UNBOUNDED OPERATORS AND THE INCOMPLETENESS OF QUANTUM MECHANICS

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ABSTRACT. A proof is presented that a form of incompleteness in Quantum Mechanics follows directly from the use of unbounded operators. It is then shown that the problems that arise for such operators are not connected to the *non-commutativity* of many pairs of operators in Quantum Mechanics and hence are an additional source of incompleteness to that which allegedly flows from the EPR paradox. Finally, it will be argued that the problem is not amenable to some simple solutions that will be considered.

It is well known to all workers in the field that unbounded operators are necessary in Quantum Mechanics (QM) and that they are, in many respects, more difficult to deal with than bounded operators. The purpose of this note is to extract some rather surprising philosophical points from the elementary postulates of QM, in particular that the use of unbounded operators leads directly to a kind of incompleteness. Indeed it is the fact that the postulates are so well known that makes the consequences so surprising. The principal reason that the conclusions have not hitherto been drawn must then lie in some rather subtle mathematical details of the unbounded operators themselves and the historical tendency to ascribe to them properties that only apply in the bounded case. This tendency goes back at least as far as Dirac (1930) and has been commented upon before (see Jauch (1972)). Before giving the main argument leading to incompleteness I will note some of the essential differences between bounded and unbounded self-adjoint operators.

We will not rehearse the elementary details of QM save where necessary for the argument. Furthermore, since in the following we are only concerned with linear operators we will drop the term 'linear' and the reader should take as understood that the operators are such. Likewise we are interested only in *complex* Hilbert spaces.

A bounded operator A defined on some Hilbert space $\mathscr{H}$ with domain $D(A) \subseteq \mathscr{H}$ is a function such that

$$\|A\psi\| \leq k\|\psi\|, \qquad \text{for} \quad \psi \in D(A), k \in \mathbb{R}^+$$

and where $\| \cdot \|$ denotes the norm of the vector enclosed, as defined by the scalar product on $\mathscr{H}$. An unbounded operator is one for which there is no such k. Informally we could say that a bounded operator maps the vectors in $\mathscr{H}$ into the interior of a finite shell of "radius" k. We will denote the scalar product on $\mathscr{H}$, for some

pair $\phi, \psi \in \mathscr{H}$, by $< \phi, \psi >$ and take it as linear in the first slot and antilinear in the second. A self-adjoint bounded operator $A : \mathscr{H} \rightarrow \mathscr{H}$ is such that

$$(1) \qquad\qquad < \phi, A\psi >=< A\phi, \psi >, \quad \text{for all} \quad \phi, \psi \in D(A)$$

Now it follows from a theorem by Hellinger and Toeplitz that if an operator obeys (1) and is defined on the whole of $\mathscr{H}$ then the operator must be bounded.[1] Consequently if an operator obeys (1) and is unbounded it cannot be defined on all of $\mathscr{H}$. But (1) is taken to be part of the definition of self-adjointness for unbounded operators so it follows that self-adjoint unbounded operators cannot be defined on every vector in $\mathscr{H}$—they are defined only on dense subspaces of $\mathscr{H}$.[2] This result should be familiar to the student of functional analysis but is often significantly underemphasized in the physical applications. Indeed, as an illustration of the way domain problems are neglected we could merely note that (1) is most often taken to be the sole defining criterion of self-adjointness. This is not the case. A densely defined operator that obeys (1) is Hermitian (symmetric) but not necessarily self-adjoint. In order to be self-adjoint it must be Hermitian and satisfy the condition that its domain $D(A)$ be the same as the domain of its Hilbert adjoint A^*, that is,

