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THE REDUCTION OF PHENOMENOLOGICAL TO KINETIC THERMOSTATICS*

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Standard accounts of the micro-reduction of phenomenological to kinetic thermostatics, based on the postulate relating empirical absolute temperature to mean kinetic energy ($\bar{u} = (3/2)kT$), face two problems. The standard postulate also allows 'reduction' in the other direction and it can be criticized from the point of view that reduction postulates need to be ontological identities.

This paper presents a detailed account of the reduction, based on the postulate that thermal equilibrium is ontologically identical to having equal mean kinetic energy. In particular, it is shown that this postulate enables reduction only in the appropriate direction, but leaves room for 'evidence transport' in the other. Moreover, it makes possible the derivation (explanation) of the standard postulate, using the existential kinetic hypothesis and phenomenological laws with which it turns out to be laden.

1. Introduction. In this paper a detailed account will be given of the micro-reduction relation between basic phenomenological laws and relevant kinetic hypotheses concerning equilibrium states of gases. Leaving out all reference to asymptotic behavior the reduction goes, according to textbook expositions, roughly as follows. The phenomenological ideal gas law states that any mole of pure gas satisfies $PV = RT$ (P : pressure, V : volume, R : the universal gas constant, T : empirical absolute temperature). Mechanical considerations lead to the kinetic hypothesis $PV = (2/3)N\bar{u}$ (N : Avogadro's number of molecules in one mole, \bar{u} : mean kinetic translatory energy). By introducing the reduction postulate, here called the kinetic temperature relation KTR, $\bar{u} = (3/2) (R/N)T =_{df} (3/2)kT$ (k : Boltzmann's constant), we are able to derive the ideal gas law from the kinetic hypothesis. Nagel's well-known account in *The Structure of Science* (1961, Ch. 11) differs from the above only in that he does not use the concept of mole. But this is of no consequence in the following two problems inherent to all standard accounts.

A moment's reflection shows that we can reverse the standard argument: from the ideal gas law and KTR we can derive the kinetic hypothesis. Hence the reduction seems to work in both directions. Intuitively,

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however, in case of micro-reduction we are inclined to expect, almost by definition, asymmetry: although the ideal gas law provides evidence for the kinetic hypothesis, the latter may not be derivable from the former and the *proper* reduction postulate. If this intuition is correct the standard bridge principle KTR must be too strong *qua* reduction postulate, although it may be valid as such.

The second problem of the standard account needs some more introduction. In his analysis Nagel pays very much attention to the cognitive status of KTR and concludes that it may be conceived either as an empirical hypothesis or as a so-called correspondence principle, depending on our willingness to accept claims to independent evidence for the reducing theory, i.e., the kinetic theory. However, in the general literature on reduction in the last two decades the ontological status of reduction postulates has come to the fore. In particular, it has been argued frequently, and in our opinion correctly, that the core of any reduction is that the reduction postulates do not represent causal connections but ontological identities. Robert L. Causey has elaborated this point of view in logical detail in a number of articles, culminating in his *Unity of Science* (1977). Although we will pay little attention to logical questions, the present study is in the spirit of Causey's work.

A main problem of the identity view is of course how to distinguish causal connections from ontological identities. There seems to be little more than what appears at first sight to be a question-begging criterion: whereas causal connections ask for further causal explanation to bridge the gap, ontological identities do not because there is supposed to be no gap. Of course, this does not mean that a reduction postulate, if it indeed states an ontological identity, cannot be empirically supported. On the contrary, if there is independent evidence for the reducing theory as well as for the theory or laws to be reduced, the total evidence supports the relevant reduction postulates, in which case they are (supported) empirical hypotheses. However, in cases of micro-reduction, independent evidence may not (yet) be available, in which case Nagel calls the reduction postulates correspondence principles. But even in this case they are not *simply* correspondence principles: if the reducing theory is interpreted realistically they state ontological identities and not causal connections.

