A Conception of Inductive Logic

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Abstract. Explication is the activity of finding a precise concept (the explicatum) that is similar to a given vague concept (the explicandum). I conceive of inductive logic as a project of explication in this sense. The explicandum is one of the meanings of the word 'probability' in ordinary language; I call it *inductive probability* and argue that it is logical, in a certain sense. The explicatum is a conditional probability function that is specified by stipulative definition. This conception of inductive logic is close to Carnap's but common objections to Carnapian inductive logic (the probabilities don't exist, are arbitrary, etc.) do not apply to this conception.

1. Explicandum

Explication consists in finding a clear and precise concept that is similar to a given vague or otherwise unclear concept. The given concept is then called the *explicandum* and the similar precise concept is called the *explicatum*.[1] Inductive logic, as I conceive of it, is a project of explication in this sense. The present section will identify and clarify the explicandum.

1.1. Identification

Suppose you have been told that a coin either has heads on both sides or else has tails on both sides and that it is about to be tossed. What is the probability that it will land heads? There are two natural answers: (i) 1/2; (ii) either 0 or 1 but I do not know which. These two answers reflect two different meanings of 'probability'.

I will use 'inductive probability' for the meaning of 'probability' in which the natural answer is (i). I will use 'physical probability' for the meaning of 'probability' in which the natural answer is (ii).

I take the explicandum for inductive logic to be the concept of inductive probability. So the explicandum is that meaning of 'probability' in ordinary language for which 1/2 is the natural answer to the above question.

In this example, the inductive probability is naturally supposed to have a precise numerical value, namely 1/2. However, inductive probabilities often have no precise numerical value. For example, the

[1] For further discussion of explication see Carnap (1950, §§2,3; 1963, §19) and Quine (1960, §53).

inductive probability that I will have an accident while driving to campus on a particular day, given what I know, is greater than 0 and less than 1/100 but it does not have a precise numerical value. Furthermore, this probability has no precise greatest lower bound or least upper bound and hence we cannot even say that its value is some interval of numbers. Thus inductive probability is a vague concept. This should not be surprising; inductive probability is a concept of ordinary language and most concepts of ordinary language are vague. The reason for this vagueness is also clear; these concepts are learned from examples of their use and the examples are commonly insufficient to determine precise boundaries for the concept in all possible situations.

1.2. Not degree of belief

I will now argue that inductive probability is not the same thing as degree of belief.

Degrees of belief are relative to a person and a time but not to evidence. For example, I might say that Jones now believes to degree 3/4 that the coin will land heads. Inductive probabilities are the opposite of this; they are relative to evidence but not to a person or a time. Thus in my first example I specified the evidence (the coin has either heads on both sides or tails on both sides and is about to be tossed) but did not need to specify a person or a time.

Perhaps some subjectivists think that, when people make statements of inductive probability without specifying the person and the time, they are making claims about their own current degrees of belief. If that were right, then if I say that the (inductive) probability of the coin landing heads is 1/2, and this statement is challenged, I could prove that my statement was correct by proving that, at the time I made the statement, I believed to degree 1/2 that the coin would land heads. But a statement of inductive probability cannot be proved in this way, so it is not a statement about the speaker's current degrees of belief.

Of course, assertions about inductive probability *express* the speaker's beliefs; but it does not follow that they are *about* the speaker's beliefs, which is what is at issue here. All sincere and intentional assertions express the speaker's beliefs but most assertions are not about the speaker's beliefs.

1.3. Rational degree of belief?

We have seen that inductive probability is not the same as *actual* degree of belief. Can we then identify it with *rational* degree of belief? More precisely, the view to be considered is:

(1) The inductive probability of H given E is the degree of belief in H that is rational for a person whose total evidence is E.

To evaluate this we need to know what is meant by 'rational degree of belief'.

Suppose that a rational degree of belief for person X is one that it makes sense for X to adopt, given X's values and information. Then if a high degree of belief in H would help X succeed in some activity, or make X feel happy, or avoid punishment by the thought police, it could for that reason be rational for X to have a high degree of belief in H, even if H has a low inductive probability given X's evidence. Thus (1) is false on this interpretation of 'rational degree of belief'.