$$(2) \qquad\qquad D(A) = D(A^*)$$

Now it is a highly non-trivial matter to find the appropriate domain for a self-adjoint operator: in general a symmetric operator may have a large number of self-adjoint extensions that satisfy (2). Indeed, proving the self-adjointness of many of the Hamiltonians that arise in Constructive Quantum Field Theory has required a great deal of collective effort.[3] Still the reader may wonder whether one can't use a Hermitian operator with an ill-defined domain and this be good enough for most practical purposes. Again the answer is "no". One reason is that only self-adjoint operators will generate the one parameter unitary groups required for the dynamics of quantum systems. A second reason is that certain physically important properties of self-adjoint operators are sensitive to the choice of domain. So if one were to have a Hermitian operator which had two different self-adjoint extensions there is no guarantee at all that, for example, their spectra would agree (Reed and Simon 1980, p. 254). Before going further with our discussion of unbounded operators we should attempt to make precise what we mean by the putative completeness or incompleteness of QM. We can begin by citing Einstein, Podolsky and Rosen's definition of completeness for a general physical theory from their ([1935], 1970).

> Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one:

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[1] The Hellinger-Toeplitz theorem can be found in most texts on Functional Analysis, for example Reed and Simon (1980).

[2] The reader will observe that the denseness of the domain does not follow from the Hellinger-Toeplitz theorem alone; the difference is made up the requirement that the adjoint A^* of A be an operator with a unique value (that is, that it properly be a function).

[3] See for example Glimm and Jaffe (1985) and the references contained therein; also Reed and Simon (1975). It is worth pointing out that the use of unbounded operators in Quantum Field Theory means that our argument is just as much a problem for QFT as it is for QM.

> every element of the physical reality must have a counterpart in the
> physical theory, (quoted from Toulmin (1970), p. 124)

As it stands this necessary condition is probably too strong: quantum mechanics should not be regarded as incomplete for its failure to have a counterpart to Minkowski space-time. What one requires is that the physical domain that the theory is intended to cover is represented by counterparts in the theory. One can allow that there may be aspects of physical reality that are not germane to the particular theory in question. This quibble aside we can readily understand what Einstein, Podolsky and Rosen intend: a theory is complete if, for every element of the physical domain that the theory covers, there is some element of the theory that is sufficiently explained by the descriptive or nomic components of the theory. The residual vagueness of this formulation (how "sufficiently explained"?) is part of the necessary trade-off with generality. However, the informal notion of incompleteness goes a little further than this. A theory may also be said to be explanatorily incomplete when it goes beyond physical reality but in which there is no procedure for generating the necessary restrictions. (Perhaps an example of this would be the Special Theory of Relativity's allowing for the existence of tachyons, however in this case one would be inclined merely to add to the theory the postulate that super-luminal velocities are everywhere uninstantiated.) We intend the word 'incompleteness' in this paper to cover both types of incompleteness. At the end of the paper we will return to these general notions and show how QM can be said to be incomplete in either one or the other of the two senses.

We are now in a position to give our proof of the incompleteness of QM The preceding discussion should have convinced the reader of two things. Firstly, self-adjoint operators are required in QM, their being merely Hermitian is not enough; and secondly, the domain of an unbounded self-adjoint operator cannot be the whole of $\mathcal{H}$. Mere notional agreement is not enough: in the past physicists and philosophers have too often paid lip service to domain restrictions and ignored them in calculations.[4]

We begin by stating two of the most fundamental postulates of QM.

Postulate 1: There is a one-to-one correspondence between the possible states of a system and the normed rays of a Hilbert space $\mathcal{H}$. The rays are equivalence classes of vectors in $\mathcal{H}$ in which any member of the class can be obtained from some other member of the same class by multiplying the latter by a complex number of unit square modulus. Thus if ψ is a vector in $\mathcal{H}$ and $[\psi]$ is its equivalence class under the definition

$$[\psi] =_{df} e^{i\theta}\psi$$

then $[\psi]$ is a ray in $\mathcal{H}$. $[\psi]$ is a normed ray if $||\psi|| = 1$. (We will henceforth drop the square brackets for $[\psi]$.)

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[4]There are many of course, who do pay heed to domain problems, for example Jauch (1968). The paper (1971) by Stein and Shimony is also notable for its care in such matters.