Hence, the crucial question is whether or not KTR can be considered as (representing) an ontological identity. The criterion mentioned above leads to the question "does KTR ask for further causal explanation?". Such a question, however, can only be answered in an acceptable way when we have found a satisfactory causal explanation, in which case we would say that KTR did indeed ask for further causal explanation and hence that KTR represents a causal connection.

We hasten to add that we did not look for a causal explanation for KTR,

let alone find one, for its being a causal connection is too counterintuitive. Nonetheless, we had a great deal of intuitive resistance, for two reasons, to accepting that KTR would be an ontological identity. As was shown before (the first problem) KTR leads directly to the 'reverse reduction' and, hence, it must be laden with (be part of) the kinetic hypothesis. Moreover, KTR uses the notions of absolute temperature T and the gas constant R , and it is well-known that these notions are based, in some way or other, on some phenomenological laws, i.e., KTR is also laden with them.

Now the obvious question arises whether 'ontological identity' is the only alternative to 'causal connection'. Or, to put it differently, a bridge principle might ask for further explanation, though no causal explanation. If this makes sense, we may call such a bridge principle a theoretical identity, i.e., an identity which can (only) be explained by appeal to one or more ontological identities and some laws from one or both sides. Hence, the particular question is whether KTR is a theoretical identity, i.e., whether it can be explained further in the described non-causal way.

In this article, it will be shown that KTR can indeed be explained on the basis of 1) the existential hypothesis that there is at least one gas satisfying the kinetic hypothesis, 2) the phenomenological laws providing the appropriate existence and uniqueness condition for the introduction of absolute temperature and the gas constant, and 3) the reduction postulate stating that thermal equilibrium between two states of gases is identical to having equal mean kinetic energy. Moreover, it will be shown that this reduction postulate is sufficient for the reduction in the appropriate direction, but insufficient for the other direction, although it transports evidence in that direction. Finally, it will become clear from the exposition that the new reduction postulate is neither laden with the reducing hypotheses nor with the reduced laws. This suggests the claim that it indeed represents an ontological identity, for the quest for a further explanation seems absurd.

Before we start the presentation some technical introductory remarks need to be made. The analysis will be given in terms of asymptotic behavior of gases because this provides a more realistic and yet sufficient foundation for the present reduction.

As Causey stresses rightly in his book (1977, p. 87) the case of reduction under study also requires a reduction postulate with respect to pressure: the identification of phenomenological and kinetic pressure. Its status as ontological identity, and hence as not further explainable, will not be disputed. Usually it remains entirely implicit in textbook expositions.

In what follows we will restrict verbal comments, supposing familiarity with elementary thermostatics. Although there is no sharp distinction be-

tween kinetic theory and statistical mechanics the present analysis certainly belongs to kinetic theory. Hence we will not enter into the statistical foundations and presuppositions underlying the primitive concepts of kinetic theory. In the final evaluation we will make some remarks about the relevance of the present analysis for the reduction of thermodynamics to statistical mechanics. Finally, predicates will be used to indicate the corresponding sets and vice versa.

2. Common Background. As common background we presuppose some macroscopic and molecular, but non-thermal and non-kinetic, notions:

- G :the set of all pure gases,
 $Z(g)$:the set of equilibrium states of an isolated mole of g in G ,
 Z :the union of $Z(g)$ for all g in G ,
 $V:Z \rightarrow IR^+$: $V(z)$ being the (measurable) volume 'of state z '.

The notion of equilibrium state is here considered as a primitive one, leaving aside the problem of how to determine that a gas is in an equilibrium state. Of course, moles of the same (pure) gas are assumed to behave in the same way, which justifies our speaking of g as if it were a (representative) mole.

