MATHEMATICAL VARIABLES AS INDIGENOUS CONCEPTS

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Abstract. This paper explores the semiotic status of algebraic variables. To do that we build on a structuralist and post-structuralist train of thought going from Mauss and Lévi-Strauss to Baudrillard and Derrida. We import these authors' semiotic thinking from the register of indigenous concepts (such as *mana*), and apply it to the register of algebra via a concrete case study of *generating functions*. The purpose of this experiment is to provide a philosophical language that can explore the openness of mathematical signs to reinterpretation, and bridge some barriers between philosophy of mathematics and critical approaches to knowledge.

1. Algebraic unknowns and indigenous concepts

Consider a series of the form

$$\sum_n a_n x^n, \tag{1}$$

where $\{a_n\}_n$ are numbers. Let's start with three different interpretations of the algebraic unknown x relevant for the context of *generating functions* to be discussed below (the term 'unknown' is used here as a generic term for the various possible interpretations of x, and not in any specialised technical meaning).

First, x can be interpreted as a variable ranging over values in a given set. This interpretation obviously raises the question, which given set? In the context to be presented below x is usually a real or complex variable. But this does not exhaust its potential for our context. As we'll see below, x can usefully stand for matrices as well. The main limitation of this approach, however, is that it excludes divergent series.

Another possibility is to view x as a transcendental element in an extension of the domain of the coefficients. This means that x is a new kind of number, which, when subjected to addition and multiplication, obeys standard algebraic rules (associativity, distributivity, etc.), and which does not nullify any expression of the form (1). Note, however, that under this interpretation x is not a variable, but a constant.

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The 'constancy' of x, however, is somehow underdetermined. Structurally speaking, if I replace the sign x by y, I'll get the exact same effects (a rose by any other name...). In that sense x and y would be the same. But to say that they are identical is risky. Indeed, if we want to use two elements transcendental with respect to the domain and to each other (x does not solve any polynomial equation with coefficients that may involve y and vice versa), then x and y will obviously be distinct, although structurally permutable. So the line between the unknown as an individuated constant and as something more general here is blurry, and may go beyond what's relevant to actual mathematical practice.

But the main constraint of this last interpretation is that it does not allow infinite series. To work with infinite sums of powers of x, and avoid convergence issues, we can use the algebra of *formal power series*. There x is not a variable or a constant, but a place holder. We can think of it informally as a comma separating the coefficients of the series (so $\sum_n a_n x^n$ is just another way or writing $(a_1, a_2, \ldots, a_n \ldots)$) , or, more formally, as a λ-operator that assigns values (the coefficients a_n) to the sequence of integers (the powers n). This point of view allows us to define and control sums of power series, products, some divisions, and even derivatives and integrals. But this interpretation must be complemented, if we are to justify other tools of the trade: exponentiation, infinite products, continued fractions, Laurent series, and, most importantly, substitutions by numbers.

What we have here is an algebraic sign that condenses various semantic and syntactic roles. This condensation might be sorted out by carefully distinguishing each use of x. But this careful, formal sorting out is not reflected in mathematical practice. As one author writes, **one of the attractions of the subject, however, is how easily one can shift gears from thinking of** power series **as mere clotheslines for sequences** (formal power series) **to regarding them as analytic functions of complex variables**[1] (Wilf 1990, 138).

Before I comment further on this useful ambiguity, I want to state the analytic framework for this paper: I would like to think of algebraic unknowns as *indigenous concepts* (why and how this might be a good idea will be argued below). In fact, I am going to think of unknowns through a specific family of indigenous concepts — a family of concepts related to what we can loosely call magic. The paradigmatic example in this family is *mana*, a Melanesian concept, which gave rise to an anthropologico-philosophical line of thinking going from Mauss through Lévi-Strauss to Baudrillard and Derrida.

Mana, like algebraic unknowns, **refers to ... beings which still have no common name, which are not familiar** (Mauss 1972, 114). Mana **is**

[1] I employ the convention of using boldface for quotations. Bolded words in this text are always quoted text. Interpellated unbolded words do not belong to the quoted texts, and the order between quotations is not necessarily that of the original.

not simply a force, a being, it is also an action, a quality, a state, that is the word mana can be **a noun, an adjective, a verb** (Mauss 1972, 108); this multiplicity may be thought of as analogous to our xs, which can be constants, λ-operators, variables. Mana has a **potentiality to do or effect results mystically** (Mauss 1972, 114), qualifying things **whose effectiveness amazes us** (Lévi-Strauss 1987, 55); analogously, professionals sometimes express wonder at algebraic proofs, qualifying them as 'hocus-pocus'.

But I must immediately emphasise that my point is not to carry out an analogy between mana or magic and unknowns or algebra. The above analogies between mana and unknowns, like those to be presented below, are very restricted, and ignore the overwhelming differences between magical mana and algebraic unknowns. There is an analogy, and as analogies go, it goes a certain way, but then again, it doesn't go terribly far. My point is not to force the analogy. My point is to use this analogy for importing ways of thinking about mana into our discourse about algebraic unknowns.

Such reading is violent. It takes thoughts from an anthropological context, and forces them onto the mathematical context. Such imported thoughts will not remain the *same*. But my purpose is not to faithfully represent anthropological thinking. My purpose is to use anthropological thinking to intervene in thinking about mathematics. I will therefore allow myself a rather brutal practice of cut-and-pasting, which will put some quoted words under a mark of erasure, and replace them by others, forcing our context on the quoted texts. I am practicing thinking through others about something that those others may have never had in mind.