Postulate 1 is probably so familiar that it requires little comment but we return to its motivation a little later. Our second postulate was given and motivated by our previous discussion.

> **Postulate 2:** The observables of the system are represented by self- adjoint operators acting on $\mathscr{H}$. These may be either bounded or unbounded. Examples of unbounded operators that represent observables are the position operator, the momentum operator and the Hamiltonian.

We could have given a stronger form of this postulate by requiring that there be a one-to-one correspondence between all self-adjoint operators on $\mathscr{H}$ and the observables of a system. This would appear less well motivated, however, and there have been objections raised to it in the past. (See, for example. Wigner (1973) p. 371.)

From these two elementary postulates we obtain our incompleteness claim (almost) directly. If every normed ray represents a possible state and if unbounded self-adjoint operators are defined only on a dense subspace of $\mathscr{H}$ then it follows that there are states of the system for which (taking the position operator as a representative example) the system cannot be operated on by the position operator. Therefore, whatever information about the system is delivered by the position operator will be unavailable in this particular case. We can see from the outset, however, that the problem here is not merely that the system is in a superposition of "eigenstates" rather than some particular "eigenstate." The real problem is that the state is simply not in the domain of the position operator.[5] Just what this signifies will depend upon what information the position operator gives about a system, a question that we will return to later in the paper. For now, however, we see the beginnings of an interpretative problem for QM which we wish to make more definite. In particular we wish to elaborate on the incompleteness of the theory and the way in which it follows from the elementary postulates.

We said that this incompleteness follows "almost" directly. This is because it is possible, as far as the above argument goes, that the domains of the unbounded operators be dense in $\mathscr{H}$ but that nevertheless each normed vector be in those domains. We now show that this is not so by explicitly producing a normed function in $L^2(\mathbb{R})$ that is not in the domain of the position operator. The position operator Q has a domain $D(Q) \subset L^2(\mathbb{R})$ defined by the condition

$$(3) \qquad D(Q) = \left\{ \psi(x) : \int_{\mathbb{R}} x^2 |\psi(x)|^2 dx < \infty \right\}$$

where $(Q\psi)(x) = x\psi(x)$. In other words the domain $D(Q)$ is defined explicitly by the condition that the range be in $L^2(\mathbb{R})$. Therefore all we need to do in order to demonstrate that the problematic conclusions given above do indeed follow is to give a normed function that is in $L^2(\mathbb{R})$ but not in $D(Q)$, that is, so that $x\psi(x)$ is

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[5]The quote remarks around the word 'eigenstate' should serve to remind the reader that such talk is strictly false: neither the position nor the momentum operators have eigenvectors or eigenvalues.

not in $L^2(\mathbb{R})$. The following function has just these properties

(4)
$$\psi(x) = \begin{cases} x^{-1} & \text{if} \quad x \geq 1 \\ \\ 0 & \text{if} \quad x < 1 \end{cases}$$

It can easily be confirmed that this function has norm 1 by evaluating the integral

$$\int_{\mathbb{R}} \overline{\psi(x)}\psi(x)dx$$

(and since the function is real valued we can ignore the complex conjugation.) In fact it is convenient to divide the domain into $(-\infty, 1)$ and $[1, \infty)$ to obtain

$$\int_{-\infty}^{1} 0dx + \int_{1}^{+\infty} x^{-2}dx.$$

The first term is then zero, of course, just leaving us with the second term, which becomes

$$F(x) = -x^{-1} + c \left. \right|_{1}^{\infty}$$

The lim $x \to +\infty$ gives $F(x) = c$. The norm is therefore 1, from $c - (-1 + c) = +1$. We have proven that the vector represents a state on Postulate 1. We now need to show that the position operator maps this $\psi(x)$ into a function that is not in $L^2(\mathbb{R})$. We do this by showing that the norm of $x\psi(x)$ is infinite. Again, the integral of the function from 1 back to negative infinity is simply zero so just consider the non-zero part of the function

$$\int_{1}^{+\infty} (x(1/x) \cdot (x(1/x))dx \quad = \int_{1}^{+\infty} 1dx \quad = \infty$$

This shows that the norm is infinite and that the function is not in $L^2(\mathbb{R})$. Hence we have found a state vector that is not in the domain of Q. The theory is therefore incomplete, though we do not yet know the significance of this incompleteness.

