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Maddy and Mathematics: Naturalism or Not

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ABSTRACT

Penelope Maddy advances a purportedly naturalistic account of mathematical methodology which might be taken to answer the question 'What justifies axioms of set theory?' I argue that her account fails both to adequately answer this question and to be naturalistic. Further, the way in which it fails to answer the question deprives it of an analog to one of the chief attractions of naturalism. Naturalism is attractive to naturalists and nonnaturalists alike because it explains the reliability of scientific practice. Maddy's account, on the other hand, appears to be unable to similarly explain the reliability of mathematical practice without violating one of its central tenets.

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1 Introduction

Consider a familiar arithmetic belief, say, the belief that there are infinitely many primes. Most of us agree that we are justified in holding this belief. Many of us have difficulty resisting the urge to ask why we are so justified. In virtue of what are we justified in holding such familiar arithmetic beliefs? For a belief such as that there are infinitely many primes, something like the following is often offered. The appropriate translation of 'there are infinitely many primes' into the formal language of arithmetic is a logical consequence of the axioms of arithmetic, and since we are justified in believing those axioms, and deductive logic preserves justification, we are also justified in believing

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that there are infinitely many primes. The troublesome among us, however, are not satisfied with this answer. We have just pushed the question back a step. For what justifies the axioms of arithmetic? A response in keeping with this line of thought is to point out that suitable translations of the axioms of arithmetic are provable from the axioms of set theory. But, of course, now we are faced with the question of what justifies axioms of set theory.

This is one way we might get worried about the justificatory status of set-theoretic axioms. Here is another. The standard axioms of set theory, i.e., the axioms of set theory currently widely accepted, are the so-called ZFC axioms—the Zermelo–Fraenkel axioms with the Axiom of Choice.[1] A set-theoretic statement σ is *independent* of ZFC if neither σ nor its negation is provable in ZFC.[2] When σ is independent of ZFC, we say that *the question of* σ is independent of ZFC. It turns out that a considerable number of questions that arise in the course of ordinary research in areas of mathematics like analysis, algebra, and topology are independent of ZFC.[3] It further turns out that we can settle these questions by supplementing ZFC with new axioms. The problem is that we can often plausibly settle a given question both positively and negatively, depending on how we supplement ZFC. Thus, though we reasonably desire to settle independent questions, doing so often requires choosing between competing axiom candidates. The natural and obvious strategy is to choose the candidate that is best justified. Thus, again we are led to ask what justifies axioms of set theory.

Penelope Maddy has advanced a view of mathematics and its methodology, *Mathematical Naturalism*, that *prima facie* intends to deliver a naturalistic way of justifying currently accepted as well as proposed new set-theoretic axioms.[4] According to this view, mathematics can be neither criticized nor defended

1 For textbook treatments of these axioms, see, e.g., (Kunen [1980]) or (Enderton [1977]).

2 It is common to use 'ZFC' ambiguously to denote both the axioms of ZFC and the closure of those axioms under logical consequence (i.e., the formal theory ZFC). I will keep with this practice; which use I intend in a given case should be clear from context.

3 Some examples: From analysis: Are there nonmeasurable Σ^1_2 sets of reals? Are there uncountable Π^1_1 sets of reals with no perfect subset? From topology: Suslin's problem. From algebra: Is every uncountable Whitehead group a free group?

4 Though Maddy tends to concentrate on new axiom candidates, she indicates in several places that the justificatory status of currently accepted axioms is not so different from that of new axiom candidates. For example: In (Maddy [1988]), she writes: 'I want to counteract the impression that [the ZFC] axioms enjoy a preferred epistemological status not shared by new axiom candidates' (p. 482). In (Maddy [1998a]), she poses the question, 'what justification can properly be offered for [a set-theoretic] axiom candidate?' and immediately thereafter notes that the problem raised by this question 'arises just as inevitably for the familiar axioms of ZFC. On what grounds do we justify the adoption of these axioms?' (pp. 161–2). And in (Maddy [1996]), she declares that her 'focus will be on the justification of set theoretic claims that are not directly supported by proofs, in particular, on *axioms and independent statements*' (p. 490, emphasis added).

on extra-mathematical grounds, philosophical or otherwise; mathematical practice is responsive only to mathematical considerations. In this article, I argue that Mathematical Naturalism does not deliver.

A tension runs through much of Maddy's work on naturalizing mathematics. On one hand, she uses explicitly epistemological language. For instance, in framing her project in the introduction to (Maddy [1997]) she writes that she is interested in the question 'what justifies the axioms of set theory?' (p. 2), where answering this question is understood as one arm of the project of giving an account of 'our much-valued mathematical knowledge' (p. 1). This naturally leads one to think that she is engaged in giving an epistemology for mathematics. On the other hand, Maddy sometimes writes as if her view is compatible with there being no fact of the matter regarding the truth and falsity of mathematical claims.[5] But if there is no fact of the matter with respect to truth and falsity in mathematics, that undermines the project of giving an epistemology of mathematics. Epistemology is centrally concerned with systematic connections between justification and truth. If there is no fact of the matter as to whether claims concerning Fs are true or false, then there simply is no question of systematic connections between what justifies our F-beliefs and the truth about Fs.[6] Maddy moves toward resolving this tension in (Maddy [2005a]), where she introduces the metaphysical positions *Thin Realism* and *Arealism*. In Sections 1–3, I will consider Maddy's position understood as an attempt to provide an epistemology for mathematics. In Section 4, I will entertain the possibility that Thin Realism enables the Mathematical Naturalist to answer my critique of her position. Finally, in Section 5, I will briefly comment on Arealism and on Maddy's position taken as unconcerned with epistemology.

2 Mathematical naturalism

Maddy's Mathematical Naturalism stems from her conviction that mathematics is not subject to criticism from without, that mathematics 'should be understood and evaluated on its own terms' (Maddy [1997], p. 184) and that 'if our philosophical account of mathematics comes into conflict with successful mathematical practice, it is the philosophy that must give' (Maddy

[5] See, e.g., the discussion of whether the Continuum Hypothesis has a determinate truth value on p. 202 of (Maddy [1997]) and (Maddy [2005a]), where Maddy suggests that 'true' applied to mathematical claims generally is merely an honorific. See also the discussion in Section 6.

[6] For more on this, see the discussion of reliability in Section 3. Notice that this point cuts across ontological views. Nominalists and realists about mathematics both think that there is a fact of the matter about truth and falsity in mathematics. They just disagree about what makes this the case.

[1997], p. 161). The proper role of philosophy of mathematics is entirely descriptive: philosophy, being external to mathematics, is in a position neither to criticize nor to defend the methods and claims of mathematics; it may only analyze actual mathematical practice. Through such analysis, philosophers of mathematics may be able to contribute to settling mathematical debates such as those surrounding the status of independent questions by revealing methodological maxims or constraints and applying them to the relevant questions, but in this capacity they function as mathematicians rather than philosophers. In Maddy's words, 'Given that a naturalistic philosopher brings no special modes of argument from philosophy, every argument she gives must be based on modes of argument available to any mathematician qua mathematician; at best, she will make explicit what is already implicit' (Maddy [1998b], p. 137).[7] In order to make explicit what is already implicit in a practice, the methodologist of mathematics (as Maddy sometimes refers to herself) constructs an amplified, naturalized model of that practice.

