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Recovering the control or implicit geometry underlying temple architecture requires bringing together fragments of evidence from field measurements, relating these to mathematical and geometric descriptions in canonical texts and proposing "best-fit" constructive models. While scholars in the field have traditionally used manual methods, the innovative application of niche computational techniques can help extend the study of artefact geometry. This paper demonstrates the application of a hybrid computational approach to the problem of recovering the surface geometry of early temple superstructures. The approach combines field measurements of temples, close-range architectural photogrammetry, rule-based generation and parametric modelling. The computing of surface geometry comprises a rule-based global model governing the overall form of the superstructure, several local models for individual motifs using photogrammetry and an intermediate geometry model that combines the two. To explain the technique and the different models, the paper examines an illustrative example of surface geometry reconstruction based on studies undertaken on a tenth century stone superstructure from western India. The example demonstrates that a combination of computational methods yields sophisticated models of the constructive geometry underlying temple form and that these digital artefacts can form the basis for in depth comparative analysis of temples, arising out of similar techniques, spread over geography, culture and time.

I.INTRODUCTION

Computational modelling provides a robust methodology for researching the genesis and evolution of geometry in temple architecture. The fragmented discontinuity of textual accounts, lack of graphical representations and heavily eroded early remains make the process of establishing the lineage of formal continuity difficult. In this context, computation presents an attractive methodology for capturing, analyzing and comparing partial geometry models from textual and graphic descriptions and specific temple sites spread over time, geography and culture. For example form models can be derived from data recovered from existing temples, two and three-dimensional idealized geometries can be reconstructed from textual canons (shastras) and these models can be analyzed and compared to yield new knowledge on the role of geometry in the genesis and evolution of temple architecture over time. This paper describes such a methodology and reports its application for recovering the surface geometry of temple superstructures.

The methodology comprises the combination of three distinct computations. First, a generic archetypal model of the superstructure surface using rule-based computation is generated. Second, detailed models of individual motif geometry from temples are recovered using close range photogrammetry. Finally, the superstructure geometry is developed by combining the first and second computations to generate a three dimensional solid model of the surface geometry. Using the superstructure of a tenth century latina temple as an illustrative example, each of the above computations is described in this paper.

2. TEMPLE GEOMETRY

From its early beginnings in the fifth century, the Brahmanic/Hindu tradition created a rich body of temples which spread across India and influenced temple building in Southeast Asia [1, 2]. The legacy of this diasporic movement evolved over time through a process of long experimentation with philosophies, form, and constructive methods. These traditions remain celebrated today in mature expressions such as Angkor Wat (Cambodia), Khajuraho (India) and Prambanan (Indonesia).

While there are thousands of variations in form, essentially every temple in the Brahmanic/Hindu tradition can be understood through principles outlined in canonic Sanskrit texts (shastras) such as Mayamata and Agni Puranas [3]. These texts provide sets of prescriptive rules which touch on all aspects of temple construction; site selection, formal types, details, and location of sculptural elements.

The architectural elements described by such shastras are based on a number of geometric figures known as mandalas, and it is from these ritual and cosmic diagrams that temple plans and superstructure have been generated.

Studies of Indic temple geometry have demonstrated the correspondence of canonical descriptions of constructive geometry with the base plans of surviving monuments. However, as these temples were built in dynamic, ever-changing cultural, physical and sectarian contexts, the actual practise of this knowledge was the subject of wide experimentation over several centuries within the regional schools of temple building [4-6]. Thus while the shastras may have been prescriptive, a multitude of interpretations and variations were possible within the canonical rules. Indeed, this ambiguous relationship between strict canon and subtle experimentation presents many challenges in relating the idealized geometry models to extant temples [7].