Now it may seem to the reader that we are merely restating something that has long been known to hold in QM, namely that (put very informally) if the momentum is sharply defined, the position is smeared out to infinity. This idea would have it that the problem that we've just given is somehow a consequence of the non-commutativity of position and momentum and therefore of the Heisenberg Uncertainty Relations. We now show that this is not the case.

It suffices to show that not only is our state function (4) not a member of the domain of the position operator, it is not a member of the domain of the momentum operator. Consider the definition of the momentum operator $P : D(P) \to L^2(\mathbb{R})$; here $D(P)$ consists of all absolutely continuous functions $f \in L^2(\mathbb{R})$ and P is defined by

$$P : \psi(x) \mapsto \hbar i^{-1}(d\psi(x))/dx.$$

It is obvious that (4) is discontinuous at $x = 1$ and so is not in the domain of P. Since (4) is not even in the domain of P it cannot be the case that—as the above

informal idea suggests—that (4) is an "eigenvector" of the momentum operator. So (4) not being in the domain of the position operator is not a consequence of the Commutation Relations—it is quite independent.

At this point it may seem to the reader that there is a rather simple solution to the problem that we have given, namely weaken Postulate 1 so that the problem functions do not represent states. Unfortunately we have good reasons for accepting Postulate 1, which, though not completely compelling, suggest that any weakening may cause trouble with quantum theory. A considerable source of difficulty is that the domain restrictions on the unbounded operators that one is interested in will not coincide—simply because the domains do not coincide. If one excludes from statehood those functions that are not in $D(Q)$ one will still have states that are not in $D(P)$. If one considers only the set of states

$$D(P) \cap D(Q) \cap D(\mathscr{H})$$

one will have excluded functions that did properly represent elements in $D(P)$ *etc*. More importantly, the operators that result from this curtailment of domains will almost certainly not be self-adjoint. But this leads on to the second problem. If one considers Mackey's axiomatic reconstruction of QM in Mackey (1963), there is a rigorous attempt to build up the usual Hilbert space presentation of the theory from more elementary postulates. Beginning with a lattice $\mathscr{L}$ of questions (or "propositions," in Jauch's terminology), the states are built up via a one-to-one correspondence between $\mathscr{H}$ and the closed subspaces of a separable Hilbert space $\mathscr{L}$. The states are obtained explicitly from Gleason's theorem. Now, since there is a one-to-one correspondence between rays and one dimensional subspaces of $\mathscr{H}$, any attempt to limit the class of rays that represent states must have a corresponding effect on the lattice $\mathscr{L}$ of questions. In other words such radical surgery on $\mathscr{H}$ can be expected to severely upset many of our foundational theorems on the structure of QM.

We will close with one final matter that requires consideration and is connected to the above point about lattices. So far we have discussed the self-adjoint operators in a particular form which we might call the functional form of the operator. By this we intend no connotations of linear functionals, merely that the operators are an explicit device for converting a function in $L^2(\mathbb{R})$ into some other function in $L^2(\mathbb{R})$, modulo domain restrictions. It is frequently useful to use an alternative form for the operators of interest, namely the spectral form, which employs projection operators. In Mackey's system these projection operators are axiomatically more fundamental. He terms them questions, or question-observables, and they are used to form measures which then allow them to be endowed with the structure of a lattice (denoted $\mathscr{L}$ in the previous paragraph). These projection operators or questions are bounded since their spectrum consists of the two points 0 and 1.