2. Unknowns and mana as instruments of condensation

So let's continue setting our opportunistic and restricted analogy, our vehicle for importing ideas from anthropology to a philosophy of mathematical practice. Mana is **mobile and fluid, without having to stir itself** (Mauss 1972, 117); analogously, the individuation of our single x, as discussed above, is rather unstable. **It is ~~impersonal~~** general, **and at the same time clothed in ~~personal~~** specific **forms** (Mauss 1972, 117); think of how a variable remains the *same* while taking *different* values. Indeed, **the normal condition of magic is** a **confusion of powers and roles** (Mauss 1972, 117), and, as Wilf testifies above (and as we'll see in greater detail below), mathematical practice depends on our capacity to **shift gears** or superpose an unknown's various possible semantic and syntactic roles.

Lévi-Strauss, to whom we will turn later, compares mana to the generic French terms *truc* or *machin*, which, like algebraic unknowns **qualify an unknown object**, one **whose function is unclear** (Lévi-Strauss 1987, 54–55) — at least as long as we suspend the decision whether x is a variable, constant or λ-operator. But Lévi-Strauss goes on to explain that **in our society these notions** (*truc* and *machin*) **have a fluid, spontaneous**

character, whereas elsewhere, e.g. Melanesian mana, **they serve as the ground of considered, official interpretative systems; a role, that is to say, which we ourselves reserve for science** ... **those types of notions** ... **occur to represent an intermediate value of signification, in itself devoid of meaning and thus susceptible of receiving any meaning at all**. The ellipsis above covers over the explicit analogy: **somewhat like algebraic symbols** (Lévi-Strauss 1987, 55).

This explicit relation between algebra and magic can be traced further back. Mauss refers to Frazer as **calling magic a kind of pre-science** (Mauss 1972, 12). Indeed, **certain branches of magic, such as astrology and alchemy, were called applied physics in Greece** (Mauss 1972, 143). But a careful reading shows that Mauss does not collapse science in general (or mathematics in particular) to magic. In alchemy, for example, Mauss considers algebraic formulas to be **bogus** (Mauss 1972, 56). Indeed, **There are algebraical formulas** ... **which once served a purpose but have since been transformed into unintelligible magical signs** (Mauss 1972, 101). In the opposite direction, turning magic into mathematical science requires, according to Mauss, that **its formulas** be **simplified** (Mauss 1972, 63) and that **all trace of mystery disappear** (Mauss 1972, 100).

While bogusness, intelligibility, simplicity and mystery may set magic and mathematical science apart, the key role of mana, according to Mauss, is something that we can predicate on algebraic unknowns. This role is to condense various meanings under a **small number of symbols** (Mauss 1972, 70). **A single lizard may, at various points in the rite, represent the curse, the person uttering the curse, and the evil contained in the curse** (Mauss 1972, 69), and **magical knots are required to represent love, rain, wind, curing, war, language and a thousand other things** (Mauss 1972, 70). Mana condenses magical representations **into distinct notions and given special terms** (Mauss 1972, 62). Here is how I'd like to borrow this thought (an act of borrowing that is indebted to Mauss, but whose credit is furnished by our evidence of the plurality of the roles of the unknown): algebra operates by condensing not only sets of values, but also a variety of interpretations and syntactic roles into **a small number** of **distinct notions and given special terms**, that is, into algebraic unknowns.

Let's articulate this condensation more explicitly by anchoring it to a specific mathematical practice: solving combinatorial problems with *generating functions*. I will not present the method in general, but only supply an example. Suppose we want to count all ways of distributing 20 identical balls into three boxes, where the first box may contain any number of balls, the second may contain up to three balls, and the third must contain an even number of balls (so 3-1-16 is one acceptable distribution of 20 balls, because 1 is not greater than 3 and 16 is even; 10-2-8 is another acceptable distribution, etc.). It turns out that the number of acceptable distributions equals the coefficient of x^{20} after unpacking and simplifying the product

$$(x^0 + x^1 + x^2 + \ldots)(x^0 + x^1 + x^2 + x^3)(x^0 + x^2 + x^4 + \ldots),$$

which equals

$$\frac{1}{1-x} \cdot \frac{1-x^4}{1-x} \cdot \frac{1}{1-x^2}, \tag{2}$$

and is referred to as the generating function of the combinatorial problem (we will go over the relation between the problem and this solution in more detail further below).

When using algebraic techniques to solve and explore such problems, the analysis is conducted on the seam line between the power series and rational function representations of the generating function. It may therefore be useful to view x as a variable, constant or an element of formal power series. As Wilf explained above, mathematical practice with generating functions requires us to **shift gears**. In order to use divergent series together with analytic techniques, and to exploit the entire arsenal of power series technologies, one doesn't choose how to articulate x. One takes advantage of the way x condenses various values and roles.

I conducted a small sample survey of textbooks and research monographs in order to probe into contemporary mathematical practice in the context of generating functions. As expected, and as I do in my own classroom, most authors either ignore the issue of the changing role of x, or wave it away with a cursory mention of some theoretic apparatus. Such comments are sometimes demoted to parenthetic remarks or footnotes, sometimes appear to be rather confused, and usually do not quite cover the entire scope relevant to the surrounding text.

One author, for instance, explains that x **is a formal variable and is used simply as an indicator**, and that therefore **there is no need to question whether the series converges** (Liu 1968, 25–26) (then, in a footnote the author suggests working **with the understanding that the value of x is set to be** 0 — an approach that would get us nowhere fast, as it renders all power series identical). But this author's practice includes substitutions and differentiations, which are not covered by the formal variable or zero value approaches.

Another form of ambiguity is manifest in juxtaposing the statement **convergence is not necessary for the series** ... **to be useful in obtaining various properties of** its **coefficients** and the qualification that in dealing with **a power series expansion which is obviously *divergent* whenever** the variable is nonzero, **we find it convenient to use** the symbol $\cong$ rather than $=$ **to indicate divergence** (Srivastava & Manocha 1984, 78). This non-rigorous notation (when exactly does divergence become **obvious**?) is already available in (McBride 1971, 2), but neither text states the conditions under which their exponentiations, derivations, Laurent series and substitutions can be controlled under a formal series approach. To back things up the latter author cites the work of E.T. Bell (as does (Eisen 1969, 70), who qualifies the unknown in a footnote as **an abstract indeterminate**, ignoring the concrete limits and substitutions shortly thereafter). However, Bell's work — a fine example of struggling for formal rigour in a setting where means had not yet fully crystallised — does not quite provide the required theory, and refers part of what is sought to Grassmann and Gibbs (Bell 1923).