The concept of mole does not presuppose any thermal theory. To be precise, the concepts of mole and pure gas are based on some macroscopic laws concerning masses of gasmixtures, some molecular hypotheses concerning types of molecules and their relative molecular-masses and the trivial, but crucial, reduction postulates that a sample of gas is a sample of molecules and that the mass of an amount of gas is equal to the sum-total of the absolute masses of the molecules. In principle it is now possible to determine whether an amount of gas is pure, i.e., is of one type of molecule. A mole amount of pure gas is then defined as an amount with mass in grams equal to the relative mass of the molecules of this gas. The molecular hypotheses, if true, assure that a mole of any pure gas contains the same number of molecules: Avogadro's number N . Of course, determination of the value of N is another story, but we will not assume to know this value.

3. Phenomenological Thermostatistics. The basic notion of any thermal theory is the empirical relation of thermal equilibrium between states. This relation happens to be an equivalence relation (zeroth law of thermodynamics). The generated equivalence classes constitute:

- S :the set of thermal states.

Further there are supposed to be unique functions:

$P:Z \rightarrow IR^+ : P(z)$ being the (measurable) pressure of z ,

$t:Z \rightarrow S : t(z)$ being the (measurable, i.e., identifiable) thermal state of z .

The following simplicity assumption will be helpful:

for all $g : \text{Range}(t \upharpoonright Z(g)) = S$.

Moreover, in the context of a particular g , z is supposed to range over $Z(g)$ and the functions V , P , t , and those to be introduced later, are supposed to be restricted to $Z(g)$.

The following two notions remain implicit in textbook expositions, but they will turn out to be of fundamental importance.

Def. 1.1 g in G is an *Asymptotic Boyle Gas* (ABG) iff there is a one-one function $B_g : S \rightarrow IR^+$ such that $P(z)V(z) \rightarrow B_g(s)$ if $V(z) \rightarrow \infty$ and $t(z) = s$,

1.2 g and g' in ABG are *Comparable* iff $B_g = B_{g'}$.

Now we are able to formulate the three empirical laws which appear to be, in conjunction, equivalent to the (asymptotic) ideal gas law. The first and the second provide the required existence and uniqueness condition for the introduction of the notions of (empirical) absolute temperature and the (universal) gas constant. Addition of the third law completes the final claim.

E-FL some G are ABG (Existential-F-Law)

C-FL all ABG are Comparable (Comparability-F-Law)

U-FL all G are ABG (Universal-F-Law).

By EC-, EU-, UC- and EUC-FL we indicate in an obvious way conjunctions of F-Laws. Note that the addition of 'E' to 'U' gives a universal law existential import.

Def. 2 For arbitrary g in ABG

2.1 the (empirical) *absolute temperature* is the function $T:S \rightarrow IR^+$ defined by $T(s) = (B_g(s)/B_g(s_3)) \cdot T(s_3)$ with s_3 and $T(s_3)$ conventional; current convention: s_3 is the (thermal state of the) triplepoint of water and $T(s_3)$ is 273.16 °Kelvin,

2.2 the (universal) *gas constant* R is the quotient $B_g(s_3)/T(s_3)$ (and hence $RT(s) = B_g(s)$).

2.3 By natural extension we define $T:Z \rightarrow IR^+$ by $T(z) = T(t(z))$ (implying $T(z) = T(z')$ iff $t(z) = t(z')$).

It is evident that E-FL provides the required existence condition for the definition of T and R and it is easy to check that C-FL provides in addition the uniqueness condition. To be precise, T is already unique on the basis of the law that all ABG are quasi-comparable, i.e., B_g and $B_{g'}$ have constant proportion. However, this law, together with E-FL, implies C-FL if R is also required to be unique. In conclusion, T and R presuppose E- and C-FL, i.e., EC-FL.

Supposing EC-FL we may now define

Def. 3 g in ABG is an *Asymptotic Ideal Gas* (AIG) iff $P(z)V(z) \rightarrow RT(z)$ if $V(z) \rightarrow \infty$.

