charges are treated as point particles (as is standardly done).[1]

As Dirac ([1938]) has shown, however, it is possible to include in a consistent way a charge's radiation field in an equation of motion for a charged particle. The stumbling block in trying to derive an equation of motion for a point charge from the Maxwell equations and the principle of energy-momentum conservation is the infinity in the charge's field, which implies that the energy associated with the field is infinite. Dirac showed that

1 If charges are modeled as extended particles, one can derive a (non-relativistic) force law involving an infinite series in powers of the radius r of the charge. Since the leading term is proportional to $1/r$, the series diverges for $r \rightarrow 0$.

one can overcome this difficulty by absorbing part of the infinite self-energy of a charge into its mass (which nevertheless is taken to be finite). With the help of this procedure, which is known as 'renormalization' (and which has become a standard technique in quantum field theories), one can derive the relativistic *Lorentz–Dirac equation* (Rohrlich [1965], equation 6–57):[2]

$$ma^{\mu} = F^{\mu} + \frac{2e^2}{3c^3}\left(\frac{da^{\mu}}{d\tau} - \frac{1}{c^2}a^{\lambda}a_{\lambda}v^{\mu}\right) \tag{1}$$

where F^{μ} is the total external force (due to the external electromagnetic field $F^{\mu\nu}$ and any non-electromagnetic forces acting on the charge), the mass term m on the left represents the renormalized finite mass, and derivatives are with respect to the proper time τ. This equation differs from familiar Newtonian equations in that it is a third-order equation, involving not only the acceleration a^{μ} of the charge, but also its derivative.[3] In renormalizing the mass one needs to assume as an asymptotic condition that the acceleration of the charge tends to zero at both future and past infinity. Thus even though (1) is a local equation in that it relates quantities at a single proper time τ to one another, it is derived with the help of a global assumption.

If the external force is zero, one class of solutions to (1) describes a free particle that (for no apparent 'reason') accelerates and continues to accelerate with an ever increasing rate. These so-called *runaway solutions* violate the asymptotic condition of vanishing acceleration at infinity and are generally rejected as unphysical. If we impose explicitly the requirement that all acceptable solutions to the Lorentz–Dirac equation have to satisfy the asymptotic condition, equation (1) can be integrated once and we arrive at the following second-order integro-differential equation of motion (*Ibid.*, equation 6–76):

$$a^{\mu}(\tau) = \int_{\tau}^{\infty} e^{(\tau-\tau')/\tau_0}\left[\frac{1}{m\tau_0}F^{\mu}(\tau') - \frac{1}{c^2}a^{\lambda}(\tau')a_{\lambda}(\tau')v^{\mu}(\tau')\right]d\tau' \tag{2}$$

where the constant $\tau_0 = 2e^2/3mc^3$. This is a non-local equation in that it relates the acceleration at τ to the acceleration at all other times after τ.

The asymptotic condition is usually motivated physically, first, by the assumption that any interaction between a charge and external fields can be modeled as a scattering process—that is, as an interaction between a charge that is 'asymptotically free' and localized external fields—and, second, by

[2] The equation is in standard four-vector notation, where indices range from 1 through 4 and repeated indices are summed over. In my presentation of the formalism and its interpretation, I largely follow Rohrlich's classic text. For another excellent discussion of Dirac's theory, see Parrott ([1987], pp. 148–169).

[3] Since position enters implicitly through the fields, the equation cannot be understood as a second-order equation for the four-velocity v^{μ}.

appealing to something like a principle of inertia according to which the acceleration goes to zero sufficiently far away from any force acting on the charge.[4] Even though this principle is weaker than the familiar Newtonian principle of inertia, adopting it has the advantage of ensuring a certain interpretive continuity between Dirac's theory and Maxwell–Lorentz electrodynamics (and Newtonian theories in general[5]). Just as in Maxwell–Lorentz electrodynamics, a charge which never experiences a force moves with constant velocity in Dirac's theory.[6]

The 'obvious' interpretation (Rohrlich [1965], p. 149), then, of (2) is that in Dirac's theory, too, forces should be taken to be causally responsible for the acceleration of a charge, where the 'effective force' is given by the expression in square brackets under the integral in (2) and includes both the external force and a force on the charge due to its own radiation field. Thus, the acceleration at τ is due to the force at τ *plus* all non-zero forces on the world line of the charge at all later times, where (due to the strong exponential damping factor in the integral) forces contribute less and less the farther they are in the future. Since, according to (2), the acceleration at τ is due partly to effective forces at times other than τ, forces can act where they are not in Dirac's theory.