Now we might imagine the following line of reasoning in response to the problem raised by this paper: the projection operators are bounded and therefore are defined on all of $\mathscr{H}$ and, since they are more fundamental, we should consider them to be our principal means of obtaining expectation values for observables; since they are defined everywhere in the Hilbert space we never have the problem of vectors not
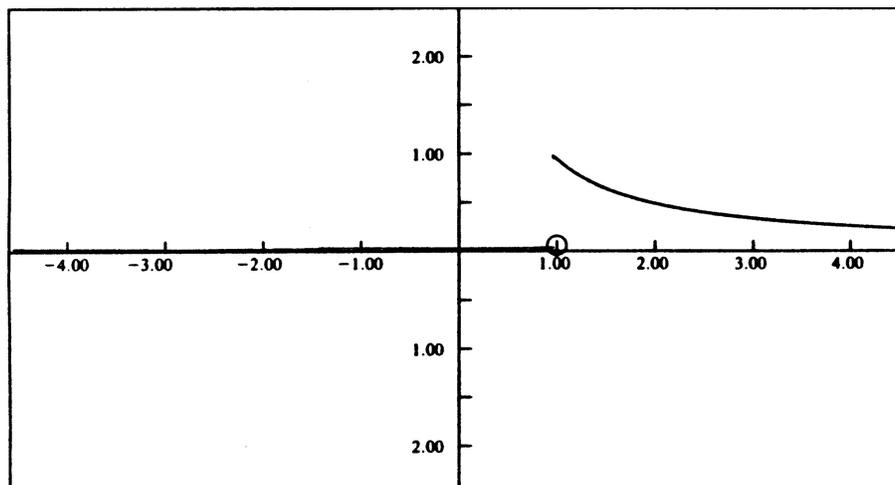
FIGURE 1. We give the graph of the function (4) so that this can be seen at a glance. It is clear from an inspection of the function that there will be many such functions not in the domain $D(Q) \cap D(P)$. It thus follows that the Commutation Relation $PQ - QP = -i\hbar I$ will not hold everywhere but at most on a dense subdomain of $\mathscr{H}$. Hence the Heisenberg Uncertainty Relations will only hold "almost everywhere."

lying in their domains, therefore there is no incompleteness. (Or rather, there is incompleteness only when one considers the operator in the wrong form, namely the functional form.)

We can give some idea of what is wrong (and also what is right) with this response only by giving some details of projection operators and their connection with self-adjoint operators in the functional form. Given a Borel subset ft of the real line IR an orthogonal projection operator P_Ω^A is a bounded, self-adjoint idempotent operator, that is,

$$P_\Omega^A = P_\Omega^{A^*}, \quad P_\Omega^A P_\Omega^A = P_\Omega^A$$

The A indicates that the projection is dedicated to this operator and P_Ω^A for some Ω is called a spectral projection of A. It is worth seeing how the projection operators work explicitly in the case of the position operator Q. The spectrum of Q, denoted $\sigma(Q)$, is equal to $\mathbb{R}$ and, like the momentum operator, contains no eigenvalues.

In general the characteristic function of A, $\chi_\Omega(A)$ is effectively a function on $\sigma(A)$. Such functions of operators are constructed in what is called the *functional calculus*. This part of the spectral theorem attempts to establish the existence of a unique map Ψ from the topological space of real-valued Borel functions to the space of operators on a Hilbert space equipped with the norm operator topology. In fact the existence of such a map can be established when the operators are self-adjoint in both the bounded and unbounded case. Since the characteristic function $\chi_\Omega(x)$ is a real-valued Borel function, that is, the function whose value is 1 for $x \in \Omega$ and