To understand the evolution of temple architecture, it is necessary to address the relationships between the construction and conception of temples through a rigorous examination of the geometries at play. To address this it is necessary to return to the formal foundations of Indian temple architecture, the early Nagara cella. The evolution of the cella embodied a progressive elaboration of this prototypical schema, using a sacred constructive geometry that conveyed the syncretic Brahmanic cosmology [8]. The formal schema of the cella comprises the base (pitha), an inner sanctum (prasada) and, later, a superstructure of distinctive form (in particular the curvilinear sikhara of the northern Indian Nâgara tradition). The morphology of the Indian temple and its progressive geometric complexity can thus be followed from the earliest extant cellae in the fifth century to entire thirteenth century complexes and temple cities across India and Southeast Asia.

Textual and graphic descriptions of mathematical and geometric constructions governing the form of temples are described in the literature. We draw upon this literature to extract two and three dimensional propositions of geometry governing the conception, composition and construction of temples.

Scholars have explained the use of geometry in temple plans by tracing their basis in canonical text, sacred diagrams and cosmogony [1-3, 9]. Specifically, the constructive and implicit relationships between geometric canon and individual monument are explained through studies of temple geometry [4-6].

In this paper, we address the surface geometry of stone superstructures. The superstructures of latina temples have a distinctive curvilinear form composed of a series of motifs as shown in Figure 1 The surface geometry of these structures, a result of intricate geometric experimentation with stereochromic techniques and carving, have been described in the literature [1, 10]. To recover the constructive principles underlying these forms, a hybrid approach combining field measurements, rule-based generation, close-range photogrammetry and parameterized models is described in this paper.

► Figure 1. Superstructure of a stone temple (960 CE) from the Maha-Gurjara school of temple building, western India. A hybrid approach that combines rule-based generation, photogrammetry and parametric modelling is used to explain the geometry of the form.



3. METHOD TO RECOVER SUPERSTRUCTURE GEOMETRY

The global model of the surface is constructed by using ruled-based generative modelling. Horizontal and vertical control geometries of the surface are derived from textual (canonical) accounts in the temple literature. The global model is then subdivided into four component motifs. Each component motif is modelled as a detailed local model using closerange architectural photogrammetry. Finally, the global surface is tiled with the motif models. The approach comprises the following steps:

- I. a global model of the superstructure using rule-based generation;
- 2. local models of motif geometry;
- 3. parametric tiling model combining the above.

The computation of each of the above is described in the following sections using the surface geometry of a tenth century stone superstructure: the temple of Ranakdevi at Wadhwan in Western India [11].

4. GENERATION OF SKELETAL GEOMETRY

Archetypal forms of superstructure can be computed from generic descriptions of geometric construction using rule-based generation. Textual and graphic descriptions of mathematical and geometric constructions described in the literature are codified in the form of shape rules and constructive methods to generate classes of formal three-dimensional geometry corresponding to the two-dimensional canonical descriptions.

The three dimensional form of the superstructure is based on encoding two control profiles: the horizontal plan profile and a vertical curved profile

(Figure 2). The profiles are computed using a set of boundary solid grammar (Heisserman and Woodbury, 1993) shape rules derived from the literature on temple geometry. The generation of geometric form with this technique allows a large class of profiles, and by extension superstructure forms to be explored.



◄ Figure 2. Left: Surface Geometry of a temple with three offsets (four faces) showing the curvature. Middle: A generated example showing a base with three offsets, central axis and geometric envelopes of the central spine (offset). Right: Top view of a generated example showing the global model of the surface geometry.

4.1. Plan Profiles

Embedded in the plan of most temples is a ritual grid diagram of $8 \times 8 = 64$ squares (mandala) prescribed for temple building in the Brhat Samhita and later texts [3]. This grid is used to generate the ground plan and control measure in the configuration of stone temples [5]. The horizontal profile depends on the number of offsets (angas) and the proportional relationships between each offset based on the subdivision of the sixty-four square grid. The grid diagram of 8×8 (64) squares, is used to generate the ground plan and control measure in the configuration of stone temples. Meister [4, 5] show how the horizontal profile depends on the number of offsets and the proportional relationships between each offset based on the subdivision of the sixty-four square grid. Following this, the horizontal profile of the temple of Ranakdevi is determined by recursive subdivision of the ritual grid of 64 squares. The basic module (mulasutra), of the ritual grid determined from field measurements, is a=660mm. Following the method reported by Meister [5], the offsets of the profile are determined based on the mulasutra (a). The horizontal profile has three offsets (four faces, caturanga) and these are sub measures of the basic module, a, a/4, a/4 and a/6 respectively. The width of the offsets in terms of the basic module (mulasutra) are 5a/4, 7a/8, a/4 and 7a/4 respectively. Using these measurements, the plan profile of the temple is computed (Figure 3).