A naturalized model $\mathcal{M}_P$ of a practice P is a description of P constructed by extrapolating from a range of cases of debate and resolution in P's history, a description from which all considerations that did not contribute to resolving a debate have been purged. Maddy refers to a model purged of methodologically irrelevant material like this as *purified*. All naturalized models are purified. By abstracting away from features specific to the content of the arguments contained in a naturalized model $\mathcal{M}_P$, one thereby makes explicit general styles of argument operative in P.[8] This abstraction process Maddy calls *amplification* (or *enhancement*). Let $\mathcal{M}_P^*$ be the result of amplifying $\mathcal{M}_P$. An amplified, naturalized model of a practice lays bare the argumentative strategies that have successfully been employed in P in the past. According to the Mathematical Naturalist, this provides a way of addressing current open questions of the practice: apply those same argumentative strategies. A toy example will help fix these ideas.

The Axiom of Choice (AC) says that for every family of nonempty, pairwise disjoint sets $\mathcal{F}$ there is a set C that intersects every member of $\mathcal{F}$ in exactly one element. Suppose that the history of the acceptance of AC is representative of the history of algebra, with respect to decisions to accept or reject controversial claims. Consider a simplified version of a decisive episode in the history of AC's adoption.

7 See also III.4 of (Maddy [1997]), especially p. 199.

8 Maddy uses this plural, but she focuses on one style of argument in her discussions, viz., means-ends reasoning.

AC has a controversial history. It is a powerful set-existence axiom which has seemed to many mathematicians considerably less obvious than the other axioms of ZFC. The feeling of unease that sometimes accompanies AC is at least in part owed to its having some highly counterintuitive consequences (e.g., the Banach–Tarski paradox).[9] This unease was so great in the middle of the last century that van der Waerden removed AC (and its consequences) from the second edition of his influential text *Modern Algebra*. However, AC proved indispensable to algebra, in the sense that algebra without AC was much impoverished compared to algebra with AC. (For example, the proof that every vector space has a basis requires AC.) This (along with the resulting outcry from algebraists) was enough to lead van der Waerden to reverse his decision and reinstate AC in the third edition of *Modern Algebra*.[10]

Looking at this episode we can identify having a rich theory as a goal of algebra and the availability of AC as a (partial) means to that goal. We can also see that AC's furthering of this goal made it rational for the algebra community to adopt AC. These observations yield a naturalized model $\mathcal{M}_{Algebra}$ of (a part of) algebraic practice. If we now abstract away from the particulars of the case, we see something like the following in play: if adopting some principle would help attain a particular goal of the practice, then that principle should be adopted. This is the result of amplifying $\mathcal{M}_{Algebra}$ to $\mathcal{M}^*_{Algebra}$. This sort of picture illustrates the method advanced by Maddy for assessing the justificatory status of set-theoretic axioms.

Based on her own examination of the history of set theory, Maddy constructs a naturalized model of set-theoretic practice, $\mathcal{M}_{ST}$, and amplifies it to $\mathcal{M}^*_{ST}$. She then recommends that we assess the justificatory status of candidate axioms in the following way. Confronted with a set-theoretic axiom candidate α, we should identify goals of set-theoretic practice (by consulting $\mathcal{M}_{ST}$) and ask whether or not there is some current goal G of the practice such that adopting α is likely to be helpful in achieving G, and more so than anything that might be adopted in place of α. We ask this because consulting $\mathcal{M}^*_{ST}$ reveals that this sort of means–ends strategy of argument has been successful in the past. If adopting α is likely to be helpful in achieving G, and more so than anything that might be adopted in place of α, then acceptance of α is justified; if not, then it is not. So according to Mathematical Naturalism we should 'frame a defence or critique of a given [axiom candidate] in two parts: first, identify a goal (or goals) of [set-theoretic] practice, and, second, argue

[9] The Banach–Tarski paradox is a theorem that implies that a solid sphere can be decomposed into finitely many pieces which can then be assembled into two spheres, each of which is equal in size to the original sphere.

[10] For more on this episode, see (Moore [1982], p. 232).

that [adopting the axiom] in question either is or is not an effective means towards that goal' (Maddy [1997], p. 197).[11,12]

3 Desiderata and the attraction of naturalism

Maddy is concerned with justifying axioms of set theory in a naturalistically acceptable way. That is, she wants to answer the *justifying axioms question*:

(JAQ) What justifies axioms of set theory?

Mathematical Naturalism is supposed to provide a naturalistic answer to JAQ. Thus we have the following two desiderata:

(D_1) Mathematical Naturalism should adequately answer JAQ.

(D_2) Mathematical Naturalism should be a form of naturalism.

Note that 'Mathematical Naturalism' is a proper name denoting Maddy's approach to answering JAQ, while 'naturalism' is a term applying to a family of philosophical views sharing a general reverence for the sciences and a rejection of Cartesian foundationalism and the supranatural, i.e., whatever goes beyond the natural. Moreover, notice that even though 'naturalism' means different things to different people, we are not involved in a superficial dispute over terminology.

Satisfying D_1 is substantive, i.e., it is not merely terminological. In order to satisfy D_1, Mathematical Naturalism must not only identify the epistemic norms and standards of set theory, it must also have the resources to explain the reliability (i.e., approximate truth-conduciveness) of judgments made in

11 Extending the terminology of Paseau ([2005]), we can say that Maddy's position is both a superstrong reinterpretation naturalism and a superstrong reconstruction naturalism. The former holds that the standard interpretation of mathematics cannot be legitimately revised or sustained on non-mathematical grounds; the latter holds that mathematics itself cannot be legitimately revised (reconstructed) or sustained on non-mathematical grounds. (Cf. Paseau [2005], p. 381.) Simplifying somewhat, if we consider mathematics as a set of statements Θ in a first-order language (say, the language of set theory), then a reinterpretation of mathematics is a change in the way the members of Θ are standardly understood (e.g., along platonist lines, along structuralist lines, or along fictionalist lines) and a reconstruction of mathematics is a change in the membership of Θ. Note that Paseau's use of 'interpretation', which I follow here, is not model-theoretic. I think it is dubious that mathematics actually has a standard interpretation in the sense intended by Paseau, but I bracket that concern here.

12 Maddy's current view differs from the set-theoretic realism she defends in (Maddy [1990]) in that the latter attempts to naturalize the epistemology of mathematics by naturalizing its ontology and deploying Quinean naturalism, in the form of theoretical virtues such as explanatoriness and simplicity, while the former rejects Quinean holism (part and parcel of Quine's naturalism) and endorses ontological neutrality for mathematics (see Section 4.2). Where set-theoretic realism exhibits a continuity between epistemology and ontology characteristic of Quinean naturalism, Mathematical Naturalism deliberately breaks this continuity with the intention of leaving the epistemological and ontological affairs of mathematics to mathematics plus the recognition that mathematics does not determine the nature of its *prima facie* ontology.

accordance with those norms and standards. This reading of D_1 echoes ([1990]), where Maddy writes:

> Even if reliabilism turns out not to be the correct analysis of knowledge and justification, indeed, even if knowledge and justification themselves turn out to be dispensable notions, there will remain the problem of *explaining the undeniable fact of our expert's reliability*. In particular, even from a completely naturalized perspective, the Platonist still owes us an explanation of how and why [our leading contemporary set theorists'] beliefs about sets are reliable indicators of the truth about sets. (p. 43, emphasis added)

I take it that Maddy's point here is independent of both reliabilism and platonism: an adequate epistemology of mathematics must at least provide for (if not outright provide) an explanation of the reliability of mathematical practice, as manifested in the (mathematical) judgments of expert practitioners.[13] Similar explanatory conditions on epistemological adequacy generally are found in (Field [1989], p. 233) and (Boyd [1973], p. 3).

Neither is satisfying D_2 merely a terminological issue. For better or worse, 'naturalism' has become an honorific of sorts. To scientifically minded philosophers, naturalistic theories are *ceteris paribus* preferable to nonnaturalistic theories. Two reasons for this stand out. First, naturalistic theories do not traffic in mysterious entities (e.g., deities) or faculties (e.g., Gödelian intuition) which tend to embarrass such philosophers. Second, naturalism in the Quinean tradition has a pretty convincing story to tell about what makes the norms and standards of scientific practice reliable, a story that, of course, does not invoke mysterious entities or faculties. Hence, satisfying D_2 goes some way toward satisfying D_1, and satisfying D_1, as already noted, is not a terminological matter.