Other authors are slightly more careful. One author provides an elementary theory of formal power series, explicitly marks transitions such as **below we treat the formal variable** s **as the *complex* variable** $s \in \mathbb{C}$ (Lando 2003, 48), and raises some issues concerning continued fractions. But the same author does not question the consistency of switching between these approaches. A complementary approach is to make sporadic parenthetic remarks concerning radii of convergence, without following this concern throughout, as in (Tucker 2002, 248). This approach is the successor of the most Stoic approach of all: keeping entirely quiet about formal rigour, as practiced by the last proponent of the 19^{th} century British school of algebra (MacMahon 1960).

In this respect Wilf is the most explicit author I found. He introduces both the formal and analytic theories, stating that under the formal interpretation **we love** x **only for its role as a clothesline on which our sequence is hanging out to dry** (Wilf 1990, 6). He points out the technical transition between interpretations (Wilf 1990, 32), and even confesses a relationship of **cheating**, **guilt** and **making peace** with his double-faced approach (pp. 32, 27, 32 respectively). But his way of making peace is only almost convincing, and lacks a formal reconciliation especially around infinite products (Wilf 1990, 91). Wilf is also the author who makes ambiguity fully explicit: **One of the attractions of the subject, however, is how easily one can shift gears** (Wilf 1990, 32).

Before I bring on the wrath of mathematical authors, I must hasten to clarify that I fully endorse most approaches taken by the authors above. The theory isn't faulty. Generating functions are mathematically robust tools. Indeed, given sufficient context, any power series manipulations cited above *can* be justified rigorously. But my point is that the justification is not an a priori framework; the justification is a cohort of ad-hoc constructions and shifting points of view that confront a developing set of tools. One can construct a theory unifying various formal power series and real or complex variable techniques, but then one could introduce new manipulations that exceed the constructed theory, which in turn would require an extension and a review of its validation.

To understand contemporary mathematical practice, however, it is crucial to observe that the mathematical literature does not bother with such tailor-made theories. If they bother considering the issue at all, authors, as we saw above, are typically happy with folk knowledge, based on experience and authority, that everything they do is rigorously justifi*able*. This justifi*ability* does not translate into an actual formal reconstruction or practiced conceptual stability, and *there's nothing wrong with this* (of course, when one reaches theoretic forefronts, one encounters manipulations that produce correct results, but don't (yet?) have fully rigorous mathematical frameworks to justify them).[2]

The point I am making is that algebraic unknowns allow to condense various semantic and syntactic roles, a condensation which enables practices belonging to different justification frameworks to be performed on the same mathematical term. **The normal condition of magic**, explains Mauss, **is an almost total confusion of powers and roles** (Mauss 1972, 88). We might want to omit the **total** if we wish to replace **magic** by algebra, but some confusion of **powers and roles** is constitutive of working with unknowns in the context of power series and generating functions. But even with magic, albeit **obscure and vague**, still, as with algebra, **the use to which it is put is curiously definite** (Mauss 1972, 109).[3]

Mauss finds it very difficult to obtain an exhaustive representation of whatever underlies magic. He is left with a **vague power** that is articulated as **the residue of the other series of representations** (Mauss 1972, 105). *Orenda* (an Iroquois equivalent of mana), for example, **is not material power, it is not the soul, nor an individual spirit, nor is it strength nor force** (Mauss 1972, 114). I appropriate Mauss' line of thinking to suggest that our instances of x above are not quite variables, constants or λ-operators. The unknown x, like orenda, **plays the role of a middle term** (Mauss 1972, 114), which binds the system together.

Mauss concludes by stating that **If our analysis is exact, therefore, we shall find — at the basis of magic — a representation which is singularly ambiguous and quite outside our adult European understanding** (Mauss 1972, 106). Given our **ambiguous** practice with unknowns in power series, perhaps these representations are not so completely **outside**. The fact that it's possible, in principle, to a posteriori rigorously reconstruct given portions of current practice, does not make the ambiguities of contemporary mathematical practice go away. Ambiguity does not mean that mathematical practice is faulty. The fault is with formal reconstruction, which fails to capture how mathematics is practiced.

Foundational/epistemological philosophies of mathematics do not acknowledge the indefinite deferral of justification. For the formalist, logicist, platonist, intuitionist and structuralist mathematical manipulations take place *in* distinctly and clearly articulated realms of rules, objects or mental constructions, rather than *through* fluid interpretive frameworks. These philosophies

[2]One example is the *replica trick*, used by statistical physicists, which, in a sense, takes an integer valued variable, and then makes it converge to zero; see Mezard et al. (1987) and Talagrand (2003).

[3]For a different discussion of productive ambiguities in mathematics see Grosholz (2007).

may acknowledge sequential 'snapshots' of mathematics, where each snapshot records a somewhat different robust mathematical system (Greek geometry vs. Cartesian algebraised geometry; formal power series vs. analytic power series); but these philosophies are not concerned with understanding the motion excluded from these snapshots. Indeed why should they? They are, precisely, foundational and epistemological, and do not care for mathematical practice.