This non-local feature of the theory is perhaps most evident in the following approximation to (2). Since τ_0 is small, the equation of motion can be approximated by (Rohrlich [1965], equation 6–84):

$$ma^{\mu}(\tau) = K^{\mu}(\tau + \xi\tau_0) \tag{3}$$

where

$$K^{\mu} = F^{\mu} - \frac{m\tau_0}{c^2} a^{\lambda} a_{\lambda} v^{\mu}$$

is the effective force. (3) almost looks like a Newtonian equation of motion, except for the fact that there is a time delay between acceleration and effective force: the acceleration at τ depends on the force at a slightly later proper time. The non-local dependence between force and acceleration could be spelled out in terms of what the consequences of counterfactual interventions into an

[4] This second assumption is often not made explicitly in the literature but is clearly needed.

[5] When I speak of Newton's laws in this paper, I intend this to include their relativistic generalization and do not mean to draw a contrast between non-relativistic and relativistic physics. Maxwell–Lorentz electrodynamics is most naturally formulated in a relativistically invariant way. Still, the theory is a Newtonian theory in that it relies on the relativistic analogues of Newton's laws of motion in order to arrive at an equation of motion for a charged particle.

[6] See Rohrlich ([1965]), sec. 6.10). But Rohrlich claims incorrectly that it follows from the fact that charges which never experience a force move with constant velocity that Dirac's theory satisfies Newton's principle of inertia. As one can see from (2), the acceleration of a charge in Dirac's theory can be non-zero even at times when the force on the charge is zero. Thus, Dirac's charges can violate Newton's principle of inertia.

otherwise closed system would be: if we were to introduce an additional external force at $\tau + \xi\tau_0$, then the acceleration at τ would have to be different. By contrast, an intervention into a purely Newtonian system would affect only the acceleration at the time of the intervening force.

Against the interpretation of Dirac's theory I am presenting here, Grünbaum ([1976]) has argued that acceleration in the theory plays a role analogous to that of velocity in Newtonian theories. If the analogy held, it would be a mistake to interpret (2) causally non-locally. Adolf Grünbaum points out that we can write down equations which allow us to *retrodict* a particle's Newtonian velocity from its final velocity (as 'initial' condition) together with future forces, but this does not show that these forces *cause* the current velocity. Similarly, we should not take the fact that (2) allows us to retrodict the present acceleration from future forces to suggest that future forces cause the present acceleration since that equation is derived with the help of the asymptotic condition as 'initial' condition. But Grünbaum misunderstands the role of the asymptotic condition in Dirac's theory and the analogy is flawed. First, the asymptotic condition does not merely play the role of an extraneous initial condition, as Grünbaum claims, since it needs to be assumed in the very derivation of the Lorentz–Dirac equation; and, second, there is of course no principle analogous to the Diracian principle of inertia in Newtonian physics which says that velocities go to zero sufficiently far away from any force. Thus, the non-local causal interpretation of Dirac's theory follows not *merely* from the fact that (2) is a non-local equation; rather it follows from (2) in light of the explicitly causal assumption that (field) forces cause accelerations, where this assumption in turn is motivated as a natural way to account for the asymptotic condition.

The fact that Dirac's theory is non-local might seem surprising, since it is a theory in which all interactions between particles are mediated by fields propagating with a finite velocity. Are not field theories local (almost) by definition? But we need to be careful here. The electromagnetic field *alone* is local in that the state of the total field (in a given frame at a certain time) is given by the state of the field in all sub-regions of space. Moreover, disturbances in the field propagate at a finite speed through the field. Non-local features arise when we consider how the electromagnetic field interacts with charged particles. The field *produced* by a charge again is locally connected to the charge: according to the Maxwell equations, radiation fields associated with a charge arise at the location of the charge and propagate away from the charge at a finite speed. But the field *affects* charges non-locally: the acceleration of a charge at an instant is due to the fields on the entire future world line of the charge. Belot says that it is a 'well-entrenched principle that classical fields act by contact rather than at a distance' ([1998], p. 532), and in this he is surely right. But it is a striking (and underappreciated) fact that in microscopic classical electrodynamics this well-entrenched principle is satisfied only by the inconsistent Maxwell–Lorentz theory, and not by Dirac's consistent theory.