We might modify this interpretation of 'rational degree of belief' by allowing only epistemic values to be taken into consideration. But a high degree of belief in H might help X succeed in some epistemic project, such as conducting a difficult experiment; it might also elicit epistemic rewards or prevent epistemic punishment by the thought police. So on this interpretation of 'rational degree of belief', it can still be rational for X to have a high degree of belief in H even if H has a low inductive probability given X's evidence. Hence (1) is still false on this interpretation of 'rational degree of belief'.

Alternatively, we might start from the idea that rational beliefs are ones that are supported by the person's evidence. The natural way to extend this idea to *degrees* of belief is to say that X's degree of belief in H is rational if and only if it equals the inductive probability of H given X's evidence. On this understanding of 'rational degree of belief', (1) is correct but trivial.

So the identification of inductive probability with rational degree of belief is either wrong or trivial. Either way, it does not clarify the concept of inductive probability.

1.4. Degree of confirmation?

Carnap (1950, §9) said that the explicandum for inductive logic is the concept of *degree of confirmation*. So I now consider:

(2) The inductive probability of H given E is the degree to which H is confirmed by E.

To evaluate this we need to know what is meant by 'confirmation'.

Carnap (1950, preface to second edition, xviii) came to think that 'confirmation' has two senses. In one sense, E confirms H if E raises the probability of H; I will call this *incremental confirmation*. In the other sense, E confirms H if H is probable given E; I will call this *absolute confirmation*.

Suppose H is a simple tautology and E is some empirical evidence. Then the inductive probability of H has the same value (namely, one) given E and not given E, so E does not incrementally confirm H to any degree, although the inductive probability of H given E is maximal. Therefore, (2) is false if 'confirmation' means incremental confirmation.

Suppose, then, that 'confirmation' means absolute confirmation. Then for E to confirm H means that H is probable given E, so presumably the *degree* to which E confirms H is the *probability* of H given E. With this identification, (2) is true but trivial.

It won't help to consider other conceptions of confirmation. If 'confirmation' means anything other than absolute confirmation then the degree to which E confirms H will not always coincide with the inductive probability of H given E and hence (2) will be false.

So the identification of inductive probability with degree of confirmation is wrong or trivial. Either way, it does not clarify the concept of inductive probability.

1.5. Logical probability

Inductive probability differs from both degree of belief and from physical probability. A probability concept like this is apt to be referred to as *logical probability*. Is inductive probability then the same thing as logical probability? That depends on what is meant by 'logical probability'.

Many writers take logical probability to be a uniquely rational degree of belief (Salmon, 1967, 68; Skyrms, 1986, 207; Gillies, 2000, 1). But we have already seen that the identification of inductive probability with rational degree of belief is either false or trivial. So let us consider other ways of understanding 'logical probability'.

Sometimes logical probability is characterized as a generalization of the logical consequence relation (Roeper and Leblanc, 1999, xi, 142), which seems to mean that the logical probability of H given E has its maximum value if H is a logical consequence of E and its minimum value if H is inconsistent with E. However, every function that satisfies the axioms of probability, and infinitely many that do not, are logical probabilities in this sense. Even physical probabilities are logical probabilities in this sense. So while it is true that inductive probability is logical in this sense, that does not tell us what inductive probability is. Some authors (Franklin, 2001) take logical probability to be the one right generalization of the logical consequence relation, without saying what it means to be 'right', but they have not said enough to tell us what they mean by 'logical probability'.

I turn now to another conception of logical probability. Let an *elementary probability sentence* be a sentence which asserts that the probability of a specific hypothesis given specific evidence has a particular value. Let a *logically determinate sentence* be a sentence whose truth value is determined by meanings alone, independently of empirical facts. I will say that a probability concept is *logical in Carnap's sense* if all elementary probability sentences for it are logically determinate. (This terminology is motivated by the characterization of logical probability in Carnap, 1950, 20, 30).

The truth value of an elementary sentence of inductive probability does not depend on the physical world (as is the case with physical probability) or on some person's psychological state (as is the case with degree of belief). So it does not depend on empirical facts at all and hence the sentence is logically determinate. Thus inductive probability is logical in Carnap's sense. But every mathematically defined function that satisfies the axioms of probability is also a logical probability in Carnap's sense. In particular, an explicatum for inductive probability, of the kind I will discuss in Section 2, is logical in Carnap's sense, though it is not the same as inductive probability. Hence the concept of logical probability in Carnap's sense is wider than the concept of inductive probability.

However, 'probability' in ordinary language has only one meaning that is logical in Carnap's sense, namely inductive probability, so we can define the concept of inductive probability as the meaning of 'probability' in ordinary language that is logical in Carnap's sense.