Wittgensteinian and constructivist approaches to mathematics are better equipped for dealing with the mathematically obscure and confused. But their respective therapeutic and sociological horizons tend to keep them from going beyond accepting such practices as given elements of our social life-worlds. Phenomenological and cognitive approaches attempt to reduce such practices to elementary components of intentionality or cognition embedded in an intersubjective or biological substrate. One may choose any of the above as one's theoretic stopping point. One is probably making a sane and academically healthy choice thereby. Nevertheless I would like to risk venturing into the *pardes* of structuralist and post-structuralist semiotics in order to relate mathematical practices to thinking that could take mathematical obscure confusions and spin them into lines of flight[4] from mathematics through anthropology and linguistics into literature. I don't pretend to record lines of flight that *exist*; this is an experiment in *writing*. Those who believe that it is good to write new lines of flight are welcome to read on.

3. From condensation to zero symbolic value

Mauss articulates magic as **a world separated from — but still in touch with — the other** (Mauss 1972, 117) — that other world on which magic operates by representing it. Here's what we get by projecting this thinking on the relation between the mathematical code and the reality it's supposed to describe. **In order to explain more clearly how the world of ~~magic~~** algebra **is superimposed on the other world without detaching itself from it, we might go further and add that everything happens as if it were part of a fourth spatial dimension. ~~An idea like *mana*~~** The algebraic unknown **expresses, in a way, this occult existence** (Mauss 1972, 117).

This last sentence is not easy to swallow. To follow Mauss in this articulation of 'two worlds' appears to lead in either of two trajectories. The first follows platonism, introducing an ideal world appended to the tangible one as a further dimension. The second trajectory is closer to that which Mauss actually takes, and is articulated in neo-Kantian terms. According to this trajectory the appended world **exists a priori before all other existence**, a world **inherent in magic in the same way that Euclid's propositions are inherent in our concepts of space** (Mauss

[4]I use this term in the sense of Deleuze & Guattari (1987).

1972, 118). **magical judgments**, for Mauss, **are** ... **well nigh perfect, *a priori*, synthetic judgments** ... **However, it must be made clear that we have no wish to imply that magic does not demand analysis or testing. We are only saying that it is poorly analytical, poorly experimental and almost entirely *a priori*** (Mauss 1972, 118). This poorly analytic, poorly experimental synthetic a priori is articulated by Mauss as (the) social, which is always a priori with respect to the individual born into it.

The **condens**ation of representations into **a small number of symbols** is, according to Mauss, inherent to intersubjective knowledge formation — the realm of the social as such. But since for Mauss this social a priori condensation is a supplementary **dimension**, and not something subject to the three 'usual' dimensions, he concludes that the actual condensations of representations in a given society depend on **value judgements** that **are the expression of social sentiments which are formed** ... **for the most part in an arbitrary fashion** (Mauss 1972, 121). Whether algebra should be considered as a priori is still a debated philosophical question (which depends, of course, on our articulation of what 'a priori' means). But even if mathematics belongs with a social a priori, it is hard to accept that mathematics expresses **sentiments** and **judgements** that are **arbitrary**. Indeed, Lévi-Strauss objects that these terms **do not shed light on the phenomena we set out to explain** (Lévi-Strauss 1987, 56–57).

Unhappy with viewing ~~mana~~ algebraic unknowns as supplementary, arbitrary, socially given condensations guided by **value** and **sentiment**, Lévi-Strauss sets out to rearticulate the relations between signs and their social use. Lévi-Strauss places the realm of signifiers on the a priori side, but not the condensations that they effect. The **two categories of the signifier and the signified came to be constituted simultaneously and interdependently, as complementary units; whereas knowledge, that is the intellectual process which enables us** ... **to choose, from the entirety of the signifier and the entirety of the signified, those parts which present the most satisfying relations of mutual agreement — only got started very slowly. It is as if humankind suddenly acquired an immense domain and the detailed plan of that domain** ... **but spent millenia learning which specific symbols of the plan represented the different aspects of the domain** ... **the progress of scientific knowledge could only ever be constituted out of processes of correcting and recutting of patterns, regrouping, defining relationships of belonging and discovering new resources, inside a totality which is closed and complementary in itself** (Lévi-Strauss 1987, 60–61).

For Lévi-Strauss the world of signifiers is still a priori, but its deployment with respect to the signified world is far from **arbitrary**. The attempt at putting signifiers and signifieds together is always work in progress. One begins with **a signifier-totality which he is at a loss to know how**

to allocate to a signified, given as such, but no less unknown for being given. There is always a non-equivalence or 'inadequation' between the two ... this generates a signifier surfeit value relative to the signified to which it can be fitted. So, in man's effort to understand the world, he always disposes of a surplus of signification (Lévi-Strauss 1987, 62). For Lévi-Strauss the world of signifiers is not equivalent to that of the **given**-but-not-for-all-that-**known** signified. Rather, it is the practical task of the semiotic *bricoleur* (or the scientist) to find ways of using signifiers as resources for expressing our signified experience. And given this **inadequation** between signifier and signified it makes perfect sense that the algebraic unknown x does not fit uniquely any single semantic or syntactic function.

Let's return to our combinatorial problem in order to see how this **inadequation** is manifest in practice. Recall that we want to count the ways of distributing 20 identical balls into three boxes, where the first box may contain any number of balls, the second may contain up to three balls, and the third must contain an even number of balls. The answer to this question is, as we stated above, the coefficient of the monomial x^{20} after unpacking and simplifying the product

$$(x^0 + x^1 + x^2 + \ldots)(x^0 + x^1 + x^2 + x^3)(x^0 + x^2 + x^4 + \ldots),$$

This solution is based on an analogy between ways of distributing balls into boxes and ways of generating monomials (terms of the form x^n) when unpacking the product. We won't justify here this solution, but let's state some elements of the analogy. The three pairs of parentheses in the algebraic model stand for the three boxes; the powers between each pair of parentheses stand for the number of balls allowed in each box; the power 20 in the target monomial stands for the total number of balls; the products between the sums stand for a conjunctive relation between the conditions set on the boxes; the summation of monomials inside parentheses stands for a disjunctive relation between the possibilities for each box. But having fitted the elements of the problem and the elements of the solution so neatly, there's something that's conspicuously left out. Indeed, what does x stand for? Well, in fact, x here does not stand *for*. The values that may be taken by x have nothing to do with the problem; x is a term in an algebraic structure.