Some reflection shows that EC-FL is now equivalent to the law:

(i) some G are ABG and all ABG are AIG.

If we use 'all G are ABG', i.e., U-FL, we can derive from (i) the law

(ii) some G are AIG and all G are AIG.

Using the fact that 'all AIG are ABG' and 'all ABG are G ' are true by definition we see that (ii) implies not only (i), and hence EC-FL, but also U-FL. Therefore (ii) is equivalent to EUC-FL. But (ii) amounts precisely to the ideal gas law if that law is considered as a universal one, restricted to asymptotic behavior and with existential import. Hence, for the reduction of the ideal gas law it suffices to reduce EUC-FL or, equivalently, its three component laws.

At the end we will show that the solution of this reduction problem leads to the reduction of the ideal gas law if that law is extended (idealized) to non-asymptotic behavior. Note, however, given the previous analysis, that this extension is in no way required for the notions of absolute temperature and the gas constant. Moreover, it will turn out that the reduction postulates to be introduced for the asymptotic case are sufficient for the extended, non-asymptotic, case.

4. Kinetic Thermostatistics. The kinetic theory considers an equilibrium state of an isolated amount of gas as a statistically stable state, i.e., as a continuous sequence of momentary states for which the following local quantities are constant in time: the local (number-) density of molecules, and the local mean kinetic (translatory) energy in all directions. It is to be noted that the definition of these local quantities needs to be related to appropriate volume-elements in order to be able to be constant in time.

Now it is possible to show that such a stable state is only possible, in

view of the mutual interactions and the interactions with the wall, if these interactions are elastic and if, in addition, the following invariance conditions are satisfied: the local mean kinetic energy is the same in all directions (isotropy), and the local total mean kinetic energy is everywhere the same (homogeneity).

On the basis of these assumptions it now follows that there are unique functions for the states of moles of pure gas:

$$\bar{u}:Z \rightarrow IR^+ \quad : \bar{u}(z) \text{ being the (total) mean kinetic energy,}$$

$$p:Z \rightarrow IR^+ \quad : p(z) \text{ being the kinetic pressure, i.e., the resultant force per surface-unit exerted by the molecules on the wall.}$$

Again a simplicity assumption will be helpful:

$$\text{for all } g, g': \text{Range}(\bar{u} \upharpoonright Z(g)) = \text{Range}(\bar{u} \upharpoonright Z(g')).$$

By a well-known argument it can now be shown that, if the effects of mutual interactions are neglected, the kinetic pressure $p(z)$ is equal to $(2/3)(N/V)\bar{u}(z)$, where N is Avogadro's number. Of course, the more dilute the gas the more realistic is the neglect of mutual interactions. This suggests the following definition:

Def. 4 g in G is an *Asymptotic Perfect Gas* (APG) iff $p(z)V(z) \rightarrow (2/3)N\bar{u}(z)$ if $V(z) \rightarrow \infty$.

Although the crucial argument about asymptotic perfect behavior seems applicable to any gas if applicable to some, it turns out, in order to discover the asymmetry of the reduction, to be of crucial importance to distinguish the existential from the universal kinetic hypothesis about perfect behavior.

E-KH Some G are APG (Existential K-Hypothesis)

U-KH All G are APG (Universal-K-Hypothesis).

By EU-KH we indicate the conjunction, i.e., the universal kinetic hypothesis with existential import.

The kinetic theory leads also of course to other specific kinetic hypotheses, e.g., the Maxwell velocity distribution. But our subject is the reduction of phenomenological laws and hence we restrict our attention to *reducing* kinetic hypotheses, i.e., hypotheses which play a role in that reduction.

5. The Core of the Reduction. In the introduction we announced the following reduction postulates:

RP1 $P(z) = p(z)$: pressure is identified with kinetic pressure, i.e.,

the pressure we measure is, ontologically speaking, the kinetic pressure,

RP2 $t(z) = t(z')$ iff $\bar{u}(z) = \bar{u}(z')$: thermal equilibrium is identified with having equal mean kinetic energy, i.e., that is what is the case, ontologically speaking, at thermal equilibrium.