The ability to account for the reliability of the norms and standards of scientific practice without trafficking in mysterious entities or faculties is one of the chief attractions of naturalism. It will be helpful to see how this goes in a bit more detail.

Naturalistic epistemologists hold that a (perhaps the) central task of epistemology is accounting for our acquisition of a reasonable theory of

[13] The distinction here between reliabilism and reliability is real and significant. Reliabilism is the position according to which (roughly) S is justified in believing that p just in case S's belief that p is produced and sustained by reliable processes. Here reliability is both necessary and sufficient for being justified. The reliability that needs to be explained (or at least be explicable) by an adequate epistemology is the reliability of the norms and standards of justification endorsed by that epistemology, in the sense that beliefs held in accordance with those norms and standards tend to be (approximately) true. Here reliability is a necessary, but not sufficient, condition on being justified. This being the case, certain problems that have been raised for reliabilism, e.g., the New Evil Demon problem (see Cohen [1984]), are irrelevant in the present context.

the world. How is it possible for us to know what we (take it we) know about the world? More specifically, the central task of epistemology according to the naturalist, is answering the *naturalist's epistemological question*:

(NEQ) How are we justified in believing what we (justifiably) do about the world?

Naturalism is able to answer NEQ because:[14] (i) the family of disciplines that fall under the heading 'science' (broadly construed to include natural and social sciences plus the mathematics and logic applied in the practice of these sciences) is large enough and varied enough that meaningful criticism of one discipline can be mustered in another while remaining within science; (ii) naturalism, in attempting to account for the success of our inductive practices, countenances a propensity in us for mapping the world—specifically, along the lines drawn by natural kinds; and (iii) combining (i) and (ii) yields an account of the reliability of our inductive practice, and hence, of scientific practice.[15]

Consider (i). While science as a whole is insulated from outside criticism on the naturalist's view, individual branches of science (e.g., physics, biology, psychology, and sociology) are not insulated from each other. So, for instance, a critique of physics mounted from within psychology would count as an intra-scientific criticism. So the naturalistic epistemologist answers NEQ 'in disregard of disciplinary boundaries, but with respect for the disciplines themselves and appetite for their input' (Quine [1995], p. 16). There is a holism here that goes beyond the meaning and confirmation holisms normally (and rightly) associated with Quinean naturalism. I call this kind of holism *disciplinary holism*. As a result of disciplinary holism, the claim that the only legitimate criticisms of the sciences are intra-scientific is not the obviously objectionable claim that the sciences are immune to criticism. Meaningful and robust criticism is afforded under naturalism. The ability to draw on multiple disciplines, each with a relatively distinct perspective, presents the potential for conflicting evidence. Allowing for conflicting evidence significantly reduces the chances of going astray. In addition, naturalism opens the way for mutual support among disciplines. This is because the lack of insulation between disciplines sets the stage for converging evidence, i.e., information from multiple disciplines all pointing to the same conclusion, and the presence of converging evidence significantly enhances the chances of getting things right.

14 These considerations were initially suggested to me by Richard Boyd.

15 See, e.g., (Quine [1969], pp. 123–9). I actually think that Quine's ontological relativity undermines the aspects of his realism that are germane to what I am about to argue. But Quine certainly writes as if it is otherwise, and there are Quinean naturalists such as Boyd and Hilary Kornblith whose positions do support the relevant arguments. This being the case, I will bracket my concern about Quine's ontological relativity.

Turning to (ii): Naturalism has it that our inductive practices are underwritten by our appreciation, conscious or not, of natural kinds. Successful inductions are those done on projectible properties of (or predicates applied to singular terms denoting) objects, and the naturalist, following Quine, holds that '[a] projectible predicate is one that is true of all and only the things of a [natural] kind' (Quine [1969], p. 116). Thus, our ability to successfully engage in induction is linked to our ability to tell projectible predicates from nonprojectible ones, which is in turn linked to our ability to track general features of the world, viz., groupings of objects by kind. (Note that I am not claiming infallibility in or awareness of these tasks.) So since naturalists are generally realists about natural kinds, naturalism, in its account of our inductive practices, takes a realist stance toward the general *prima facie* subject matter of the sciences.[16]

As to (iii): The meaningful cross-discipline criticism and support allowed for, indeed encouraged, by disciplinary holism, together with the realist underpinnings of our inductive practices, enables us to explain the reliability of those practices. Projectibility judgments are made on the basis of theory. Theories are refined in part as a result of the inter-disciplinary cross-talk facilitated by disciplinary holism. Thus, projectibility judgments are in part a result of this cross-talk. This is the contribution of (i). Furthermore, those judgments reflect our appreciation of natural kinds. This is the contribution of (ii). Consequently, when we engage in induction we engage in a process that has significant purchase on the world. That is why the deliverances of induction tend to be (at least approximately) true, i.e., why inductive, hence scientific, practice is reliable. Since naturalists tend to be reliabilists,[17] this yields an account of why we are justified in believing the deliverances of science, i.e., what we do believe about the world. That is all the naturalist needs to answer NEQ.

As an example, consider how are we justified on this view in believing, for instance, that a mushroom of a certain size and shape and with certain markings is hallucinogenic.[18] The size, shape, and markings of the mushroom indicate that it is of kind K, which is associated with a stable cluster of properties $\mathcal{C}$. The stability of $\mathcal{C}$ is a product of the particular microstructure of mushrooms of kind K. That this is the case and that mushrooms of kind K, in general, have a similar microstructure are theoretical findings involving (at a minimum)

16 Notice that I am not here identifying naturalism and realism. Rather, I am observing that naturalists tend to be realists and arguing that their realism contributes in an important way to the success of their naturalism. For more detailed arguments in this vein, see the first chapter of my ([2005]).

17 See, e.g., (Boyd [1980], [1988]; Goldman [1976], [1979], [1986]; Kitcher [1983], [1992]; Kornblith [1993], [1994]).

18 I here draw on (Boyd [1988], [1991]; Kornblith [1993]).

physics, chemistry, biology, botany, physiology, and psychology. This theory-dependent stability of $\mathcal{C}$ underwrites the judgment that the properties in $\mathcal{C}$ are projectible, i.e., that they support induction. Thus, our belief that a mushroom is hallucinogenic, when inferred from its having the appropriate size, shape, and markings, is likely to be correct and so is reliable, or, in other words, is naturalistically justified.

Some terminology will be convenient going forward. Call an epistemology for a practice *P dissident* if and only if it not only addresses questions concerning the modes of justification (i.e., epistemic norms and standards) operative in *P* but also has the resources to address the question of the reliability (i.e., approximate truth-conduciveness) of those modes of justification. Call an epistemology for a practice *P quietist* if it is not dissident. In these terms, we have just seen that naturalism provides a dissident epistemology for science. To satisfy D_1, Mathematical Naturalism must provide a dissident epistemology for mathematics.

By reflecting on naturalism's answer to NEQ, we can isolate the elements of naturalism that are central to its ability to supply a dissident epistemology of science. The first of these elements is explicitly identified in discussing (i): disciplinary holism. For the naturalist, while science as a whole is a closed system, the disciplines collectively constituting science are open to and engaged with one another. Intra-scientific cross-checking is part of good scientific methodology: no branch of science is allowed to insulate itself from other branches of science. When the results of different branches of science appear to be in conflict, all branches concerned will move to resolve the conflict (perhaps by showing that it is merely apparent). When the results of different branches of science appear to be converging on a single view, all branches concerned derive some support from the convergence.