Lévi-Strauss claims that notions such as mana, **somewhat like algebraic variables** function to **fill a gap between the signifier and the signified, or, more exactly, to signal the fact that in such a circumstance ... a relationship of non-equivalence becomes established between signifier and signified, to the detriment of the prior complementary relationship** (Lévi-Strauss 1987, 56). Like mana, the unknown fills a gap between the combinatorial problem and the algebraic techniques of solution, and it can do so precisely because it is not bound to any specific signified, and can serve various functional roles. Now the sign, magical or mathematical, is no longer an instrument of arbitrary social condensation, but an instrument for sorting out the difficulties arising from attempts to map signifiers onto signifieds (which, recall, enjoy an originary **inadequation**). Indeed, given this originary **inadequation**, the signifier and signified (here, the mathematical model and combinatorial problem) could not be put together, unless some **signifier surfeit** such as x provides that flexibility, that **distribution of a supplementary ration**, which is **absolutely necessary to ensure that, in total, the available signifier and the mapped-out signified may remain in the relationship of complementarity which is the very condition of the exercise of symbolic thinking** (Lévi-Strauss 1987, 63).

Note what it is that's put together. For Mauss mana amalgamated representations, or, as Lévi-Strauss put it, Mauss' mana **enjoys the same role which the copula plays in a grammatical clause** (Mauss 1972, 122). But if only Mauss **had been able to formulate the problem of thinking, instead, in terms of relational logic**, writes Lévi-Strauss (Lévi-Strauss 1987, 50), this faulty conception of **weld**ing together different things (e.g. smoke and cloud) would have been undone in favour of **restoring their unity** (Lévi-Strauss 1987, 59). This unity Lévi-Strauss refers to **a deeper level of thinking** (Lévi-Strauss 1987, 50), which does not yet tell them apart. Following this way of thinking, we can observe that the algebraic unknown does not a posteriori weld together the role of value carrier (variable) and place holder (λ-operator), but restores a deeper *thought* relation between the cardinal and the ordinal. Rather than condense representations in a world of adequately related words and objects, the unknown restores relations suppressed by necessarily imperfect attempts to establish a simple complementarity between signifier and signified.

But what structural position allows a **surfeit signifier** to restore such relations? To implement the semantic and syntactic condensation required for this task ~~mana~~ the algebraic unknown must be **a symbol in its pure state, therefore liable to take on any symbolic content whatever** (Lévi-Strauss 1987, 64). ~~Mana~~ The algebraic unknown can reconstruct relations because it is ***zero symbolic value*****, that is, a sign marking the necessity of a supplementary symbolic content over and above that which the signified already contains, which can be any value at all, provided it is still part of the available reserve** (Lévi-Strauss 1987, 64). This ***zero symbolic value*** is differentiated from other signs, but does not relate to any proper expression.

The structural ***zero symbolic value*** introduces the flexibility to redeploy signifiers and signifieds, condense and expand their rapports, and thus enforce limited but useful representations of the world. It's as ***zero symbolic value***, not committed to any specific role, that algebraic unknowns can force the thinkable algebraic structure on the combinatorial signified. This is not Mauss' **arbitrary** social **judgements** and condensations; this is the action of the sign as a tool for projecting the *a priori* signifier order of thought on the incommensurable given world (indeed, Lévi-Strauss endorsed Ricoeur's articulation of his approach as **Kantism without a transcendental subject** (Lévi-Strauss 1969, 11)). The ***zero symbolic value*** algebraic unknown, which does not appear in the original combinatorial problem, enables the realm of mathematical signifiers to represent our given mathematical experience.

The algebraic unknown is so incredibly useful, because it does not require us to decide once and for all what it stands for. We can, of course, stop at any given moment, and reconstruct a more-or-less stable signified for the algebraic unknown. But **even though scientific knowledge is capable, if not of staunching it, at least of controlling it partially**, the ***floating signifier*** remains **the disability of all finite thought** (Lévi-Strauss 1987, 63): a reconstructible, reinterpretable, but never finitely containable carrier of an a priori order.

We should reiterate the optimistic undertone of Lévi-Strauss' **inadequation** and ***floating***. The algebraic unknown is the felicitous **expression of a semantic function, whose role is to operate despite the contradiction inherent in it** (Lévi-Strauss 1987, 63) — the contradiction of **inadequation** between the *a priori* order of signifier and our *changing* signified life-worlds. It's the leeway of reworking signifier/signified relations through ***floating signifier***s and ***zero symbolic value*** that enables us to more-or-less successfully represent **despite the contradiction**. The ***floating signifier*** is not a pathology, it is a constitutive condition for signification. As such, **The notion of ~~*mana*~~** unknowns **does not belong to the order of the real, but to the order of thinking** (Lévi-Strauss 1987, 59).

we shall not go along with Mauss, writes Lévi-Strauss, **when he proceeds to seek the origin of the notion of *mana* in an order of realities different from the relationships that it helps to construct: in the order of feelings, of volitions and of beliefs, which, from the viewpoint of sociological explanation, are epiphenomena, or else mysteries** (Lévi-Strauss 1987, 56). In fact, Lévi-Strauss accuses Mauss of using this unscientific grounds as his own mana — his own floating signifier used for putting his theory together. Where Mauss traced a limited world of signifiers that a priori condense various representations based on **mysteries** and **arbitrar**iness, Lévi-Strauss insists that the relations projected by floating signifiers are the hinges that join the a priori language of thought to an a posteriori experience — two strata that simply cannot be fitted together once and for all.