A second sense in which Dirac's theory is non-local is that the theory allows for superluminal causal propagation. On the one hand, the present acceleration of a charge is determined by future fields according to (2). On the other hand, an accelerated charge produces a so-called *retarded* radiation field which affects the total electromagnetic field along the forward light cone of the charge. The combination of the backwards causal effect of an external field on a charge and the forward causal influence of a charge on the total field can result in causal propagation between space-like separated events. If the radiation field due to a charge q_1 at τ_1 is non-zero where its forward light cone intersects the world line of a charge q_2, then the acceleration of q_2 at τ_2 will be affected by the field due to q_1, even when the two charges are space-like separated. Again one could make this point in terms of interventions into an otherwise closed system: if q_1 were accelerated by an external force, then the motion of a space-like separated charge q_2 would be different from what it is without the intervention. In principle (if τ_0 were not so extremely small) the causal connection between space-like separated events could be exploited to send superluminal signals. By measuring the acceleration of q_2, an experimenter could find out whether the space-like separated charge q_1 was accelerated or not, and therefore it should in principle be possible to transmit information superluminally in Dirac's theory.[7]

I can think of two objections to my claim that Dirac's theory allows superluminal signaling. First, one could argue that since the effect of the acceleration field of q_1 are 'felt' on the entire world line of q_2 prior to the point where the world line intersects the future light cone of q_1 at τ_1, one cannot really speak of signaling between space-like separated points, since no information that was not already available on the world line of q_2 is transmitted from q_1. But while I take it that signaling implies a causal connection, the converse does not hold. So even if it were impossible to send signals between space-like separated charges, this does not imply that the two charges cannot causally affect each other. Moreover, we can imagine a scenario in which two experimenters could use two charges to signal to each other. We only need to assume that the experimenter who is to receive the signal at τ_2 has a detection device that is not sensitive enough to detect the influence of q_1 on q_2 at times prior to τ_2.

[7] Dirac's theory is a relativistic theory since it is Lorentz-invariant. Thus, contrary to what is sometimes said, the special theory of relativity does not prohibit the superluminal propagation of causal processes. See Maudlin ([1994]) for a detailed discussion of the claim that the theory of relativity prohibits superluminal signaling or the superluminal propagation of causal processes.

Second, one could object to my appeal to counterfactual interventions.[8] Counterfactual interventions are strictly speaking miracles and one should not, it seems, draw any consequences concerning a theory's interpretation from what happens if miracles violating the theory's laws occur. We would, for example, not wish to draw any consequences for the causal structure of the theory from a miracle that created additional charges: even though the effect of the additional charge will be felt instantaneously over space-like intervals, this does not show that traditional Maxwell–Lorentz electrodynamics is non-local. But the intervention in the situation I am considering differs from one creating a charge in that it does not require an *electromagnetic* miracle, for the additional force accelerating the charge could be a non-electrodynamic force. Since the laws of electrodynamics are not violated in such an intervention, it appears legitimate to appeal to these laws in assessing the effects of the intervention.

Dirac's theory, then, violates two causal locality conditions: the condition that all action is by contact, and the condition that causal propagation takes place with a finite velocity. Both conditions are causal in an intuitive sense: the action-by-contact principle restricts causes to be contiguous to their immediate effects. And the propagation condition states that causal influences propagate at a finite speed. If the maximum velocity of causal propagation is the speed of light, this means that causes and their effects cannot be space-like separated.[9] Now, can either of these two causal locality conditions be explicated in non-causal terms? In the next section I will introduce a non-causal locality condition due to Belot that looks like a promising candidate for such a non-causal explication.

3 Belot's diachronic locality and electromagnetism

Belot ([1998]) distinguishes the following two locality conditions:

> (i) *Synchronic Locality*: the state of the system at a given time can be specified by specifying the states of the subsystems located in each region of space (which may be taken to be arbitrarily small).

[8] This objection was raised by an anonymous referee for this journal.

[9] One can distinguish several different locality conditions concerning the velocity with which effects propagate. Earman ([1987], p. 451) distinguishes the principle that all causal propagation takes place with a finite velocity from the strictly stronger principle that there is a fixed, finite limiting velocity for all causal propagation. The condition that there is no superluminal causal propagation is a specific version of the latter principle. Moreover, for each causal propagation principle there is a corresponding principle concerning signaling speeds, such as the principle that there is no superluminal signaling. Since signaling is a type of causal process, the principles concerning causal processes are strictly stronger than the respective signaling principles. Since the differences between these conditions are of no importance to what follows, I will for ease of exposition continue to speak as if there were just one locality condition restricting the velocity with which causal propagation takes place.