0 otherwise, the unique map Ψ associates it with a function of an operator $\chi_\Omega(A)$, which is equivalent to a projection operator P_Ω^A. Thus we have $\chi_\Omega(A) = P_\Omega^A$. Its action is to select some portion of the spectrum $\sigma(A)$, namely that lying in Ω, and to construct a new operator which is 1 for $\lambda \in \Omega$ and 0 for $\lambda \notin \Omega$. In effect this operator takes a snapshot of the vector $\psi \in \mathscr{H}$ on Ω and relegates the rest of the function to 0. In the case of Q, which is the function from $\psi(x) \rightarrow x\psi(x)$, for $\psi(x) \in D(Q)$, we can think of P_Ω^Q as a composition of the functions $\chi_\Omega(x)$ and $f(x) = x$, that is $(\chi_\Omega \circ f)(x)$. A wave function $\psi(x)$ is then mapped to

$$P_\Omega^Q \psi(x) = \begin{cases} \psi(x) & \text{if} \quad x \in \Omega \\ \\ 0 & \text{if} \quad x \notin \Omega \end{cases}$$

We return to the physical interpretation of this shortly. In general, the range of such projection operators is always a closed subspace of $\mathscr{H}$.

The connection between a self-adjoint operator A and the projection operators is the content of the spectral theorem. We do not need to give the full details here but merely note that for every bounded and unbounded self-adjoint operator A there is a family P_Ω of spectral projections of A which forms a projection valued measure (p.v.m.). This p.v.m. can be used to form a standard Borel measure on $\mathbb{R}$ by taking $\phi \in \mathscr{H}$ and forming $< \phi, P_\Omega \phi >$. The resultant measure can be used in integration so that one obtains

$$(5) \qquad\qquad < \phi, A\psi > = \int_{-\infty}^{+\infty} \lambda d < \phi, P_\lambda \psi >$$

for A unbounded, and

$$(6) \qquad\qquad < \phi, A\psi > = \int_{-a}^{+a} \lambda d < \phi, P_\lambda \psi >$$

for A bounded and some interval $[-a, a]$. For our purposes the important thing to note is that the right hand sides, as well as the left hand sides, are only defined for $\phi, \psi \in D(A)$. In the case of bounded A, $D(A) = \mathscr{H}$, so the measure on the left hand side is defined for all $\phi, \psi \in \mathscr{H}$ but in the unbounded case this is, of course, not true. So passing to a p.v.m. via the spectral theorem, cannot be a way of eliminating domain restrictions.

The import of this is that one cannot escape domain restrictions by passing to projection valued measures. They are fully inherited. This, I suggest, is exactly what one should expect. The spectral theorem gives an equivalence between a self-adjoint operator and a certain integral over a function defined on a measure space obtained from projection operators. This equivalence will not permit what is undefined on one side of the equal sign to be defined on the other.[6]

What of the projection operators themselves and the fact that they are defined everywhere on ? In order to answer this question we must look at the physical

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[6]This version of the spectral theorem is essentially taken from Reed and Simon (1980) and Naimark (1960) but can also be found (in a more quantum mechanical context) in Emch (1984) or Jauch (1968). Robert Geroch's (1985) is also excellent on the spectral theorem.

interpretation of the function (4) and the various computations on it. Calculating that the norm of the function was equal to one was effectively establishing that a probability distribution exists for the system in question. It corresponds to a particle localized "rather badly" in the interval $[1, \infty)$. When we attempt to compute the expectation value by traditional means, that is, by:

$$\mu_\psi = <\psi, Q\psi> = \int_{-\infty}^{+\infty} x|\psi(x)|^2 dx$$

we see that it is undefined because $(Q\psi)(x)$ is not an element of the Hilbert space. We may hope to overcome this problem by passing to projection operators. If we consider the projection P_Ω^Q for some $\mathscr{H}$ and we have it operate on the function (4) it is equivalent to asking the question: is the particle in the set Ω? This is the source of the identification of projections and questions in Mackey's axiomatics. The measure defined by $<\psi, P_\Omega^Q>$ gives the probability of finding the particle in Ω. Clearly as Ω is increased across $\mathbb{R}$ to cover $[1, +\infty)$ the probability measure must go to 1. This is effectively just restating the fact that a probability distribution exists for our function $\psi(x)$. However, none of this helps in obtaining an expectation value, since that is given by (6) and is only defined for some $\psi(x) \in D(Q)$. This expectation value, or mean value, is undefined precisely because it is infinite. The lack of compact support for the function (4) combined with the slow decay of the function at infinity shifts the mean of the probability distribution to infinity.