One might balk at the suggestion that disciplinary holism is necessary for giving a dissident epistemology of science. Might not some branches of science be capable of internally accounting for the reliability of the epistemic norms deployed in them? For instance (so the worry goes), might not we explain the reliability of perception in terms of fundamental physics and physiology, even though those disciplines themselves are ultimately founded on perception, and in doing so give an account of the reliability of physics and physiology in terms of themselves?[19] There are multiple responses to this worry.

First, notice that even if the reliability of perception could be accounted for in terms of only physics and physiology—leaving out basic logic and mathematics, psychology, chemistry, and biology, to name just a few likely contributors to such an account—a healthy measure of disciplinary holism is still in play. Neither physics nor physiology alone is sufficient and both

[19] Thanks to an anonymous referee for raising this possibility.

arguably incorporate and deploy multiple sub-disciplines in giving the sort of account under discussion. Physics will deploy, e.g., optics as well as fundamental results about the microstructure of matter and surfaces. Physiology will deploy, e.g., parts of biology, chemistry, and neuroscience. Indeed, in the case of physiology, one might argue that the field itself, by its very nature, incorporates a measure of disciplinary holism.

Second, even if physics and physiology are in some sense founded on perception, it does not follow that an account of the reliability of perception yields an account of the reliability of the norms or methods of physics and physiology. Those methods include, at a minimum, ways of choosing between empirically equivalent theories, which underdetermination considerations show go beyond straightforward matters of perception.

Third, an account of the reliability of perception must bridge theory and the world. This bridge is provided by a causal theory of detection (about which more below). The naturalist can avail herself of such a theory owing to the *in media res* character of naturalism plus the fact that a causal theory of detection plays a central role in science. But this role, as well as its centrality, is supported by the (often tacit) reliance on a causal theory of detection in every branch of empirical science. This is the sense in which it is reasonable to say that physics and physiology, in addition to biology, chemistry, psychology, neuroscience—even sociology and economics—ultimately depend on perception. The experience on which empirical science depends is perceptual experience, broadly construed to include detection (indirect perception) by instruments, but empirical science only fulfills its primary mission, i.e., to tell us about the world, if that experience is causally connected to the world. The belief that this is so is both of chief importance to an account of the reliability of perception and available largely due to its widespread, entrenched use across scientific disciplines. Thus, a naturalistic account of the reliability of sense perception incorporates disciplinary holism.

This leads to the second central element of the naturalist's answer to NEQ: a causal theory of detection. Theories are formulated, tested, and refined on the basis of our causal interactions with the world. Our confidence in the accuracy of observations of medium-sized objects of the sort that prompt the initial formulation of a theory depends (often tacitly) on a causal theory of perception, the ground level of detection. We formulate a theory to help explain why water boils at 100 degrees Celsius under standard pressure because we believe that water boils at 100 degrees Celsius under standard pressure. We believe that water boils at 100 degrees Celsius under standard pressure on the basis of observing water to boil at 100 degrees Celsius under standard pressure and the belief that such observations typically result from a causal interaction between us and the world. Testing and refining a theory of any sophistication invariably involves the use of detecting instruments, beliefs in the reliability of which are based on our beliefs that such instruments do what they do in virtue of causally interacting with the relevant part of the world. The naturalist's account of theory testing and confirmation would not get off the ground without a causal theory of detection. Moreover, it is due to the causal component of the naturalist's theory of detection that theories are reasonably taken to be concerned with facts about and features of the world that are independent of us and our theorizing. It is this that gives us confidence that our theories manage to reach beyond mere artifacts of cognition and culture and that underwrites the deference to nature noted below.

The third element of naturalism central to its dissident answer to NEQ is a conception of natural kinds according to which (a) kind definitions are theory-dependent and (b) natural kinds reflect or align with the causal structure of the world.[20] A dissident answer to NEQ, i.e., an answer to NEQ, links the norms and standards of science to (approximate) truth. According to naturalism, this linkage is made with the help of natural kinds. The worrisome component of scientific practice is induction. The norms and standards of science license induction on predicates we judge to be projectible.

Projectibility judgments are informed by theory: they are judgments of theoretical plausibility in the sense that a hypothesis is projectible 'just in case it is supported by plausible inductive inferences from the "observational" and "theoretical" claims embodied in previously well established theories' (Boyd [1991], p. 136), and projectible predicates are those that figure in projectible hypotheses. Thus, which predicates count as projectible depends on the 'well established theories' one has on hand. Furthermore, projectible predicates correspond to natural kinds, since the classification of things into kinds '[reflects] a strategy of deferring to nature in the making of projectability [sic] judgments' (Boyd [1991], p. 139). In addition, the deference to nature mentioned here is such that the resulting kinds 'reflect the actual causal structure of the world' (Boyd [1991], p. 139). Lightly modifying some terminology found in (Millikan [1999]), we can say that the causal structure of the world is the *ontological ground* of induction (in the sciences), in the sense that it is in virtue of that structure being aligned with natural kinds that inducing on kinds results in (approximate) truths. Were kinds not to answer to anything ontologically robust, successful induction would be merely accidental. So, according to the naturalist, the norms and standards of science license induction guided by theory-dependent classifications of things into kinds which reflect the causal structure of the world, and in this way

[20] See, e.g., (Kornblith [1993]; Boyd [1980], [1988], [1991]).

our inductive practices align with the causal structure of the world so as to make those practices generally (approximately) truth-tracking. In short, induction (typically) works because it is ontologically grounded in the causal order.

Notice that that this account of the reliability of scientific practice requires a commitment to a realist conception of causation, one such that causal powers and processes exist in the world independently of us and our theorizing and may legitimately be appealed to in our philosophical theorizing independent of any reductive analysis. Call this conception of causation *causal realism* (CR). By taking CR on board, the naturalist gains a license for explaining the reliability of certain types of interactions in causal terms without thereby lashing that explanation to a future (eliminative) analysis of causation. More importantly, accepting CR clears the way for the adequacy of such an explanation. Reliability has to do with truth—not perceived truth or judged truth, but truth *simpliciter*. An adequate explanation of the reliability of certain types of interactions in terms of causation (i.e., causal powers, processes, or structure) must give us reason to think that beliefs formed as a result of the right types of interactions are true in a robust sense. A realist conception of causation can do this; a neo-Kantian conception of causation, according to which the causal order is in some sense imposed on the world by our theorizing, cannot. Hence, a realist conception of causation is key to naturalism's ability to provide a dissident epistemology of science.

4 Assessment: Naturalism and names

4.1 Taking 'naturalism' seriously

To begin, we would like to know whether or not Mathematical Naturalism satisfies D_2, the desideratum that Mathematical Naturalism actually be a form of naturalism. If satisfying D_2 requires endorsing disciplinary holism, then Mathematical Naturalism fails to satisfy D_2. According to Maddy, the Mathematical Naturalist 'extends the same respect to mathematical practice that the Quinean naturalist extends to scientific practice' (Maddy [1997], p. 184). More specifically, she continues:

> Where Quine holds that science is not answerable to any supra-scientific tribunal, and "not in need of any justification beyond observation and the hypothetico-deductive method" (Quine [1975], p. 72), the mathematical naturalist adds that mathematics is not answerable to any extra-mathematical tribunal and not in need of any justification beyond proof and the axiomatic method. Where Quine takes science to be independent of first philosophy, my naturalist takes mathematics to be independent of both first philosophy and natural science (including the naturalized

> philosophy that is continuous with science)—in short, from *any external* standard. (Maddy [1997], p. 184, emphasis added)

So the Mathematical Naturalist sees and raises the Quinean naturalist. What is good for the natural sciences (broadly construed) is good for mathematics, and the latter separates itself from the former and prior philosophy just as the former separates itself from prior philosophy alone. But this is to explicitly reject disciplinary holism.