> (ii) *Diachronic Locality*: in order to predict what will happen *here* in a finite amount of time, Δt, we need only look at the present state of the world in [a] finite neighbourhood of *here*, and the size of this neighbourhood shrinks to zero as $\Delta t \rightarrow 0$. (p. 540)

The first criterion, according to Belot, is meant to capture the non-holist intuition that the properties of a (classical) system ought to be reducible to the properties of its parts. The second criterion is meant to be equivalent to the condition of finite signaling speed. Newtonian gravitational theory, for example, is diachronically non-local, Belot says, 'since gravitational effects propagate with infinite velocity' (*Ibid.*, p. 541), while (Maxwell–Lorentz) electrodynamics under the traditional interpretation is diachronically local, since 'electromagnetic radiation propagates at a fixed speed' (*Ibid.*). According to Belot, diachronic locality implies synchronic locality. Since he also says that if a magnetic field were allowed to act where it is not synchronic locality would be violated, it follows that, according to his account, the condition of diachronic locality implies the condition of action-by-contact.

Before investigating the relations between Belot's conditions and the two causal conditions in detail, I want to digress for a moment and comment on the case Belot himself discusses. Belot is interested in comparing various possible interpretations of Maxwell–Lorentz electrodynamics. According to the standard interpretation, the theory's basic ontology is taken to consist of electromagnetic fields and charged particles, which results in a theory which is both synchronically and diachronically local, as Belot says. The theory's mathematical formalism also makes use of what are known as *vector* and *scalar potentials*, but in the standard interpretation the potentials are treated as mere mathematical fictions, since they are not determined uniquely by the observable fields but only up to what is known as a *gauge transformation*. Thus, if we want the theory to be deterministic (in the sense that the evolution of the state of an electromagnetic system is completely determined by what, according to the theory, we can know about the state of the system), then the potentials cannot be interpreted realistically.

However, Belot argues that since the quantum mechanical Aharanov–Bohm effect shows that differences in the vector potentials can have empirically observable consequences, we should, for reasons of interpretive continuity, abandon the standard interpretation. The Aharanov–Bohm effect has taught us, he says, that we had 'misunderstood what electromagnetism was telling us about the world' (*Ibid.*, p. 532). Two alternative interpretations are suggested by the Aharanov–Bohm effect. Either, we can take the electromagnetic potential to represent a real physical field. Or, we take the state of an electromagnetic system to be given partly by the values of all closed-loop integrals—so-called *holonomies*—over the vector potential.[10] The second alternative has the advantage of retaining determinism, since the holonomies, unlike the potential, are gauge-invariant and are uniquely determined by the values of the fields, but it has the disadvantage of rendering classical electrodynamics radically non-local. If we interpret the holonomies, but not the vector potential, realistically, then the theory is synchronically (and hence, Belot says, also diachronically) non-local: since holonomies are 'spread out' in space, the state of the system at a given time cannot be specified by specifying the states of spatially localized subsystems.

Belot also claims that interpreting the potentials realistically results in a general diachronically non-local theory, but this claim is mistaken. As an example of how the potential supposedly depends non-locally on a change in current-density, Belot cites the case of a very long solenoid—a conducting wire coiled around a cylinder. If a constant current is running through the wire, then (according to the Biot–Savart law) there will be a constant magnetic field inside the device, while the magnetic field outside will be zero. Since the holonomy around a closed curve which loops around the solenoid is equal to the magnetic flux through the area enclosed by the curve, Belot argues that 'it follows that the vector potential propagates with infinite velocity' and that 'If we switch the thing on, then the values of the vector potential at some point arbitrarily far away must change instantaneously' (*Ibid.*, p. 549, n. 29). But of course no such thing follows, since the Biot–Savart law is a law of magnetostatics and does not apply to a case of time-dependent currents. The math in this case is a lot more complicated, since the magnetic field depends not only on the current density but also on the derivative of the current density. What we would find is that if we switch the solenoid on at $t=0$, there will be a non-zero electromagnetic pulse spreading out from the solenoid which ensures that the flux through any area with radius greater than ct is zero.[11]

Belot's mistake here might be partly responsible for the fact that he does not see his claim that synchronic non-locality implies diachronic non-locality to be in need of much discussion. However, on one natural construal of what

[10] The holonomies over the vector potential are, according to Stokes's theorem, equal to the magnetic flux through the area enclosed by the loop.