Rules describe the operations involved in elaborating a profile from the simple square grid. By recursive subdivision, offsets are generated on each side and proportional relationships attached to each of the offsets.

geometry is controlled by two horizontal closed profiles in plan and an open curved profile in section. The rules for computing these profiles are described. The horizontal profile (caturanga) is offset into four faces based on a proportionate subdivision of the ritual 8x8 grid. The curve measure (caturguna) is derived by joining points of intersection in the vertical grid in the XZ-plane. Note: By changing the control parameters of the vertical XZ-grid, different curvatures can be obtained.

► Figure 3. The superstructure



4.2. Curve measure

The vertical profile is based on the extrusion of the profile of the ground plan in the vertical direction [12]. The extrusion in the vertical direction is based on a curved profile (rekha). This profile establishes the degree of curvature of the superstructure and controls the overall geometry of the superstructure (Figure 2). This profile establishes the degree of curvature of the superstructure. Following Kramrisch [3] and Dhaky [10], Datta [13] shows how this curve measure can be computationally generated based on textual descriptions.

The procedure is dependent on the height of the superstructure, the number of vertical units chosen for each offset and the choice of an integer for controlling the curvature (3, 4, 5, 7). In actuality, each offset has a different number of units, and hence a different rhythm. For simplicity, we treat the entire superstructure as a single unit (Figure 3). Following Kramrisch [3], the rules for deriving the rekha are summarised as:

If the base profile (ritual grid) is divided into ten parts, then the width of the top of the superstructure or skandha, is six parts, the height being given (multiple of mulasutra). This establishes the extent of the bottom and top profiles.

If the integer chosen for the curve is 4 (caturguna), the height is H, and then the successive vertical divisions are: H/4, I/4 (H - H/4), I/4(H - 3H/4), up to n terms, where n is the number of units [11-13].

As described above and shown in figure 3, the global geometry of the superstructure can be thus characterized by the following:

- I generating a horizontal base profile in the XY-plane based on rules for dimensioning the 8x8 grid and its proportionate subdivision into offsets;
- generating a vertical curved profile based on rules for dimensioning a vertical grid in the XZ-plane and its proportionate subdivision into stone units.

As the profile creation process is computed from parametric rules based on canonical description, a large class of profiles, and by extension, superstructure geometry, can be explored. The advantage of this rule-based generation of profiles based on parameterized rules is that the same set of rules can be used to generate "best-fit" constructive models that correspond to field measurements and observations of surviving monuments. In this way, the computability of a global geometry can form the basis for comparative analyses of a multitude of temple superstructures that are derived from the same constructive canon. Further, the comparison of generated global models allows for a rapid evaluation of the geometric similarities and differences between multitudes of samples. The application of this method to comparing the base and superstructure of several samples from South and Southeast Asia is currently being undertaken by the authors.

5. RECOVERY OF MOTIF GEOMETRY

The next stage of the method is the recovery of the individual motif geometries. Recall that the form of the superstructure is based on an extrusion of the caturanga plan profile following the vertical curve profile (caturguna rekha). The offsets of the superstructure (Figure 1) are tiled with distinctive units of carving that progressively diminish through self-similar copies, following the surface geometry of the superstructure (Figure 3). The geometry of these motifs can be accurately derived using control points determined from field measurements and digital photogrammetry.