4.2 Second philosophy (or what's in a name)

Though Maddy readily acknowledges that her position owes a debt to Quine and his naturalism,[21] she does not claim that her position tracks Quinean naturalism in all its details. She merely wants to '[extend] the same respect to mathematical practice that the Quinean naturalist extends to scientific practice' (Maddy [1997], p. 184), viz., free it from the baleful influence of philosophy (and the non-mathematical sciences) by giving an account of the epistemology of mathematics (in particular, of set theory), where the relevant norms and standards are internal to mathematics. One might be tempted to think that this means Maddy subscribes to some version of the Quine–Putnam indispensability argument.[22] This would be a mistake. Maddy quite explicitly rejects indispensability to science as a basis for ratifying or criticizing mathematics. Indeed, she thinks that the chief claim of the set-theoretic realism she advanced in ([1990]), viz., that perceptually accessible numerical facts are facts about sets, is false precisely because that claim, she says, 'depends on an indispensability argument [$\cdots$] that I [$\cdots$] now reject' (Maddy [1997], p. 152, fn. 30). Mathematical Naturalism cannot both deploy an indispensability argument and at the same time endorse the independence of mathematics from natural science (i.e., reject disciplinary holism), which Maddy does and which is exactly what separates Mathematical Naturalism from Quinean naturalism.[23]

Maddy has recently tried to separate questions concerning the merits of her view from questions concerning its naturalistic status; she now prefers to call her position *Second Philosophy*.[24] By moving the focus away from naturalism, she judges she can 'avoid largely irrelevant debates about what "naturalism" should be' (Maddy [2003], fn. 8). So what should we say about Maddy's position judged solely on its merits, setting aside the question whether or not

[21] See, e.g., (Maddy [1997], p. 161).

[22] See, e.g., (Quine [1948], [1954]; Putnam [1971], [1975]).

[23] For more on Maddy's misgivings about and rejection of Quine–Putnam-style indispensability arguments, see (Maddy [1992]; [1997], II.6, II.7, and p. 109, fn. 5; [2005b]).

[24] See, e.g., (Maddy [2003]).

it is naturalistic? This is really to ask how Mathematical Naturalism[25] fares with respect to D_1. I will argue that Mathematical Naturalism does not fare well in this respect.

Satisfying D_1 requires that Mathematical Naturalism provide for an account of the reliability of the epistemic norms and standards of mathematics as Mathematical Naturalism holds them to be. But those very norms and standards are supposed to be codified in the method of Mathematical Naturalism. So what Mathematical Naturalism really needs is an account of its own reliability as a method for justifying mathematical claims (more precisely, set-theoretic axioms). In other words, we are looking for an argument that Mathematical Naturalism itself constitutes a dissident answer to JAQ. Since Mathematical Naturalism is modeled on naturalism, it is reasonable to look to naturalism's dissident answer to NEQ as a model for Mathematical Naturalism's dissident answer to JAQ.

Recall that naturalism owes its ability to answer NEQ to its embrace of disciplinary holism, a causal theory of detection, and a theory-dependent conception of natural kinds. If Mathematical Naturalism is going to answer JAQ on analogy with naturalism's answer to NEQ, it needs to countenance mathematical counterparts to these views. I maintain that Mathematical Naturalism is unable to consistently endorse the appropriate counterparts and that this inability is due to a central feature of Mathematical Naturalism which makes it impossible to offer any argument for the claim that Mathematical Naturalism satisfies D_1 without at the same time undermining Mathematical Naturalism. If this is right, then an account of the reliability of the method of Mathematical Naturalism analogous to the account of the reliability of scientific practice available to the naturalist is out of reach for the Mathematical Naturalist. It follows that, absent an argument for some other way of accounting for the reliability of mathematical practice accessible to the Mathematical Naturalist, giving a dissident answer to JAQ is also out of her reach.

We have already observed that Maddy abandons disciplinary holism, and also that the naturalist's ability to give a dissident answer to NEQ largely depends on her acceptance of disciplinary holism. If the Mathematical Naturalist is going to likewise avoid quietism, she needs to offer something in place of disciplinary holism which will do the work in her answer to

[25] Second Philosophy encompasses the natural sciences as well as mathematics and logic, but it makes recommendations specific to each (Maddy [2003]). For mathematics, Second Philosophy coincides with Mathematical Naturalism. Hence, I use 'Mathematical Naturalism' to denote Second Philosophy of mathematics. 'Second Philosophy' denotes Second Philosophy generally unless indicated otherwise. The reader should bear in mind that, despite its name, whether or not Mathematical Naturalism is a form of naturalism is of only secondary interest for the remainder of this article.

JAQ that disciplinary holism does in the naturalist's answer to NEQ. The obvious thing to try is a mathematical analog of disciplinary holism according to which the sub-disciplines of mathematics (algebra, real and complex analysis, topology, etc.) are engaged with one another in such a way that they regulate mathematical beliefs similar to how interplay between branches of science regulate scientific beliefs on disciplinary holism. Call such a view *mathematical holism*. Can Mathematical Naturalism put mathematical holism into play? I suspect not, since by Maddy's standards the various sub-fields of mathematics—individuated largely by their respective goals—arguably count as distinct practices. If they do count as distinct practices, then the same sort of argument in favor of insulating mathematics from the non-mathematical sciences and philosophy offered by the Mathematical Naturalist should work for each sub-field of mathematics. The result is a splintering of mathematics into multiple distinct practices, each unable to criticize or support the others. If this is right, then mathematical holism is not available to the Mathematical Naturalist. Determining whether this is in fact right would require a more detailed investigation of what differentiates practices than I propose to give here. But nothing hinges on whether or not my suspicion is correct; Mathematical Naturalism has more serious problems.[26]

Leaving aside the question of whether or not Mathematical Naturalism can consistently countenance mathematical holism, what would an appropriate analog to a causal theory of detection have to be like? At a minimum, it would have to do for Mathematical Naturalism what a causal theory of detection does for naturalism in answering NEQ; i.e., it would have to engender confidence in us that our mathematical theories are on to the structure of mathematical reality[27] and not just delivering artifacts of our cognizing and culture. Maddy recognizes that naturalism is committed to realism concerning the *prima facie* subject matter of the sciences: '[N]aturalism counsels us to second the ontological conclusions of natural science, and natural science ratifies a commitment to objectively existing physical objects of many kinds' (Maddy [1995], p. 251). We can add that science ratifies a commitment to an objectively existing causal order. But the Mathematical Naturalist is in no position to take a similar stance—or indeed any metaphysical stance—toward the *prima facie* subject matter of mathematics.

[26] I am not claiming here that there is nothing like mathematical holism in fact operative in mathematics; I think there is. I am claiming that given Mathematical Naturalism's conception of practices, what individuates them and how easily insulated they are, it's doubtful that any mathematical holism actually found in mathematics would be usable by the Mathematical Naturalist.

[27] By this I mean whatever it is in virtue of which true mathematical statements are true. No particular answer to the question of mathematical realism is assumed, though a dissident answer to JAQ modeled on naturalism's answer to NEQ most likely requires some fairly robust version of mathematical realism.