[11] In fact, whether or not the theory with realistically interpreted potentials is local depends on the choice of gauge. In the *Coulomb gauge*, in which the vector potential A is divergence-free—that is, satisfies $divA=0$—the scalar potential Φ is the instantaneous Coulomb potential due to the charge density ρ. Thus, in the Coulomb gauge Belot's criterion seems to suggest that the theory is diachronically non-local, since in order to find out how the scalar potential will change here we have to 'look at' the charge density arbitrarily far away from here. (The vector potential, however, is diachronically local even in the Coulomb gauge.) But in the *Lorentz gauge*, which treats scalar and vector potential on an equal footing (and, thus, is the natural gauge to use in a relativistic setting, where both potentials are components of a single four-vector potential A^{μ}), both potentials satisfy a wave equation and propagate at a finite speed c. Thus, in the Lorentz gauge the theory is diachronically local.

it is 'to predict what will happen *here*', one can only predict how local quantities change *here*, since non-local quantities, like holonomies, do not 'happen *here*' at all. But then synchronic non-locality does not entail diachronic non-locality. Since a synchronically non-local theory can be concerned with local quantities as well (witness electric fields on the holonomy interpretation of electromagnetism), such a theory can be diachronically local in the sense that what will happen *here* to the values of localized quantities can be determined locally. Moreover, even if we are willing to accept that holonomies are the kind of things that can happen *here*, electrodynamics is still local across time in an intuitive sense that Belot's criterion does not capture: even though holonomies are 'spread out' through space, changes in their values do not propagate instantaneously. Since the vector potential propagates at a finite speed, holonomies around paths far away from a disturbance in the field will change only after a finite time, when the disturbance has reached some point on the path.

4 Locality: let me count the ways

Now what are the relations between Belot's condition of diachronic locality and the two causal conditions? And how, if at all, are the two causal conditions related? Belot takes his condition to be equivalent to the principle of finite causal propagation. Interestingly, Belot's condition is identical to a principle proposed by Erwin Schrödinger as a non-causal analysis of the prohibition against action-at-a-distance. Since Schrödinger took this prohibition to be at the core of the very notion of causation, he calls his principle the 'principle of causality':

> The exact situation at *any* point P at a given moment is unambiguously determined by the exact physical situation within a certain surrounding of P at any previous time, say $t-\tau$. (Schrödinger [1951], p. 28)

Schrödinger adds that 'the "domain of influence" [that is, the 'surrounding' of P] becomes smaller and smaller as τ becomes smaller' and he maintains that '[c]lassical physics rested entirely on this principle' (*Ibid.*, p. 29).

Putting Belot's and Schrödinger's proposals together, we get the view that the condition that all action is by contact and the condition that causal propagation takes place with a finite velocity are equivalent. And an initial survey of physical theories might seem to support this view. Dirac's theory, as we have seen, violates both conditions, as does the paradigm example of a non-local classical theory—Newton's gravitational theory. Pure field theories, by contrast, which are paradigmatic examples of local classical theories, satisfy both conditions.

In fact, however, the two causal conditions are distinct and neither implies the other. On the one hand, the principle of action-by-contact does not imply the finite propagation condition, as the example of non-relativistic rigid body mechanics shows. In this theory, forces act only by contact, but since extended bodies are treated as rigid the action of forces on a body are transmitted instantaneously throughout the entire body.

On the other hand, Wheeler and Feynman's infinite absorber theory ([1945]) suggests that the converse implication fails as well. In Wheeler and Feynman's theory (which is a pure particle theory of electrodynamics), forces between distant particles are not mediated by an intervening field and are transmitted across gaps between particles, but nevertheless the force associated with the acceleration of one charge reaches the world line of another spatially-separated charge only after a finite time. Fields are treated in the theory as mere calculational devices (analogously to the treatment of gravitational fields in Newton's theory). Now, the Wheeler–Feynman theory does not itself provide us with a straightforward counterexample to the claim that the finite propagation condition implies the action-by-contact principle, since the equation of motion for a charge in the Wheeler–Feynman theory is the Lorentz–Dirac equation. Thus, the theory does permit superluminal signaling through the combination of forward causal and backward causal effects in the way I have discussed above. Still, the theory suggests how one could 'cook up' a theory that satisfies the finite propagation condition while violating the principle of action-by-contact. For example, a pure particle version of standard Maxwell–Lorentz electrodynamics which by analogy with the Wheeler–Feynman theory treated fields as mere calculational devices would be such a theory. Clearly, that causal influences take a finite time to propagate across spatial distances does not by itself imply that there is a medium in which the propagation takes place.

We can appeal to the same two examples to show that neither of the two causal principles implies Belot's condition of diachronic locality. A pure particle version of standard electrodynamics is diachronically non-local, even though the effects of one charge on another propagate at a finite speed. Since there is no field that transmits the effects that charges have on one another, they do not 'show up' in a small neighborhood of a test charge before they are felt by that charge. Thus, the present state of the world in a small, finite neighborhood of a charge does not allow us to predict what will happen to the charge next.