4. Closure

This open-endedness not withstanding, Lévi-Strauss' theory postulates **a totality which is closed** (Lévi-Strauss 1987, 61). And here Mauss and

Lévi-Strauss are in agreement. **Compared with the infinity of possible symbolic actions, or even those actually found throughout the world, the number used in a single magical system is singularly limited** — so limited indeed, that Mauss goes as far as hypothesising that **We would be able to assert that symbolic systems are always limited by codes** (Mauss 1972, 51) in **a closed circuit, in which everything is *mana* and which is itself *mana*** (Mauss 1972, 112).

But should we a priori close this way? Should we confine ourselves to a metaphysical assumption of totality? Or should we regard the multifarious unknown, witnessed above, as an expression of the opposite of closure? (and what is this opposite? openness? dis-closure?) This problem is especially subtle, as it is never any specific set of sign relations that's a priori and closed (that would be obviously false); it's the structure of signifying thought writ large, the order of the signifier *as a whole*, which is supposed to be **a totality which is closed**.

In order to rethink this closure, let's return to our case study. When working with generating functions, their rational function representation, e.g. (2), may lead to algebraic problems such as finding the coefficients a and b for which

$$\frac{1}{(1-x)(1-2x)} = \frac{a}{1-x} + \frac{b}{1-2x},$$

or equivalently

$$1 = a(1-2x) + b(1-x).$$

Here, one of the simplest ways to get a and b is to substitute the values 1 and $\frac{1}{2}$ for x, which yield $1 = a(-1) + 0$ and $1 = 0 + b(1/2)$, so $a = -1$ and $b = 2$. Note that this requires the unknown x to become a substitutable variable, even if so far we have used it as a formal series term. More importantly, note that x is substituted for by the very values that render the first equality above undefined.

As with the zig-zagging between formal series and analytic functions above, this glitch too can be rigorously circumvented (e.g. substitute values tending to 1 and $1/2$ from below, and take the limit — which, by the way, would still be problematic, if we went back from the rational function to the power series, one of which is undefined around 1). But that's precisely what mathematicians usually do not do, even when teaching or writing for first year students, who are not likely to note the gap and patch it up by themselves. Which, I emphasise again, is a practice that I endorse. Indeed, if this weren't our practice, how would our students ever get to actually practicing mathematics? One doesn't train mathematicians by drowning them in mathematical foundations.

The point here is no longer to demonstrate the non-rigorous superposition of various approaches, or the floating of unknowns over a non-equivalent signified. The point here is to bring up openness.

This openness has several dimensions. The first dimension is that of the ever expanding roles taken by algebraic unknowns. For example, if our manipulation would have led us to an equation of the form

$$x^2 - 1 = a(1-x) + b(1-x)^2,$$

then no number substituted for x could nullify the b term without nullifying the a term as well; but a quick glance shows that one can find matrices that do the job. However, if we allowed matrices in this practice, the range of x would exceed the previous confines of commutative algebra without zero divisors, which includes both formal power series and analytic functions. The first dimension of openness is the openness of the algebraic unknown to assuming new roles and reforming the justification framework of its practice (note also that if we allow x to take matrix values, then the number 1 becomes the matrix I — so is this 1 a constant or a variable?).

The next dimension of openness is more radical than the above referential and syntactic openness. This second dimension of openness is the mathematician's practice of not deciding, either in advance or after the fact, what x is. This openness is the openness to semi-rigorous entities in mathematical practice. In the last example above x is not quite an element of the formal algebra of power series, nor is it a variable standing for values. It is not simply a middle term either. There, x is a line of flight from rigorous formality. Indeed, this line of flight can (to an extent even should) be reintegrated through a formal reconstruction of the algebraic setting, which would render consistent the substitution of 1 and $1/2$ for x. I have no wish to deny that such reintegration is a constitutive aspect of contemporary mathematical practice. But I insist on asserting that lines of flight are no less constitutive in their turn.

But even that second dimension is somehow contained inside a certain **totality which is closed** or **a closed circuit** of mathematics. Indeed, the dialectic between informal thinking and post-hoc reconstructions of mathematical manipulations is confined to the question of consistency, leaving the question of efficacy outside its scope. The question of efficacy, which never receives, indeed can't receive *mathematical* justification, is the question of **mutual agreement** between a practical, not-yet-mathematical problem (such as distributing balls into boxes), and the algebraic model used for solving the problem (set theoretic modelling can only serve as mediator here; the gap between the pre-mathematical problem and the model remains intact).

Opening this gap requires much care, as, unless we accept a simplistic empiricist division into actual phenomena vs. mathematical models, it is not at all obvious where mathematics ends and the not-yet-mathematical problem begins. Wittgenstein, for example, draws the line between the mathematical and the empirical in an original way. Suppose I say 'the pentagram has 10 vertices'. Whether this is a mathematical or empirical observation depends, for Wittgenstein, on what happens if, when I recount the vertices, I obtain 9 rather than 10. If this leads me to conclude that my counting was wrong, or that what I have before me is not a proper pentagram, then 'the pentagram has 10 vertices' is used mathematically, that is, as something against which to measure our practice. If, by contrast, I simply go and fetch 9 candles to place on the 9 vertices of the pentagram, then 'the pentagram has so-and-so many vertices' is indeed an empirical claim (Wittgenstein 1975, 116–117, 245–246).

On Wittgenstein's terms, then, the problem of counting distributions of balls may already be mathematical through and through. So if we would like to assert openness, it's not the model/phenomenon border that should be breached, but the border between mathematical problems and their non-mathematical contexts.

In fact, it is because we articulate the intra- and extra-mathematical as apart, that we feel some sort of mana is required to put them together (e.g. Wigner's famous articulation of the effectiveness of mathematics in natural sciences as **unreasonable**). This was Lévi-Strauss' reproach against Mauss' postulation of copulas rather than a relational logic. Lévi-Strauss needs no copula because for him the a priori signifier and the given, changing signified are simply non-equivalent. Between a 'signifying' hammer and the 'signified' nail there's relational impact, not a copula. If we manage to structure the latter with the former, it is by belaboured trial and error, not through an originary adequation or some **unreasonable** miracle. But the price we pay for this view is the articulation of the signifier-signified system as **a totality which is closed and complementary in itself**. If we follow this path, we enclose mathematics as well.