The Mathematical Naturalist settles questions of mathematical ontology by appeal to a methodological directive operative in contemporary mathematics that counsels set-theoretic reduction: 'mathematical existence means existence in the world of sets' (Maddy [1993], p. 19).[28] According to Maddy's view, such maxims appear at the methodological level of mathematical practice, and so are internal to mathematics. She admits that, as a result, 'only the barest ontological questions about mathematics can be naturalized as mathematical questions' (Maddy [1997], p. 192), and this becomes clear when one realizes that Mathematical Naturalism has nothing to offer on the question of set existence. More precisely, the Mathematical Naturalist merely parrots set theorists when it comes to set existence, and while sets exist according to set theory, the nature of their existence—whether or not they are mind-independent, spatially located, causally inert, etc.—goes unremarked upon. Thus, the nature of mathematical existence is ultimately left open, and it is in this sense that no more than 'the barest' traditional questions of mathematical ontology are amenable to treatment as mathematical questions. To Mathematical Naturalism, ontology is extra-mathematical, and as such it is methodologically irrelevant and a distraction. The upshot is that 'the [Mathematical Naturalist] must carry on without appeal to ontology' (Maddy [1995], p. 262). Given this, the Mathematical Naturalist cannot put metaphysics into play as the naturalist puts realism into play in answering NEQ without revising the core of her position, i.e., without in effect abandoning Mathematical Naturalism.

The ontological neutrality just noted renders moot the question of whether or not Mathematical Naturalism admits an analog of the theory-dependent conception of natural kinds deployed by naturalism in answering NEQ. Recall that the naturalist uses this conception of natural kinds to bridge theory and the world, since for her natural kind definitions are informed by theory in such a way that they align with the causal order. But we just saw that the Mathematical Naturalist can countenance nothing to play a role in the epistemology of mathematics analogous to that of the causal order in naturalistic epistemology. So even if she can make out a conception of mathematical kinds which are informed by mathematical theory, that conception cannot do the same work for her that the naturalist's theory-dependent conception of natural kinds does in answering NEQ.

One might think that the Mathematical Naturalist could substitute logical kinds for natural kinds. After all, is not logic ontologically neutral? And if it is, might not the Mathematical Naturalist be able to tell a story according to which mathematical kinds are logical kinds without violating the ontological

[28] Maddy is somewhat less forthcoming with regard to this position in (Maddy [1997]). There she is at pains to emphasize that while mathematical methodology ratifies the existence of mathematical entities, it is silent on the nature of this existence (see p. 192.)

neutrality of her position? This suggestion is problematic in various ways. Let us suppose that there are logical kinds very much like natural kinds. What sort of story should the Mathematical Naturalist tell that will help her? What sort of story *can* she tell? An obvious story to tell is a logicist one, in some sense reducing mathematical entities (i.e., objects and properties) to logical entities. However, logicism is not ontologically innocent; it makes claims about what mathematical entities *are*. One might object that since logic is ontologically innocent and mathematical entities, according to logicism, just are logical entities, mathematics is ultimately ontologically innocent as well. Such an objection appears to trade on a confusion. Logic is arguably ontologically innocent in that no particular thing or sort of thing must exist for it to do its work *at the object-language level*. But it does not follow that nothing need exist *simpliciter* for logic to do its work. If there are logical properties, and there had better be if logic is doing anything at all, then there are properties. And while properties are not objects, they nonetheless are part of ontology. Similarly for concepts (in case one considers them different from properties). Logical entities are entities, whether objects or properties, with ontological standing.[29] Suppose I am wrong about this. Suppose an extreme case in which there is a fully nominalist interpretation of logic. Even this would not help the Mathematical Naturalist. The reduction of mathematics to even a nominalized logic is still a metaphysical story, and the Mathematical Naturalist can not avail herself of *any* metaphysical story concerning mathematics, realist or not, without violating her ontological neutrality. The Mathematical Naturalist can only access what is internal to mathematics, and neither positive nor negative judgments concerning the nature of mathematical entities are internal to mathematics. Given this, it is very hard to see how the Mathematical Naturalist could access any other (nonobvious, nonlogicist) story grounding the reliability of mathematical practice in logical kinds, even if such were ready to hand.

By maintaining her ontological neutrality, Maddy denies Mathematical Naturalism an objective ontology of mathematics with which to anchor the reliability of mathematical practice; she denies mathematics an ontological ground. Consequently, she undermines the ability of Mathematical Naturalism to account for the reliability of mathematical practice along the lines followed by the naturalist in answering NEQ. The naturalist answers NEQ by telling a story about how we formulate, test, and refine theories in response to causal interactions with an objectively existing (causally ordered) world. This story has it that we causally detect features of the world in such a way that our theories incorporate natural kinds that reflect the causal order (at

[29] It is worth noting that many prominent logicists are mathematical realists. See, e.g., (Frege [1884]; Wright [1983]; Hale [1988]; Hale and Wright [2001]).

least to a good approximation), and that as a consequence inductions (and explanations) carried out in light of these theories yield (approximately) true results. Mathematical Naturalism cannot model a story about the reliability of mathematical practice on the one the naturalist gives for NEQ because it has nothing to play the role of the causal order, since it countenances no ontology for mathematics at all. Moreover, since any story about the reliability of mathematical practice the Mathematical Naturalist might tell, whether modeled on naturalism's answer to NEQ or not, would involve judgments concerning mathematical ontology—which by Maddy's own lights would be stepping outside mathematics—there is no story of the relevant sort consistent with Mathematical Naturalism available to the Mathematical Naturalist.[30] So no argument in favor of Mathematical Naturalism satisfying D_1 can be given without at the same time undermining Mathematical Naturalism.[31] It follows that Mathematical Naturalism yields only a quietist epistemology of mathematics.

The Mathematical Naturalist might respond by arguing that naturalism is ultimately no better off with respect to quietism.[32] The idea is that natural science taken in its entirety cannot account for the reliability of its methods internally, and so naturalism must yield a quietist epistemology of science after

30 On this view, a satisfactory story about the reliability of a practice requires making judgments about the ontology of that practice. Again one might wonder if logic provides a counterexample: since logic is ontologically neutral, we can tell a story about the reliability of logical inference without getting involved in ontology (say by invoking necessary truth-preservation). For reasons rehearsed in the previous paragraph, I do not think this is a counterexample. In general, reliability concerns truth and truth concerns truth-makers (i.e., whatever it is in virtue of which the relevant statements are true). Whether one holds a deflationary or inflationary view of truth, one's account of the reliability of a practice will deploy judgments about truth-makers for that practice. But whether one holds a deflationary or inflationary view of the relevant truth-makers, those judgments are about the ontology of the practice in question. I am not claiming that we can never make such judgments, even about mathematics and logic. I am claiming that the Mathematical Naturalist, by her own lights, cannot make such judgments about mathematics without going outside mathematics and thereby trespassing against her own position.

31 One might be tempted to defend Mathematical Naturalism by arguing that mathematical truth is coherence regulated by whatever maxims emerge from the operative naturalized model of mathematical practice. If mathematical truth were this sort of coherence truth, then the norms and standards of mathematical practice would trivially be truth-conducive. But aside from the *prima facie* inadequacy of a coherence conception of truth where reliability is concerned, such a defense is not open to the Mathematical Naturalist. The claim that truth in mathematics is a type of coherence truth is a distinctly philosophical claim that would need to be argued for on distinctly philosophical grounds. So advancing such a claim in defense of Mathematical Naturalism would constitute an extra-mathematical defense of mathematics of the sort explicitly rejected by Mathematical Naturalism. Not only this, but such a defense of Mathematical Naturalism is inconsistent with the motivations of Mathematical Naturalism, which are modeled after those of philosophical naturalism. The naturalist takes the sciences at face value and follows the endorsements of the sciences. The Mathematical Naturalist purports to do the same with respect to mathematics. But mathematicians do not commonly take truth in mathematics to be a type of coherence truth.