Similarly, rigid body mechanics is diachronically non-local, since in order to predict, for example, what will happen next to one end of a rigid rod, one always has to look at the entire rod and at whatever other objects are in its immediate vicinity. Thus, the size of the neighborhood of *here* at which we have to look does not shrink to zero as $\Delta t \rightarrow 0$. Yet the theory only allows for action-by-contact. So the condition of action-by-contact does not imply diachronic locality either.

One might think that, nevertheless, some logical connection between the different conditions exists and that the condition of diachronic locality is strictly stronger than both the condition of action-by-contact and that of finite signaling speeds. But as I want to argue now, the condition of diachronic locality implies neither of the two causal locality principles, since, perhaps surprisingly, Dirac's theory is diachronically local.[12]

5 Diachronic locality in Dirac's theory

It might appear that Dirac's theory is diachronically non-local simply because equation (2) is non-local, since (2) appears to indicate that one needs to look at the state of the world at all times later than τ in order to determine the acceleration at τ. However, this appearance is deceptive for two reasons.

First, (2) is not the only way of writing down an equation of motion in Dirac's theory. Since in the derivation of (2) one needs to assume that the field is an analytic function of the proper time τ, the field and acceleration functions can be expanded in a Taylor series and the non-local equation (2) is mathematically equivalent to the local equation (Rohrlich [1965], equation 6–87):

$$ma^{\mu}(\tau) = \sum_{n=0}^{\infty} \tau_0^n \left[F^{\mu}(\tau) - \frac{1}{c^2} a^{\lambda}(\tau) a_{\lambda}(\tau) v^{\mu}(\tau)\right]^n \tag{4}$$

Equation (4) involves only local quantities, and thus Dirac's theory appears to be diachronically local. According to the condition, a theory is local if it is *possible* to determine what will happen *here* by looking at the present state of the world close to *here*, and (4) shows that this is possible. But the possibility of representing the motion of a charge in terms of equation (4) instead of (2) does not affect the causal interpretation of the theory: even though it might be possible to calculate the acceleration of a charge from the Taylor expansion of the effective force, the force—and not any of its derivatives—is understood to be the cause of the acceleration. Thus (4) notwithstanding, Dirac's theory is *causally* non-local in the two senses I have distinguished.

One might want to object to this line of argument by claiming that the appeal to analyticity involves some kind of illegitimate trick. The fact that the field is represented by an analytic function, one might say, is merely an artifact of the mathematical formalism and has no physical significance. Thus, that the values of the derivatives of the field *here* allow us to determine

[12] The only logical connection that might exist between the notion of diachronic locality and the two causal principles is this: the conjunction of the action-by-contact principle and the condition of no superluminal propagation may imply diachronic locality.

the value of the field elsewhere does not imply that Dirac's theory is diachronically local. A consideration in favor of this response is that physicists treat the analyticity condition rather loosely, when they apply the theory. For example, two applications of Dirac's theory discussed by Rohrlich are the motion of a charge subject to a delta function field-pulse and a step function pulse, both of which violate the analyticity condition. If the analyticity condition was physically significant, then looking at situations in which the condition is not satisfied and in which, therefore, the theory does not apply is arguably not a good way of investigating what the theory tells us about the world. Moreover, we already know that relying too heavily on the analyticity condition is problematic: analyticity gives us determinism on the cheap (see Earman [1986], p. 15). Thus, since diachronic locality is a principle of determinism, it is (not surprisingly) automatically satisfied once we assume analyticity.

Yet even if in the end we want to reject arguments that rely solely on the analyticity condition, doing so without further discussion skirts some important issues. Given that the very derivation of Dirac's equation of motion relies on the analyticity condition, why is it that we should be allowed to rely on the condition in certain circumstances but not in others? Are there reasons for why considerations of analyticity can sometimes be discounted? Why should only (2) but not (4) be a guide to whether Dirac's theory is diachronically local, given that the two equations are mathematically equivalent? I believe that the correct answer to the last question is that (2) is privileged in that it represents the causal structure of the theory accurately. If I am right, this means that the condition of diachronic locality could not provide a non-causal explication of either of the two causal conditions, since it would have to be supplemented by the requirement that in predicting what happens *here* next we have to use an adequate causal representation of the phenomena.