As shown in figure I, the complexity of the surface patterns and the difficulties associated with direct manual measurements of these intricate motifs, necessitated the use of close-range architectural photogrammetry for recovering the motif geometry. The global model described previously serves as the "base" geometry for analysis, recovery and tiling of motif data from calibrated images of individual monuments.

► Figure 4. Models of individual elements of superstructures are generated using close-range architectural photogrammetry, supported by control points from field measurements. Over the last decade, hybrid approaches, combining close-range photogrammetry with model-based methods have been proposed in the literature for recovery and reconstruction of geometry from photographs. For example, DIPAD combines digital photogrammetric methods with CAD models in a priori and a posteriori modes [14]. The Facade system combines photogrammetry with a model-based stereo algorithm to develop architectural scenes and renders these scenes with view-dependent texture mapping [15].

In our approach, the 2D and 3D surface information is extracted from calibrated photographs of superstructures using close range architectural photogrammetry (Figure 4) supported by control points from field measurements of temples [11]. This process recovers the elemental geometric information in vector form in the form of points, lines, curves and planar surfaces. Control points from field surveys are used to add accuracy to the model information. These elemental geometries are then converted into closed 2D profiles in a CAD modelling environment. Here, the accuracy of the data is further improved and validated. Finally the profiles are converted into solids through standard constructive geometry (surfaces and solids) operations (extrusions, Booleans) into motifs.



The derivation of the motif geometry of the central offset is described here using three models. Based on measurements of the control points of the motif, a global bounding box model is established. This bounding box information is necessary from two perspectives, to accurately apply metric information to the photogrammetric process and to provide a graphic

"handle" for creating scaled copies of the unit, described in the next section. The bounding box is then subdivided into a number of equal horizontal bands (see back plate in Figure 5). Finally, the units of carving are superimposed onto the horizontal band model (Figure 5 and Figure 6).



The faces of the carving motifs are simplified into planes for easing the construction of the three dimensional model (Figure 5). In the actual motif, the gavaksha pattern is a smoothly curved surface. Further work is necessary to accurately represent the subtle curvature in the stonework.

◄ Figure 5. The motif geometry of the central spine is recovered into a three dimensional tile made of two interlaced segments, a "back plate" of horizontal bands and superposed units composed of carved patterns.

Figure 6: The local geometry of the central spine is computed into a three dimensional tile made of a bounding box and two interlaced segments, a band model and units composed of carved pattern (gavaska patterns).



6.TILING OF SURFACE GEOMETRY

In the final step, the model obtained from rule-based generation is combined with the detailed models of motifs. Each of the offsets has a separate motif and distinct sequence of constituent units forming their surface. This change in the surface geometry, and hence in the proportional series, imbues the surface with a complex rhythm. Taken together, the skeletal global model, three dimensional motif geometries and the series formulation of proportionate tiling defines the surface geometry of a latina temple superstructure.

To demonstrate this process, we focus on a single part of the superstructure, the central offset. The computing of the final geometry is broken into three parts, a skeletal model of the central offset, a parametric tiling rule for subdividing this model into constituent units and a tiling function for substituting the constituent units with individual motifs [16]. The bounding (skeletal) geometry of this offset is tiled with scaled copies of the unit of carving described in the previous section. The parametric surface is developed using the global model as a skeletal surface and this skeletal surface is tiled with a sequence of scaled units using the local geometry of the motif (Figure 6). The shape and appearance of model entities are derived by parameterization and generative modelling to support multiple variations from the same motif geometry. This forms the basis for the repetitive tiling of the surface using a scaling function based on the curve profile shown in Figure 3.A parametric model of the three dimensional surface is developed based on geometric sequences measured from the example shown in Figure 2.