By RP we will indicate the conjunction of RP1 and RP2. Note that RP1 implies its structural analogue to RP2: $P(z) = P(z')$ iff $p(z) = p(z')$, and hence that both also relate states of different gases. Note also that RP2 suggests directly the definition of the unique one-one function of *thermal energy* of a thermal state:

$$u:S \rightarrow IR^+ : u(s) = \bar{u}(z) \text{ for arbitrary } z \text{ with } t(z) = s.$$

The *basic theorem* for the reduction is:

Th. 1 RP implies: all APG are Comparable ABG.

Proof: Suppose g is an APG; with RP1 we get that $P(z)V(z) \rightarrow (2/3)N\bar{u}(z)$ if $V(z) \rightarrow \infty$. The condition $t(z) = s$ in the definition of ABG corresponds, according to RP2, with $\bar{u}(z)$ is constant, viz. $u(s)$. Hence $B_g(s) =_{df} (2/3)Nu(s)$ provides the one-one function guaranteeing that g is an ABG. Because this B_g does not depend on g , all APG are even mutually comparable ABG, *q.e.d.* It is important to note, and easy to check from this proof, that, assuming either RP1 or RP2, the other is not only sufficient to derive the basic claim, but also necessary.

The following *reduction theorems* are direct consequences of Th. 1.

Th.2.1 E-KH RP-implies E-FL,

Th.2.2 U-KH RP-implies UC-FL.

They state that the F-laws are derivable (reducible) from (to) the K-hypotheses, using RP. The central reduction claim, viz., the reduction of the asymptotic ideal gas law, which was shown to be equivalent to EUC-FL, is obtained by conjunction:

Th.2 EU-KH RP-implies EUC-FL.

Similar to the proof of Th.1 is the proof of the *evidence transport theorem*:

Th.3 E-KH and UC-FL RP-imply U-KH (and then trivially EU-KH and EUC-FL)

It states that UC-FL, i.e., all G are Comparable ABG, is sufficient evidence, assuming RP, for the claim that all G are APG (i.e., U-KH), provided there is at least one APG (i.e., E-KH). However, it is clearly

not possible to derive E-KH from the F-laws and RP. Hence, in combination with Th.2 we see that RP leaves an essential asymmetry between the F-laws and the K-hypotheses: the former can be reduced to the latter, but not vice versa.

We conclude this section by indicating some points that could have been included in the analysis. To begin with, given the symbolic formulation of RP2, which uses t , it is trivial to claim that RP2 implies that thermal equilibrium is an equivalence relation (the zeroth law). From the verbal formulation, however, it is clear that we could have avoided t , in which case the zeroth law would have been a non-trivial consequence, enabling the introduction of S and t .

The second point is that the relation on S , defined by $u(s) > u(s')$, corresponds to the empirical linear order relation 'higher thermal state than' on S . The latter is based on the empirical relation 'warmer than' on Z , the former on the relation 'higher mean kinetic energy than' on Z . Again we claim that these relations are ontologically identical. If we had included this identity as a reduction postulate it would have been possible to derive the additional F-law that all ABG are monotone, i.e., $B_g(s) > B_g(s')$ iff s is in a higher thermal state than s' .

6. The Kinetic Temperature Relation (KTR). One way of formulating KTR, the bridge principle used in standard expositions, is the following:

$$\text{KTR} \quad RT(s) = (2/3)Nu(s) \text{ or, equivalently, } u(s) = (3/2)kT(s), \text{ with } k \stackrel{\text{def}}{=} R/N, \text{ i.e., Boltzmann's constant.}$$

Note that KTR in this formulation not only presupposes EC-FL (because T and R occur in it) but also RP2, because the thermal energy function u (with argument s) occurs in it. A second formulation is

$$RT(t(z)) = RT(z) = (2/3)N\bar{u}(z)$$

which also presupposes EC-FL, but not RP2. Nonetheless, it implies RP2 because $T(z) = T(z')$ iff $t(z) = t(z')$. This, however, is not the case for the first formulation. For this reason and because it may even be a more adequate interpretation of the standard expositions we will assume the first formulation from now on.