1.6. Review

The concept of inductive probability is one meaning that 'probability' has in ordinary language. This meaning can be distinguished by examples of its use, as I did when I introduced the concept. It can also be distinguished by the observation that inductive probability is logical in Carnap's sense. It can be further clarified by noting what it is not. For example, inductive probability is not physical probability, or a person's degree of belief, or degree of belief that it would make sense for a person to adopt, or degree of incremental confirmation.

I think that these observations are sufficient to identify the concept that I take to be the explicandum for inductive logic. Of course, I have not given a clear and precise account of this concept but that is because the concept is neither clear nor precise. It is the task of explication to find a corresponding clear and precise concept, a matter to which I now turn.

2. Explicatum

Inductive logic, on my conception, proposes an explicatum for the concept of inductive probability. Explicata are not right or wrong but they may be judged better or worse according to the following criteria. First, an explicatum should be similar to the explicandum in those respects that are important for the usefulness of the explicandum for the ends that we have in view. (This does not preclude significant differences between explicandum and explicatum. There must be differences, since the explicandum is vague and the explicatum is precise.) Second, it is desirable that the explicatum be theoretically fruitful, which means that it satisfies general laws. Third, it is desirable for the explicatum to be simple, since this makes it easier to grasp and use.

Any proposal of an explicatum for inductive probability would count as inductive logic, on my view. But on the approach that I favor, the explicatum is a conditional probability function, which I will call 'p'. Thus p is a function which takes two sentences (or alternatively, propositions) as its arguments, has real numbers as its values, and satisfies some version of the mathematical laws of conditional probability. I will use '$p(\sigma_1, \sigma_2)$' to denote the value of p for arguments σ_1 and σ_2.[2] The function p is to be defined by specifying those σ_1 and σ_2 for which $p(\sigma_1, \sigma_2)$ exists and giving rules that logically determine the numeric value $p(\sigma_1, \sigma_2)$ for all such σ_1 and σ_2.

The definition of p will be chosen with the aim of having the numbers $p(\sigma_1, \sigma_2)$ be good explicata for the inductive probability of σ_1 given σ_2. So, for example, if the inductive probability of σ_1 given σ_2 has a precise numerical value, we would normally want $p(\sigma_1, \sigma_2)$ to be equal to that value. If the inductive probability of σ_1 given σ_2 is greater than that of σ_3 given σ_4 (which can be true even if these probabilities lack precise numeric values), then we would normally want to have $p(\sigma_1, \sigma_2) > p(\sigma_3, \sigma_4)$. One can easily specify other similar desiderata.

I said that p is required to satisfy the mathematical laws of conditional probability. In my view, there are two reasons for this requirement. First, inductive probabilities that have numerical values obey these laws and a good explicatum is similar to its explicandum, so the values $p(\sigma_1, \sigma_2)$ should satisfy these laws when the inductive probability of σ_1 given σ_2 has a numerical value. Second, the desiderata of fruitfulness and simplicity are advanced by requiring that the same laws also hold when the inductive probabilities lack numerical values.

[2] I separate the arguments of p by a comma, rather than the more customary vertical bar, in order to conform to the general convention for writing functions in logic and mathematics.

There is a large literature that can be interpreted as doing inductive logic as I conceive it, i.e., proposing conditional probability functions as explicata for inductive probability. However, the authors of this literature have not usually understood what they were doing in quite this way and their proposals often have unsatisfactory features. A simple example of an explication that I am willing to defend, and that consciously applies the conception of inductive logic that I have described, is at http://patrick.maher1.net/coileg.pdf.

3. Objections

The conception of inductive logic that I have been describing is similar to Carnap's (1950) conception, so I don't claim much originality for it. But philosophers today generally believe that Carnap's conception is demonstrably untenable. More generally, they take Carnap to advocate a logical theory of probability and they believe that no such theory is tenable. So in this section I consider common arguments against logical probability and show that they are not sound objections to the conception of inductive logic that I have described.

3.1. Ramsey's argument

Ramsey argued that logical probabilities do not exist because people "are able to come to so very little agreement" about them (1926, 27). Ramsey's argument was directed at Keynes's theory but many recent authors cite it as a powerful objection to any logical theory of probability (Gillies, 2000, 52; Percival, 2000, 365; Hacking, 2001, 144). I will now show that it is not a sound objection to the conception I have described.