Baudrillard radicalises Lévi-Strauss' reformulation of the signifier-signified system (his so called 'order of the sign') by challenging its totality. He presents this order as just one form of social existence, and as conditioned by a more radical **symbolic ambivalence** (unlike Lévi-Strauss, Baudrillard reserves the term **symbolic** for whatever exceeds the order of the sign). The signifier-signified distinction is for Baudrillard **the very principle of the sign's rationality; it functions as the agent of abstraction and universal reduction of all potentialities and qualities of meaning (sens) that do not depend on or derive from the respective framing, equivalence, and specular relation of a signifier and a signified. This is the directive and reductive rationalization transacted by the sign — not in relation to an exterior, immanent "concrete reality" that signs would supposedly recapture abstractly in order to express, but in relation to all that which overflows the schema of equivalence and signification; and which the sign reduces, represses and annihilates in the very operation that constitutes it** ... **The rationality of the sign is rooted in its exclusion and annihilation of all symbolic ambivalence on behalf of a fixed and equational structure** (Baudrillard 1981, 149).

Focusing on the signifier-signified divide distracts our attention from the suppression of whatever the **sign reduces, represses and annihilates**. If we attempt to challenge this divide, we find that **the referent does not constitute an autonomous concrete reality at all; it is only the**

extrapolation of the excision (decoupage) established by the logic of the sign onto the world of things (onto the phenomenological universe of perception). It is the world such as it is seen and interpreted through the sign ... The "real" table does not exist. If it can be registered in its identity (if it exists), this is because it has already been designated, abstracted and rationalized by the separation (decoupage) which establishes it in this equivalence to itself (Baudrillard 1981, 149). This statement does not express an anti-realist stance; this is a statement that challenges the forcing of the sign structure onto the world by articulating the latter into referents. In our mathematical context, this stance translates into a challenge to the forcing of formal thinking on algebraic models and combinatorial problems, which threatens to enclose mathematics.

Indeed the boundary of mathematics is not the problem-model boundary, but the boundary that attempts to enclose the practice of articulating problems in terms of models. But this boundary too is permeable through and through. Consider, for instance, Kant's **object=x**. Clearly, this statement lies outside mathematics. It carries the mathematical x into metaphysics only as metaphor. But we can also think this metaphor the other way round. It then takes the metaphysical notion of 'object in itself', and carries it into mathematics, imposing it on our understanding of the algebraic unknown x. This reading obviously bears on our understanding and practice of mathematics, and affects mathematics from what is supposedly 'outside'. It projects on mathematics the clear and distinct order of objectivity and signs, covering over a more ambiguous **symbolic** practice.

What's excluded, according to Baudrillard, by the economy of the sign is ambivalent exchange that can't be reduced to discrete objective units of value or meaning. **Only ambivalence (as a rupture of value, of another side or beyond of sign value, and as the emergence of the symbolic) sustains a challenge to the legibility, the false transparency of the sign** (Baudrillard 1981, 150) — and this ambivalence may have a lot to do with the suppressed ambivalence of the algebraic unknown x, covered over by the rigour of post-hoc reconstruction.

Baudrillard's statements above are quoted from *The political economy of the sign*, and I quote them here to bring up a politics of knowledge and representation (after all, accepting or rejecting totality and closure is first and foremost an ethico-political act). Something is suppressed in contemporary images of mathematics. Ambivalent exchanges with symbols whose practice is not reducible to a signifier-signified or model-phenomenon opposition are suppressed. And while it is a great exaggeration to claim with Baudrillard that **All the repressive and reductive strategies of power systems are already present in the internal logic of the sign**, and while I can't follow him in hoping that **total revolution, theoretical and practical, can restore the symbolic in the demise of the sign and of value. Even signs must burn** (Baudrillard 1981, 163), nevertheless I believe it is good for contemporary philosophers of mathematics to invoke **the symbolic that continues to haunt the sign, for in its total exclusion it never ceases to dismantle the formal correlation** (Baudrillard 1981, 161).

We can't be any more precise about this symbolic that Baudrillard is invoking. We can't, because **the symbolic, whose virtuality of meaning is so subversive of the sign, cannot, for this very reason, be named except by allusion, by infraction (effraction). For signification, which names everything in terms of itself, can only speak the language of values and of the positivity of the sign** ... **Of what is outside the sign, of what is other than the sign, we can say nothing, really, except that it is ambivalent, that is, it is impossible to distinguish respective separated terms and to positivize them as such** (Baudrillard 1981, 161). And here we've already crossed the bounds of objectivity, which is expected from an academic scholar.

Here I, a mathematician and analyst of mathematical practice, can't know or argue for a resolution. We keep reconstructing our mathematics in a post-hoc effort to contain an apparatus that keeps exceeding the limits we impose. But to inquire about a **totality** or an end here leads us where **the analyst** ... **can no longer describe or objectify the programmed development of ~~ritual~~** mathematical practice. Indeed, that **the analyst in fact discovers limits to his work of scientific objectivation, that is quite normal: he is a participant in a process which he would like to analyse** (Derrida 1995, 23).[5] The one analysing the algebraic unknown is the one manipulating the algebraic unknown, resisting as creative manipulator the limits that as analyst one (who by now is at least two) set(s). Note that this **limit** of **objectivation** is, again, not a pathology, but, rather, what **furnishes positively the condition of his intelligence, of his reading, of his interpretation** (Derrida 1995, 23). Indeed, to think, **read** and **interpret** algebra one must manipulate the algebraic unknown, be open to creative reformulations, and thus risk this **limit** of **objectivation**.