It is not difficult to check that Th.1, and hence the reduction theorem Th.2, can be proved more easily with KTR, although it is still necessary to use RP2. That is, if S(tandard) RP indicates the conjunction of RP1 and RP2 and KTR, we have

$$\text{Th.4} \quad \text{EU-KH SRP-implies EUC-FL.}$$

But, with SRP we can also prove the 'reverse reduction':

Th.5 EUC-FL SRP-implies EU-KH (where E-FLS RP-implies E-KH is crucial).

For its proof KTR is not only useful but even essential, just as RP2 (and RP1). Hence we may conclude that KTR is responsible for this reverse reduction and that it is, in view of Th.2, not necessary for the reduction in the appropriate direction.

From the foregoing, it may *not* be concluded that we dispute the validity of KTR. On the contrary, we have

Th.6 E-KH and EC-FL RP-imply KTR.

The proof is again elementary. From that proof it is clear that the premises E-KH and C-FL are essential (note that E-FL is, according to Th.2.1, an RP-consequence of E-KH). Hence, our conclusion is straightforward: KTR can and must be explained by appeal to E-KH, in addition to EC-FL and RP. Therefore, KTR does not represent an ontological identity, and hence it is not adequate as a reduction postulate, but follows as an explainable bridge principle, to be called a theoretical identity, i.e., an identity laden with a reducing hypothesis (E-KH) and also with reduced laws (EC-FL).

This leads to the following general picture of micro-reduction of macro-laws. The proper reduction postulates, being ontological identities, lead to asymmetric reduction. That is, they enable only the derivation of the macro-laws from the relevant part of the micro-theory, the so-called reducing micro-hypotheses, notwithstanding evidence transport in the other direction. However, it may be that the reduction postulates, combined with a reducing micro-hypothesis (and some macro-laws, but this may not be essential) generate additional (or stronger) bridge principles, called theoretical identities, which conceal the asymmetry. The conjunction of ontological and theoretical identities may enable the derivation of the reducing micro-hypotheses from the macro-laws, i.e., the 'reverse reduction'. In this description 'the derivation . . . from' may be replaced by 'the reduction . . . to' and it will be clear how to amend it if there is a macro-theory on the macro-side.

7. The (Extended) Ideal Gas Law. We remarked already that Th.2 states in fact the reduction of the asymptotically interpreted ideal gas law. In view of the definition of an Asymptotic Ideal Gas (AIG, Def. 3) this theorem may be restated as:

Th.7 EU-KH RP-implies $\emptyset \neq G = ABG = AIG = APG$,

that is, if the kinetic theory is true, at least EU-KH, and if RP is correct, then, as a matter of theory, the notion AIG has the same extension as the

notion APG, viz. G , i.e., all gases. Hence, Th.7 provides the legitimation of the practice of equating asymptotic ideal and perfect behavior.

In philosophical expositions the ideal gas law is usually stated without the restriction to asymptotic behavior, i.e., as a strong idealization indeed. It will be shown, in an indirect way, that our reduction postulates are sufficient for the reduction of this extended ideal gas law.

The following notions, laws and hypotheses are all straightforward generalizations of the asymptotic ones.

Def. 5.1 g in G is a *Boyle Gas* (BG) iff there is a one-one function $B_g: S \rightarrow IR^+$ such that $P(z)V(z) = B_g(s)$ if $t(z) = s$,

2 g in G is an *Ideal Gas* (IG) iff $P(z)V(z) = RT(z)$,

3 g in G is a *Perfect Gas* (PG) iff $p(z)V(z) = (2/3) N\bar{u}(z)$.