In my conception of inductive logic there are two kinds of probability, both of which are logical in Carnap's sense. One is the explicandum, inductive probability, and the other is the explicatum, the function p. Now the values of p are fixed by definition, so there cannot be any doubt that p does have values and hence that *this* kind of logical probability exists. The only hope for a sceptic about the existence of logical probability is therefore to show that inductive probabilities do not exist.

However, with inductive probability, the facts are the opposite of what Ramsey says; there is actually a great deal of agreement about its values. For example, practically everyone assents to the following sentence:

WB. The probability that a ball is white, given that it is either white or black, is 1/2.

(I have discussed sentences like this with students in many classes and the students always think the sentences are obviously true.) The reference in WB to evidence, and the lack of any indication of an experimental setup, make it clear that this is a statement of inductive probability, not physical probability.

Ramsey himself conceded that “about some particular cases there is agreement” (28). He asserted that “these somehow paradoxically are always immensely complicated” but WB is a counterexample to that.

Ramsey gave the following argument to show that there is disagreement about logical probabilities of simple propositions:

> If [. . .] we take the simplest possible pairs of propositions such as ‘This is red’ and ‘That is blue’ or ‘This is red’ and ‘That is red,’ whose logical relations should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them. (28)

Ramsey is right that no one pretends to be sure of a numerical value for these inductive probabilities but this merely illustrates the point that many inductive probabilities lack a precise numerical value. It does not show disagreement; on the contrary, it shows *agreement*, since *no one* pretends to know numerical values here. Furthermore, there are qualitative judgments in this case that would secure wide agreement; for example, the inductive probability of ‘This is red’ given ‘That is red’ is higher than the inductive probability of ‘This is red’ given ‘That is blue’ (assuming nothing else is given).

Ramsey continued:

> Or, perhaps, they may claim to see the relation but they will not be able to say anything about it with certainty, to state if it is more or less than 1/3, or so on. They may, of course, say that it is incomparable with any numerical relation, but a relation about which so little can be truly said will be of little scientific use and it will be hard to convince a sceptic of its existence.

Although the inductive probabilities that Ramsey is discussing lack precise numerical values, they are not “incomparable with any numerical relation.” Since there are more than three different colors, the a priori inductive probability of ‘This is red’ must be less than 1/3 and so its inductive probability given ‘That is blue’ must likewise be less than 1/3. I believe that these relations would also command widespread agreement, at least on reflection.

I don’t deny that there are disagreements about the values of inductive probabilities. What I deny is Ramsey’s claim that there is “very little agreement" about inductive probabilities. The existence of some disagreement is only to be expected and it can be attributed to human error and to the vagueness of the concept of inductive probability. There are also disagreements about the extension of logical truth and logical consequence, though few take this to show that there are no logical truths or logical consequences.

3.2. Ramsey reversed

I've argued that the opposite of Ramsey's premise is true: there is a great deal of agreement about inductive probabilities. I will now use this to argue for the opposite of Ramsey's conclusion, namely that inductive probabilities exist.

Consider, for example, the sentence WB. It is an elementary inductive probability sentence and therefore logically determinate. So if WB is not true, the people who endorse WB are using ordinary language incorrectly. But competent users of ordinary language usually use their language correctly, so the fact that most people endorse WB is strong evidence that WB is true and hence that at least one inductive probability exists. And since WB is just one of many statements of inductive probability that most people agree about, we likewise have good reason to believe that many inductive probabilities exist.

3.3. The Principle of Indifference

The Principle of Indifference, as it is usually formulated, says that if evidence does not favor one hypothesis over another then those hypotheses are equally probable on this evidence. A popular argument for the non-existence of logical probabilities begins by assuming that the existence of logical probabilities depends on deriving them from the Principle of Indifference. It is then pointed out that, in some cases, the Principle of Indifference can be applied in different ways that seem equally natural and yet give different values for the same probability. This fact is taken to show that the Principle of Indifference is untenable and hence that logical probabilities do not exist (van Fraassen, 1989, ch. 12; Gillies, 2000, 37–49).

To consider how this argument might apply to my conception of inductive logic, I begin by observing once again that my conception involves two different logical probability concepts, namely inductive probability (the explicandum) and the function p (the explicatum).

The values of p are fixed by stipulative definition, so their existence does not depend on deriving them from the Principle of Indifference.