32 Thanks to an anonymous reviewer for raising this issue.

all. Since we are not upset by this state of affairs in the case of naturalism, the response continues, why should we be upset by an analogous state of affairs in the case Mathematical Naturalism? It seems to me that this response misses the force of the crucial difference between naturalism and Mathematical Naturalism, the robust ontological stance of the former and the ontological neutrality of the latter. By countenancing a certain sort of ontology and a grounding connection between scientific practice and that ontology, naturalism is able to give an account of the reliability of scientific practice, and since the relevant ontology and grounding connection are countenanced by science itself this account is internal to science, i.e., it uses methods and findings of science itself. Mathematical Naturalism, on the other hand, restricted to the methods and findings of mathematics alone, which are neutral on the ontological status of mathematical entities according to the Mathematical Naturalist, can not give a similar account of the reliability of mathematical practice. She can not even give a deflationary account (e.g., along constructivist lines) because such an account still takes a stand on the ontological status of mathematical entities, albeit a negative one.

Moreover, the Mathematical Naturalist accepts the naturalist's account of the reliability of scientific practice. If she wants to repudiate that story for the natural sciences, thereby repudiating her self-conscious adherence to naturalism with respect to non-mathematical sciences, and argue that naturalism yields a quietist epistemology of science, she will need an argument against the account offered by the naturalist. Otherwise, her repudiation is no more than an *ad hoc* response to her own difficulties. At best, the idea floated above amounts to the claim that naturalism yields a quietist epistemology of science because there are no first principles, principles independent of science, from which to argue that the methods of science taken as a whole are reliable. But the naturalist will not deny this, of course. The naturalist argues that despite this being the case, we can generate the requisite argument using the methods of science, *and that this is legitimate*. Pushing the line under consideration requires the Mathematical Naturalist to counter this, and it is hard to see how she could without at the same time calling her project into question on the same grounds she uses against the naturalist.

5 A way out?

(Maddy [2005a]) can be seen as an attempt to deal with the issues raised here—in particular, with the reliability of mathematical practice. There Maddy considers a view of mathematical existence she calls *Thin Realism*. The details of the view need not concern us. What is important for present purposes is that, according to Thin Realism, there is no 'gap at all between set theoretic methods and sets,' in the sense that 'that sets can be known about in these ways is part of what sets are,' and so 'the reliability of [set theoretic] methods is [· · ·] conceptual' (Maddy [2005a], p. 368). This might lead one to offer the following argument in favor of Mathematical Naturalism's satisfying D_1:

> (AFSP) In order to satisfy D_1, Mathematical Naturalism must be able to account for the reliability of the epistemic norms and standards of set-theoretic practice. Those norms and standards are part of the methods of set theory. It is a conceptual truth that the methods of set theory are reliable, since sets *just are* the sorts of things we come to know about using those methods. Hence, the relevant norms and standards are reliable (and it is a conceptual truth that this is so).

I hope to address this thought-provoking article at greater length elsewhere; here I restrict myself to some brief observations on AFSP.

Observation 1: Anyone who attaches importance to whether one's epistemology of mathematics is naturalistic or not and who also follows Quine in holding that naturalism and *a priori* knowledge are incompatible, will deny that Mathematical Naturalism supplemented with Thin Realism is naturalistically acceptable. According to AFSP, Mathematical Naturalism plus Thin Realism satisfies D_1 by making it conceptually true that the epistemic norms and standards operative in set theory are reliable. Since conceptual truths are paradigmatically *a priori* knowable, it follows that Mathematical Naturalism plus Thin Realism makes the reliability of those norms and standards *a priori* knowable. This directly contradicts one of the widely accepted tenets of naturalistic epistemology, expressed by Philip Kitcher as the thesis that '[v]irtually nothing is knowable *a priori*, and, in particular, no epistemological principle is knowable *a priori*' (Kitcher [1992], p. 76). It does not follow from this, of course, that Mathematical Naturalism plus Thin Realism fails to satisfy D_1; however, if D_1 is so satisfied, it is arguably satisfied at the expense of naturalism.

Observation 2: The conception of mathematical truth that emerges from the Thin Realist's account of reliability strongly resembles a coherence conception of truth, in the sense that to be a true set-theoretic belief is just to be a member of the class of set-theoretic beliefs regulated by the norms and standards of set-theoretic practice. This is problematic, since such a conception of truth is at least *prima facie* inadequate for an explanation of the reliability of the norms and standards of set-theoretic practice. If we are concerned with what reason we have to think that the epistemic standards operative in some practice get us on to the truth about the subject matter of the practice, it does not help to be told that those standards get us on to the truth because being true just is a matter of being ratified by those standards.

One might respond to this worry by arguing that it is not appearing in the class of set-theoretic beliefs arrived at and regulated by the norms and standards of set-theoretic practice that makes a set-theoretic belief true, but rather that, as a matter of fact, the beliefs formed and regulated by the relevant norms and standards coincide with the true set-theoretic beliefs. This response has significant shortcomings. First, if this is right it is not at all clear what to make of the claim that the reliability of the methods of set theory is a conceptual matter. Second, this response requires an argument for the coincidence of the class of beliefs formed and regulated by the norms and standards of set theory and the class of true set-theoretic beliefs. But such an argument is an argument for the reliability of the epistemic norms and standards of set theory. So this response takes us in a circle. Moreover, even if a noncircular version of the relevant argument were in the offing, it is doubtful that it would be available to the Mathematical Naturalist, since it is hard to see why such an argument would not be extra-mathematical.[33]

Observation 3: One can understand the question of whether or not an epistemology for a practice *P* is dissident as a challenge. To meet the challenge, an epistemology must take seriously the question of the connection between the standards of belief acquisition and maintenance internal to *P* and the truth concerning the subject matter of *P* and also have some means of answering this question that yields a connection sufficiently tight to support the claim that the standards internal to *P* are reliable. AFSP attempts to meet this challenge by positing a conceptual connection between set-theoretic truth and the epistemic standards of set theory: it is part of the concept *SET*, part of what it is to be a set, that truths about sets are accessible via the methods of set theory. But one might reasonably wonder why this approach does not easily extend to practices of which we are rightfully suspect when it comes to the truth of the relevant subject matter. To take one such example, consider Christian theology.[34]

We might extend the strategy of Thin Realism to an argument in favor of a dissident epistemology for Christian theology as follows:

> (AFCT) A dissident epistemology of Christian theology must account for the reliability of the epistemic norms and standards of Christian theological practice. Those norms and standards are part of the methods of Christian theology. It is a conceptual truth that the methods of Christian theology are reliable, since God *just is* the sort of thing we come to know about using those methods. Hence, the relevant norms and standards are reliable (and it is a conceptual truth that this is so).

[33] Cf. fn. 31.

[34] Which theology we consider is irrelevant for present purposes. I choose Christian theology simply for definiteness.

Of course, AFCT is entirely unsatisfactory, leading as it does to an 'overeasy' (in the terminology of Yablo [2000]) argument for the existence of God. So why should we be moved by AFSP?

Gideon Rosen raises a closely related worry in his ([1999]). There he calls a practice *authoritative* 'if, whenever we have reason to accept a statement given the proximate goal of the practice, we thereby have reasons to believe that it is true' (Rosen [1999], p. 471). The Authority Problem (for Naturalized Epistemology) is the problem of being able to tell authoritative practices from nonauthoritative practices in some principled way (Rosen [1999], p. 471). Our problem—the problem of why we should accept AFSP when we should plainly reject AFCT—is a localized version of the Authority Problem, the problem of saying why Mathematical Naturalism plus Thin Realism is authoritative while Christian theology plus the suitably adapted version of Thin Realism deployed in AFCT is not. Since we can think of the dissidence of an epistemology for a practice *P* as having the resources to argue that *P* is authoritative, solving this localized Authority Problem *within Mathematical Naturalism* (i.e., using only the resources of and without violating the tenets of Mathematical Naturalism) would show that Mathematical Naturalism provides a dissident epistemology of set theory.