A different question that arises in this context is this: precisely what quantities characterize the local state of a system *now* and can legitimately be used as inputs to predict future states? Belot's talk of properties which can be looked at is rather vague and is obviously meant only metaphorically, but perhaps one might try to respond to the difficulty raised by the equivalence between (2) and (4) in the following way: since derivatives represent *changes* of quantities, derivatives of the field function are not genuinely local quantities[13] and thus cannot be used as inputs in Belot's condition. But the problem with this suggestion (aside from the worry that it is far from clear why we cannot think of derivatives as genuinely local quantities) is that we now get non-locality too easily: even Newtonian mechanics would come out

[13] Albert ([2000], pp. 9–10) argues for such a view.

as diachronically non-local, since Newton's laws require velocities, which are derivatives, as inputs.

Dirac's theory comes out as diachronically local for a second reason—one that is independent of the analyticity requirement. Looking at the present state of a system *here* presumably reveals not only the charge's position and velocity but also the local value of the acceleration function to us.[14] Since disturbances in the electromagnetic field propagate at a finite speed, we can in addition determine the fields on the world line of the charge during a time interval Δt into the future by determining the fields now in a finite neighborhood of *here*, where this neighborhood shrinks to zero as Δt goes to zero. But then we can use equation (1), the Lorentz–Dirac equation, to determine the trajectory of the charge during the time interval Δt. The state of the charge *here* together with the field in a finite neighborhood of *here* allows us to predict what will happen *here* in a finite amount of time. This does not conflict with the fact that the theory is causally non-local in the two senses I have distinguished, since the effects of future fields—such as that of the field of a signaling charge q_1—are already encoded in the present acceleration of the charge q_2 here. Thus, even though we can determine the local evolution of the system from local data, this does not imply that what happens to the system is due only to locally-acting causes whose effects propagate at a finite speed.

6 Localizations of global models and an objection

I have argued that Belot's condition of diachronic locality is logically distinct from both the condition that causal propagation occurs at a finite speed and the condition that all action is by contact. Belot's condition is what one might call *a condition of local determinism*: we can predict what will happen here next, if and only if the evolution of a localized subsystem is completely determined by the local state of the system—the state of the system in a finite neighborhood of *here*. For if the evolution of a local subsystem is not completely determined by the local state, we cannot know what will happen *here* next without looking elsewhere; and similarly, if the evolution of the subsystem is determined by the local state, then we can use our knowledge of the local state to predict what will happen *here* next.[15]

[14] Since one could in principle measure the acceleration by determining the radiation field of the charge, there is a rather straightforward sense in which the acceleration *here* can be looked at. Presumably everything that can be measured can be looked at, but the converse does not hold.

[15] Could it not be that it is impossible for us to know what the local state of a system is and that, therefore, we are unable to predict what will happen *here* next, even though the state is determined locally? But Belot's criterion is clearly not meant to be an epistemological condition and his talk of looking and predicting must be meant metaphorically. The way Belot uses the terms, I take it, every real property of a system can be looked at and can be used to predict what will happen next.

I now want to discuss very briefly another condition that might be thought to provide a non-causal explication of one of the two causal locality conditions. John Earman has proposed the following condition as a possible explication of the action-by-contact principle: 'Every localization of a global model of T is again a model of T' (Earman [1987], p. 455), where a localization is a restriction of a model of T to a neighborhood U which is a subset of the manifold M on which the models are defined.[16] If a localization of a global model is again a model of the theory, then it does not matter to local properties of the system whether these properties can 'see' the values of quantities far away. That is, if a localization is itself a model of the theory, then all properties of a local subsystem are completely determined by the values of local quantities. Thus Earman's condition, like Belot's, is a condition of local determinism. Yet Earman's condition is distinct from the principle of diachronic locality, as the example of non-relativistic rigid body mechanics shows: the theory is, as we have seen, diachronically non-local, yet it satisfies Earman's condition.

Does Dirac's theory satisfy Earman's condition? The answer is: that depends, once more, on which equation we take to be the fundamental equation of motion of the theory. If the non-local equation (2) is taken to be the fundamental equation of motion, then Dirac's theory does not seem to obey Earman's condition. According to (2), the acceleration of a charge which only 'sees' the fields in a finite neighborhood U of the charge is in general different from the acceleration in a global model (and hence in a localization of the global model), in which the acceleration also depends on future fields outside of U.[17] Thus, a localization of a global model will not in general be a model of Dirac's theory. If, however, the local equations (1) or (4) were to be taken to be the fundamental equation of motion, then the theory would satisfy Earman's condition. Since according to (4) the acceleration is determined from the local values of all the derivatives of the field, a localization of (4) is itself a model of (4). Dirac's theory would come out as local, according to Earman's condition, even though it violates the action-by-contact principle. Now, we have already seen that one way to decide which of the two equations is fundamental is to appeal to the causal structure of the theory: (2) is more fundamental, since it gives the acceleration in terms of its causes, that is, the electromagnetic fields. But if Earman's

[16] Earman himself says that the condition 'captures a good part of the content of the action-by-contact principle' (*Ibid.*).