Thus far we have concentrated on the position and momentum operator since they are the most computationally tractable and familiar. We can, however, put an additional argument for the substantial incompleteness of QM. It is this: consider the Hamiltonian operator H given by $H = 2m^{-1}P^2 + V$, where V is the self-adjoint operator for the potential energy and P^2 is the self-adjoint square of the momentum operator. In general the domain of H is $D(P^2) \cap D(V)$, although we should note that certain restrictions must be placed on V in order for this to make sense. Now our function (4) is not in the domain of Ha fact we state without proof since it is easily checkedand this means that the expectation for the energy is undefined. But if the product $<\psi, H\psi>$ is undefined, because infinte, for some ψ in $\mathscr{H}$, then we have states that would require an infinite amount of energy to prepare. Since an infinite amount of energy would be required to prepare such states the functions for which $<\psi, H\psi>$ is infinite are physically impossible. Indeed, projection operators will not ameliorate this problem at all, as will be clear from a comparison of this case with the infinite expectation for the position operator.

The Hamiltonian operator is physically far more significant than the position operator in that it governs the time dependent evolution of the system. The Schrödinger equation $i\hbar d\psi(x,t)/dt = H\psi(x,t)$ is only available for those $\psi(x)$ in the domain of H. If all systems are governed by this evolution then we should clearly hold that only such $\psi(x)$ as are in $D(H)$ represent states. This will clearly exclude functions that are admitted as states by postulate 1. We have good reason, therefore, to believe that L^2 spaces are strictly too large—they contain normed functions which cannot be realized as states of any system, contrary to postulate 1.

Our incompleteness dilemma can now be stated thus: either Postulate 1 must be altered—clearly Postulate 2 is too well-motivated to be changed very much, if at all—or there are states of physical systems whose expectation for position, momentum and total energy are generally undefined or infinite. If Postulate 1 must be altered then QM could be said to be incomplete in the sense that we do not have within the theory an algorithm for generating the appropriate restrictions on H. On the other hand, if Postulate 1 is retained, then QM is incomplete in the sense that it gives us no account of those systems that are able to have a particular wave function but no mean finite position, momentum or energy—*and, if the latter, no Schrödinger evolution.* Either way QM is incomplete.

What, then, should we take this incompleteness result to signify? We cannot of course, with any propriety, give a definitive answer to this question. At one level we could simply add it to the list of interpretive problems for quantum theory. My own view, however, is that it signifies the existence of inherent limitations in the use of Hilbert space as the appropriate construction for representing physical states. This suggestion is not without precedent. It has been known for many years that the Hilbert space of interacting quantum fields, Fock space, has rather severe inadequacies. The problem with Fock space led to renewed interest, in the 1960s and 70s, in purely algebraic generalizations of quantum mechanics and quantum field theory, following the work of Segal and Haag (see Emch (1970)). The current status of this work seems rather unclear.[7] However, it seems clear that there are problems with Hilbert space. Since our incompleteness result applies to all unbounded self-adjoint operators it spans both quantum mechanics and quantum field theory. We should take it as further evidence that we are not yet in possession of the correct mathematical formalism for micro-systems.[8]

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[7]In recent years there has been a convergence of opinion that the quantum logic (QL) approach to QM is probably wrong. The thought is that, though the idea of a QL might be a good one, there is something wrong with the particular logic that we have traditionally worked with. Since standard QL arises from the lattice of all closed subspaces of $\mathscr{H}$ our suggestion that $L^2(\mathbb{R})$ is too large and that some 1-dimensional subspaces may not represent states can be seen to entail the falsity of standard QL.

[8]I wish to express my gratitude to Alan Carey for correspondence on the subject of this paper.