Maddy addresses something very much like Rosen's Authority Problem for Second Philosophy (hence, for Mathematical Naturalism) in (Maddy [2005b], II.1). There she recognizes that the following question arises for Second Philosophy: Once we allow that mathematics uses methods distinct from those of the natural sciences, what separates mathematics from other practices, such as theology and astrology, which also use methods distinct from those of the natural sciences and which we rightfully reject? Her answer:

> [M]athematics is used in science, so the [Second Philosopher's] scientific study of science must include an account of how its methods work and how the theories so generated manage to contribute as they do to scientific knowledge. Astrology and theology are not used in science—indeed, in some versions they contradict science—so the [Second Philosopher] needs only to approach them sociologically or psychologically. (Maddy [2005b], p. 449)

It is certainly correct to say that mathematics plays a role in science that neither astrology nor theology do; the usefulness of mathematics to science is beyond question. But does this response really get to the heart of the question it is intended to answer? Not if the Authority Problem lies at the heart of that question.

This response identifies a feature that sets mathematics apart from other nonscientific practices, such as astrology and theology,[35] viz., usefulness to natural science. However, just setting mathematics apart from the other nonscientific practices is not enough to solve the Authority Problem. We can motivate setting mathematics apart from the other nonscientific practices by noting that it is the most notation-heavy among nonscientific practices, but this does not tell us why we should think it is authoritative. Mathematics is certainly special in its usefulness to the (non-mathematical) sciences. However, absent an argument that usefulness to science is evidence for truth, being special in being useful to science is simply irrelevant to the Authority Problem. One might deploy an argument based on disciplinary holism to bridge usefulness and truth, but this move is not available to the Mathematical Naturalist since she rejects disciplinary holism.[36]

6 Or out of the way?

In proffering and advocating Mathematical Naturalism, Maddy does not give the impression that she intends her position to be a quietist one. With respect to Second Philosophy not applied to mathematics, this impression proves accurate. Of NEQ, Maddy says:

> The Second Philosopher thinks she has at least the beginnings of an answer to this question, in her account of how and when perception is a *reliable* guide, in her study of various methods of reasoning, and her efforts to understand and improve them. (Maddy [2003], p. 87, emphasis added)

[35] From here on I will leave out astrology and concentrate on theology, understood as Christian theology.

[36] This argument has points of contact with Dieterle ([1999]). The argument there can be presented as a dilemma: If the value of mathematics resides merely in its usefulness to science, then only applied mathematics is valuable and Mathematical Naturalism collapses into scientific naturalism. If the value of mathematics consists in something other than its usefulness to science, then the Mathematical Naturalist has to accept that other, obviously unpalatable forms of naturalism (e.g., theological and astrological naturalisms) are legitimate. Maddy grasps the first horn, arguing that pure mathematics is valuable for its potential future use to natural science (see, e.g., Maddy [2000]). My argument runs from the antecedent of the first horn, where usefulness includes potential future usefulness, to the consequent of the second horn. From the point of view of the Authority Problem, usefulness to science distinguishes Mathematical Naturalism from obviously unpalatable forms of naturalism only if it is evidence of truth. But an argument that this is the case seems unlikely to be forthcoming, which leaves Mathematical Naturalism on no firmer ground than obviously unpalatable forms of naturalism. It's also worth noting that my project in this article is only superficially connected to the one taken up in (Paseau [2005]). There Paseau argues in favor of the authority of philosophy, even over mathematics and so contra Mathematical Naturalism, as a default position. He is not concerned with Rosen's Authority Problem and so has a target quite distinct from that of this section.

In the case of natural science, the Second Philosopher countenances a conception of epistemic justification which requires the reliability of justificatory norms. Thus, the Second Philosopher aspires to a dissident epistemology when mathematics is not at issue.

As we have seen, Mathematical Naturalism understood as an epistemological position fails to be dissident *by design*. The Mathematical Naturalist self-consciously rejects disciplinary holism and embraces an anti-metaphysical position that robs it of any ontological ground for mathematical practice, so that in the end the epistemology of mathematics codified in Mathematical Naturalism cannot escape quietism. But there is another reading of Mathematical Naturalism according to which that position is intended to be neither quietist nor dissident, as it is not intended to be or yield an epistemology of mathematics at all. (To help us keep straight Mathematical Naturalism understood as an epistemological position and Mathematical Naturalism understood as this alternate, non-epistemological position, 'Mathematical Naturalism$_e$' and cognates will denote the former and 'Mathematical Naturalism$_{alt}$' and cognates will denote the latter.)

Though Second Philosophy of non-mathematical sciences is straightforwardly intended to give a dissident epistemology, Mathematical Naturalism$_{alt}$ is not. According to Mathematical Naturalism$_{alt}$, mathematics is of special interest or significance owing to its usefulness (or potential future usefulness) to the non-mathematical sciences. But we get no epistemic mileage out of this usefulness. What we do get is an answer to the question: Why do we care about mathematics? (or, perhaps, Why should we care about mathematics?), an answer grounded in pragmatic rather than alethic concerns. This allows us to make sense of the *prima facie* inadequate response to the Authority Problem considered above. That response seemed simply off the mark, disconnected from the question it was supposed to answer. But if we read Maddy's mathematical project as unconcerned with epistemology, i.e., as advancing Mathematical Naturalism$_{alt}$, the Authority Problem evaporates; being authoritative is an epistemic matter.

In addition to Thin Realism, Maddy ([2005a]) considers a position on mathematical existence called *Arealism*. According to Arealism, 'mathematical things do not exist' and 'pure mathematics is not in the business of discovering truths' (Maddy [2005b], p. 364). If we make ontological judgments in accordance with science and decline to admit mathematics into the pantheon of sciences (on grounds that science is essentially empirical and mathematics is not), then we are led to Arealism. Though Arealism seems particularly well-suited to Mathematical Naturalism$_{alt}$, whether one ultimately supplements Mathematical Naturalism$_{alt}$ with Thin Realism or Arealism supposedly 'comes down to matters of convenience, taste, and preference in the bestowing of honorific terms (true, exists, science)' (Maddy [2005b], p. 368). According to

Maddy, any difference between Thin Realism and Arealism is ‘essentially cosmetic’ (Maddy [2005b], p. 373). This assessment notwithstanding, Thin Realism and Arealism seem to differ considerably in the extent to which they help with the problems raised in this article: Thin Realism allows the Mathematical Naturalist$_{alt}$ to escape virtually none of these problems, while Arealism allows her to escape virtually all of them. Supplementing with Thin Realism yields a position with something to say on the reliability of set-theoretic methods, i.e, an epistemological position. So supplementing Mathematical Naturalism$_{alt}$ with Thin Realism, if coherent, takes us back to Mathematical Naturalism$_{e}$ with all its problems. Supplementing Mathematical Naturalism$_{alt}$ with Arealism, on the other hand, allows it to sidestep the problems I have raised, moves it out of the way of those problems, since the problems I have identified only affect positions that claim to be epistemic.[37] Mathematical Naturalism$_{alt}$ is self-consciously nonepistemic.

I think that Arealism has problems of its own, independent of what a satisfactory epistemology of mathematics requires. However, I will not take up those problems here. Suffice to say that if one is interested in an epistemology of mathematics, whether naturalistic or not, one should reject Mathematical Naturalism, read as Mathematical Naturalism$_{e}$. Of course, one may eschew the project of giving an epistemology for mathematics, in which case Mathematical Naturalism, read as Mathematical Naturalism$_{alt}$, is still potentially a viable position—though to what purpose is less than clear.

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[37] Though it should be noted that Mathematical Naturalism$_{alt}$, supplemented with Arealism or not, lacks a key attraction of epistemological naturalism, viz., dissidence, and so garners little support from the success of epistemological naturalism.

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