What would lie beyond the **limit** of **objectivation**? The scientist can't know, as this question takes us from science to literature. **Something of literature will have begun when it is not possible to decide whether, when I speak of something, I am indeed speaking of something (of the thing itself, this one, for itself) or if I am giving an example of something or an example of the fact that I can speak of something, of my way of speaking of something, of the possibility of speaking in general of something in general, or again of writing these words, etc.** (Derrida 1995, 142–143). Indeed, the algebraic

[5]Derrida comments on Lévi-Strauss in Derrida (1976), where he also remarks on algebraic writing. The not necessarily fulfilled potential of algebraic writing to undermine logocentrism is also mentioned in Derrida (1981, 35). For the path I'm following here, however, I found it best to draw on Derrida's later work, where his notion of writing 'springs off the page' into a less strictly textual world.

unknown has been remaining undecided[6] for as long as it's been inscribed. **Something of literature will have begun when it is not possible to decide whether** the algebraic unknown has been a name, an object, a place holder or an assertion of the capacity to represent. And this indecision is not restricted to the mathematical sign, it encompasses mathematical practice as well. When I solve an exercise, am I necessarily solving *this* exercise? Or am I presenting an example of a *general* technique of solution? Or am I demonstrating my capacity to do mathematics? Or am I demonstrating that working with algebraic unknowns is possible in general as a way of solving problems? It is not always the case that one can decide between these possibilities. Occasionally, there is **Something of literature** with mathematical practice. Occasionally, algebraic unknowns do not manifest the closure of a semiotic system by zero symbolic value. Their un-dis-closure can occasionally **testify to a secret that is without content**, that is, without objectively knowable content, **without content separable from its performative experience, from its performative tracing** (Derrida 1995, 24).

For my violated Mauss algebra is a social a priori, yet arbitrary gluing of adequate representations. For my coerced Lévi-Strauss algebra is a closed system of forcing an a priori signifying structure onto a non-equivalent signified experience via ***floating*** elements. For my appropriated Baudrillard algebra should challenge the reduction of symbolic ambiguous exchange to the rigid order of the sign. But for Derrida the floating of the algebraic unknown, its secret, its functionality, would be in excess of *any* given or give-able system.

Can the unknown exceed *any* system? More modestly put, what can a hypothesis of such universal excess tell us about the algebraic unknown? To exceed *any* system this unknown, the algebraic unknown's practice of non-disclosure, its functionality, could not be an **artistic or technical secret reserved for someone** (Derrida 1995, 24), not even a genius' incommunicable knowledge of algebraic manipulations, as such knowledge would be *confined* to those in the know, contradicting our hypothesis of exceeding *any* system. This unknown, this practice of non-disclosure, if it were to exceed *any* system, could not be a **representation dissimulated by a conscious subject, nor ... an unconscious representation** (Derrida 1995, 24), for our hypothesis precludes any agency or subjectivity ruling over the unknown so as to be able to hide it. To exceed *any* system the unknown could not be **a deprived interiority ... to which one would have to respond by accounting for it and thematizing it in broad daylight** (Derrida 1995, 25) — assuming that something operates the unknown and rules over it from an 'inside' would be the precise opposite of respecting the unknown's excess

[6]For a comparison between Derridean indecision and the mathematical/logical concept related to the work of Gödel and Turing see Plotnitsky (1994, 214–223). For a analysis of the semiotic undecidability underlying Gödel's work see Wagner (2008) and Wagner (2007).

(**is there any worse** epistemic **violence than that which consists in calling for the response, demanding that one *give an account* of everything, and preferably *thematically*?** (Derrida 1995, 25)). Nor could the secret be **that to which a revealed religion initiates us nor that which it reveals** (Derrida 1995, 25), as assuming excess precludes enclosing mathematical unknowns in a Pythagorean or any other esoteric mysticism.

Here's how I could trace the algebraic unknown's excess. Here's what I could testify to, disclose, but never know or prove. **We testify to a secret that is without content, without a content separable from its performative experience, from its performative tracing** (Derrida 1995, 24). The unknown would lie bare on the surface, it would not hide a value or functionality. It would be **Heterogeneous to the hidden** ... **It simply exceeds the play of veiling/unveiling** ... **Its reserve is no longer of the intimacy that one likes to call secret** (Derrida 1995, 26). This 'secret' is not behind it, it's its repetition. **One can always speak about it**, of course, but **that is not enough to disrupt** or contain its capacity to exceed any closure through repetition (Derrida 1995, 26).

The secret of the unknown **would also be homonymy, not so much a hidden resource of homonymy, but the functional possibility of homonymy and *mimesis*** (Derrida 1995, 26). The algebraic unknown would always be variable. It would be able to mimic and repeat values, objects and syntactic roles without committing to a context, not even a mathematical context, where an analyst may wish to confine it. The algebraic unknown would always be variable. It would be what remains of a material symbol that's gone through replication — a leftover simulacrum with no guarantee of identity or closure. **Consequently, if the simulacrum still bears witness to a possibility which exceeds it, this exceeding remains, it (is) *the* remainder, and it remains such [il (est) *le* reste, il le reste] even if one precisely cannot here trust** either me **or any definite witness** (Derrida 1995, 31).

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The anonymous reviewer of this paper raised, among various useful remarks for which I'm grateful, the following questions: **is there a certain particular mathematical specificity to the working of Derrida's 'secrecy' in mathematics? In other words, how are the workings of Derrida's 'secrecy' different in mathematics than in other domains? And how does this specificity affect and make possible the technical aspects of mathematics?** ... **is mathematics a special field,**

which might, for example, prevent the possibility of metaphors in the strict sense, or would distinguish the way structures work in algebra as opposed to the case of indigenous magic concepts? I could not consider these questions in this text, but I would like to highlight them as important questions for future research.

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