\bar{E} -FL some G are BG,

\bar{C} -FL all BG are Comparable,

\bar{U} -FL all G are BG,

\bar{E} -KH some G are PG,

\bar{U} -KH all G are PG.

Again it is easy to see that the law:

some G are IG and all G are IG

is not only a straightforward interpretation of the extended ideal gas law, with existential import, but also that it is equivalent to $\bar{E}\bar{U}\bar{C}$ -FL.

Once we have agreed upon the localization of the asymmetry in the reduction and the presuppositions of KTR (according to Th.6:E-KH, EC-FL and RP) we may of course use SRP (i.e., RP and KTR) to legitimate extended 'reduction' claims. Analogous to Th.1,2 (or 4),3,5 and 7 we get:

Th.8.1 SRP implies: all PG are Comparable BG,

2 $\bar{E}\bar{U}$ -KH SRP-implies $\bar{E}\bar{U}\bar{C}$ -FL, i.e., the extended ideal gas law,

3 \bar{E} -KH and $\bar{U}\bar{C}$ -FL SRP-imply \bar{U} -KH,

4 $\bar{E}\bar{U}\bar{C}$ -FL SRP-implies $\bar{E}\bar{U}$ -KH,

5 $\bar{E}\bar{U}$ -KH SRP-implies $\emptyset \neq G = \text{BG} = \text{IG} = \text{PG}$.

It is to be noted, however, that the role of KTR in the respective proofs may be quite different. In Th.8.1 KTR is helpful, but not essential. It is

only essential in Th.8.4; to be precise, in view of the premise, it is necessary to use E-KH. In Th.8.2,8.3 and 8.5 KTR is already RP-implied by the premises. In particular, from the fact that $\overline{E\bar{U}}$ -KH obviously implies EU-KH and Th.2 and 6 we may conclude that Th.8.2 may be strengthened to

Th.9 $\overline{E\bar{U}}$ -KH RP-implies $\overline{E\bar{U}C}$ -FL,

and, hence, that RP is also sufficient for the reduction of the extended ideal gas law.

8. Concluding Remarks. The foregoing analysis may not be surprising for physicists. In particular, we guess, on the basis of the verbal comments in some textbooks, that many physicists already share the conviction that the identification of thermal equilibrium with equal mean kinetic energy is crucial for the reduction, notwithstanding the introduction of KTR as a postulate in the technical part of textbook presentations. Our analysis legitimates this conviction.

But apart from the value of a detailed account of the present case of reduction for its own sake, it provides a non-trivial illustration of, and hence support for, the ontological-identity-view on (micro-) reduction, including some further articulations of that view. To summarize once more; specifically for micro-reduction:

- a. the reduction postulates are (or represent) ontological identities, and hence are not further explainable,
- b. they enable only the reduction in the appropriate direction,
- c. notwithstanding transport of evidence in the other direction,
- d. and the possibility of theoretical identities, laden with the micro-theory, enabling the 'reverse reduction'.

Still outstanding is the problem of the reduction of (equilibrium) thermodynamics to classical and quantum (equilibrium) statistical mechanics. In fact, the present research was started as a small preparatory study. We do not claim that our analysis is directly applicable to the big reduction problem. Nonetheless, a detailed analysis, in the spirit of the present one, may well show a similar structure. For, after all, the textbook bridge principles also allow here the reverse reduction, and it is difficult to accept them as ontological identities. If we find a similar structure it will also become clear what the precise relation is with the present case. We may expect at least compatibility. At the moment it seems to make little sense to write down our further speculations on all these points. For, from the present paper, it will be clear that the process of trying to prove such speculations leads to many corrections, until the puzzle is solved. More-

over, we do claim that the present analysis does not depend on the solution of the big reduction problem.

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