[17] Since (2) only gives the acceleration as a function of the fields *everywhere* on the future world line, I am assuming that the restriction of the fields to a finite neighborhood U could be obtained by multiplying the field function by a step function whose value is one inside of U and zero outside. In fact, it is not obvious how one should apply Earman's criterion to theories with 'global' equations of motion like equation (2). One cannot simply restrict (2) by changing the limits of integration, but if one 'localizes' the fields by multiplying them by a step function, the resulting field-function is no longer analytic.

condition needs to be supplemented by considerations concerning the causal structure of the theory, then it cannot provide us with a strictly non-causal explication of the action-by-contact principle.

The failure of Belot's and Earman's principles of determination to provide adequate explications of the two causal locality conditions suggests that there is more to causation than determination. And this in turn implies that in the case of those scientific theories that make causal claims, the job of an interpretation cannot be exhausted by stipulating how the mathematical formalism maps onto the ontology of the theory. For a theory's causal structure will generally be constrained by the mathematical formalism but not uniquely determined by it. In classical electrodynamics, for example, it is part of the causal structure that force (and not any of its derivatives) is the cause of acceleration; but this cannot be inferred from the theory's formalism alone. Whether a theory is causally non-local in either of the two senses I have distinguished depends crucially on the causal interpretation of the theory. Yet whether a theory satisfies a condition of local determinism depends only on the theory's mathematical formalism (and the associated ontology).

At this point the following response suggests itself: the fact that the intuitive causal claims associated with a scientific theory can outrun what can be legitimately inferred from the theory's mathematical formalism and the fact that taking the causal locality principles too seriously can lead to rather strange results (as in the case of the putatively backwards causal theory of Dirac) only further support Russell's view that a rich notion of causality which cannot be reduced to that of functional dependency should have no place in science. But what this reply misses is that a theory's causal interpretation can play a significant methodological role. In the case of Dirac's theory (as we have seen above) causal assumptions play an important role in motivating various steps in the derivation of the theory's equation of motion.

First, the assumption that field forces are the cause of a charge's acceleration makes plausible the adoption of what I have called *the weak principle of inertia*, according to which the acceleration of the charge should vanish far away from any forces. This principle then helps to motivate the asymptotic condition of vanishing accelerations at infinity. Without the causal framework, the asymptotic condition can only be given a purely mathematical motivation: one can renormalize the mass only if the condition is presupposed. Assuming that fields cause charges to accelerate provides a physical reason for the condition as well.

Second, the causal assumption helps to motivate the rejection of the runaway solutions as unphysical. For given that fields cause a charge to accelerate, a charge should not accelerate in the absence of any external fields. We can contrast the case of the runaway solutions in the absence of any external fields with that of a charge in a step-function field. In the latter case the charge begins to accelerate even before the field turns on. This, on the standard interpretation, is an example of the backward causation allowed by Dirac's theory. Even if backward causation might strike one as somewhat problematic, the fact that one can point to the future field as cause of and reason for the acceleration of the charge makes this situation physically more palatable than that of the runaway solutions.

7 Conclusion

One of my two aims in this paper was to correct what seems to be a widespread misconception of classical electrodynamics. By contrast with Newton's gravitational theory and quantum theories, classical electrodynamics is thought to satisfy various (often only vaguely characterized) locality principles and is generally taken to be the paradigm of a well-behaved classical theory. I have argued that this characterization is highly misleading. In fact, we must distinguish between two quite different approaches to microscopic classical electrodynamics, neither of which measures up to our ideal of a classical theory. Maxwell–Lorentz electrodynamics, on the one hand, is indeed causally well-behaved and is local in every interesting sense. But that theory is inconsistent. Dirac's consistent microscopic electrodynamics, on the other hand—the theory on which I have focused here—is backward causal and causally non-local.

My second aim was to distinguish several different locality conditions and to argue that there are two distinct irreducibly causal principles of locality—the principle that all causal propagation takes place at a finite velocity and that all action is by contact. In particular, I have argued in detail that neither of the two causal principles can be reduced to Belot's non-causal condition of diachronic locality. Since any non-causal explication of the two *prima facie* causal principles would apparently have to invoke some principle of local determinism, and since the concept of causation does not seem to be reducible to that of determinism, the prospects for a successful empiricist reduction of either of the two causal locality principles appear to be dim.

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