The values of inductive probabilities are fixed by the concept of inductive probability. This concept is learned from examples, without reference to any general principle such as the Principle of Indifference. So the existence of inductive probabilities also does not depend on deriving them from the Principle of Indifference.

Hence the above argument is not a cogent objection to my conception of inductive logic. It does, however, raise a question: What are we to say about cases in which different equally natural applications of the Principle of Indifference lead to incompatible probability assignments? Answer: We should say that precise numerical inductive probabilities do not exist in such cases. This does not prevent us defining precise values for p in these cases, but different choices for that definition are compatible with the desideratum that p be similar to inductive probability. So we could, for example, choose one way of applying the Principle of Indifference in order to assign values to p, while recognizing that this choice may be to some degree arbitrary.

3.4. Arbitrariness

Another common objection to logical probability is that the choice of a logical probability function requires arbitrary choices. We have just seen one example: The decision to apply the Principle of Indifference one way rather than another way might be arbitrary. Another commonly cited example of arbitrariness is the choice of a value for λ in Carnap's (1952) continuum of inductive methods. Those who raise this as an objection seem to be thinking that logical probability cannot really depend on an arbitrary choice and hence logical probabilities don't exist (Howson and Urbach, 1993, 66–72; Hájek, 2003, sec. 3.2).

It should already be clear why this is not a sound objection to the conception of inductive logic I have described. The explicatum, the function p, can indeed be arbitrary to some degree. This does not prevent p being logical in Carnap's sense; that status follows simply from the fact that the values of p are fixed by definition.

The explicandum, inductive probability, is not arbitrary. In cases where choice of any precise value seems arbitrary, the natural conclusion is that inductive probability does not have a precise value. So, for example, if the choice of any precise value of λ in Carnap's continuum is arbitrary, that would be because inductive probability is not sufficiently precise to fix a definite value.

4. Conclusion

I have presented a conception of inductive logic that is similar to Carnap's and I have shown that it is not subject to objections that are routinely hurled against Carnap's conception. If this is accepted then the focus should now shift to the work of actually constructing adequate explicata for inductive probability in non-trivial contexts. Much of the technical work in what is generally recognized as "inductive logic" can be interpreted as aimed at this goal, even if its authors did not see it in quite this way. I hope that this paper, by showing how inductive logic can be conceived in a way that is philosophically defensible, will encourage more people to do constructive work of that kind.

References

Carnap, Rudolf (1950), *Logical Foundations of Probability.* Chicago: University of Chicago Press. Second edition 1962.

Carnap, Rudolf (1952), *The Continuum of Inductive Methods.* Chicago: University of Chicago Press.

Carnap, Rudolf (1963), "Replies and Systematic Expositions", in Paul Arthur Schilpp (ed.), *The Philosophy of Rudolf Carnap.* La Salle, IL: Open Court, 859–1013.

Franklin, J. (2001), "Resurrecting Logical Probability", *Erkenntnis* 55: 277–305.

Gillies, Donald (2000), *Philosophical Theories of Probability.* London: Routledge.

Hacking, Ian (2001), *An Introduction to Probability and Inductive Logic.* Cambridge: Cambridge University Press.

Hájek, Alan (2003), "Interpretations of Probability", in Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy.* http://plato.stanford.edu/archives/sum2003/entries/probability.

Howson, Colin and Peter Urbach (1993), *Scientific Reasoning: The Bayesian Approach*, second edition. Chicago: Open Court.

Percival, Philip (2000), "Probability", in W. H. Newton-Smith (ed.), *A Companion to the Philosophy of Science.* Oxford: Blackwell, 358–372.

Quine, Willard Van Orman (1960), *Word and Object.* Cambridge, Mass.: MIT Press.

Ramsey, Frank P. (1926), 'Truth and Probability', in Henry E. Kyburg, Jr. and Howard E. Smokler (eds.), *Studies in Subjective Probability*, second edition. Huntington, New York: Krieger, 25–52.

Roeper, P. and H. Leblanc (1999), *Probability Theory and Probability Logic.* Toronto: University of Toronto Press.

Salmon, Wesley C. (1967), *The Foundations of Scientific Inference.* Pittsburgh, PA: University of Pittsburgh Press.

Skyrms, Brian (1986), *Choice and Chance: An Introduction to Inductive Logic*, third edition. Belmont, CA: Wadsworth.

van Fraassen, Bas C. (1989), *Laws and Symmetry.* Oxford: Oxford University Press.