The tiling function is based on the sequential subdivision of the curved surface. The tiling geometry of the central offset of the superstructure in this temple comprises 27 units. The first (lowest) unit is enclosed in a bounding box of size, $4a/3 \times a/3$ and the last unit has a bounding box of 4a/7x a/7, with the remaining 25 units falling within these limits in a series progression. The tiling of the central spine is computed by recursive subdivision of the global geometry using this series formulation of the scaled motifs [11, 13]. The bounding box of each unit is computed from a set of parameters that control the global model such as the initial starting unit, number of units, scale factor and type of progression. These are then tiled within the enclosing geometry of the central (bhadra) spine (Figure 7). This process rationalizes the degree of curvature derived from the rulebased curve generation into planar facets that approximate the curvature. Thus the explicit derivation of the curvature of the form as shown in Figure 4 now replaced by a family of polygonal tiles related by a function of the underlying series mathematics.

It is now possible to derive the motif geometry directly from this model by a simple substitution rule that maps the bounding box of each unit to the specific geometric size of the tile shown in Figure 5. The final parametric

model comprises the three dimensional surface and a collection of scaled repetitions of the original motif sequences measured from the example shown in Figure 2. The resultant model gives the final superstructure geometry where each tile in the series is a self similar scaled version of the motif model geometry (Figure 7).



◄ Figure 7. Left: The skeletal geometry of the central spine is subdivided into constituent units based on a series progression. Right: A tiling function replaces each constituent unit in the series with self similar motif geometry.

7. EXTENSIONS

The computational approach described in this paper results in the creation of multiple partial three dimensional models of superstructure geometry. It is envisaged that these models will be useful for:

- I. supporting the comparative analysis of superstructure geometry of temples from related temple building traditions (e.g. within South and Southeast Asia);
- piecing together the genesis and evolution (over time) of the geometric experimentation within specific schools of temple building (e.g. Maha-Gurjara, Chandela, etc.); and
- 3. explaining the complex and problematic linkages between canonical prescriptions of ideal form with the analysis of data recovered from surviving monuments.

The methods described in this paper will be extended through a research project that aims to trace the genesis and evolution of temples in India and Southeast Asia from early experiments to mature expression. The geometric

basis of this architecture will be pieced together from diagrams and canonical descriptions, rule-based generation of idealized form models and close-range architectural photogrammetry of temple remains. Through a comparison of the relationships between cosmology, geometry and physical form using computational methods, in these early sites with both Indian and Southeast Asian models, it is intended that the generative role of geometry within the architectural historiography of Brahminic temples can be clarified and more fully developed.

The digital models of the tiles are manually created, and motif geometry is particularly tedious and time consuming to reconstruct due to the complexity of the shapes. There is scope to further automate this process by using laser scanning techniques. To address more complex superstructures based on tiling motifs, further work is necessary in this domain. Tools that combine interactive manual modelling (direct manipulation) with formal methods for solving tiling problems (solvers) offer one possible direction to extend the computational tools used in this work.

8. DISCUSSION

The paper describes a computational technique for reconstructing the surface geometry of stone temple superstructures. The reconstruction present new possibilities for interpreting the formal and geometric basis of temple form. The combination of techniques described here in our hybrid approach, shows a dramatic improvement from past results [11-13]. The advantage of this process over manual methods is partly a matter of speed, both of data collection and of making geometric comparisons. It is difficult to pinpoint exactly how the surface geometry of a particular monument relates to canonical descriptions in the Sanskrit texts. As this research has shown, the answers nevertheless seem embedded in the architectural remains and our research attempts to 'read' temple remains through computational means to derive a position as to their formal derivation from canon. In this aspect, our approach may be compared in a propositional sense to Robin Evans' theories on geometry in architectural making [17]. Evans uses a series of translations to track the development of architectural form through projective geometry. In his work, the building as object is cast, through a series of drawings, to the finished product, a projection informed by the subjective experience of buildings. While Evans develops a proposition about how architecture develops through the translation of drawing into building, of representation into actuality, our particular challenge is the opposite, the translation of building through the geometric and proportional clues present in its form back to its description. In this reversal, computational means such as rule-based generation, photogrammetry and parametric modelling become useful methods for reconstruction.

The digital reconstruction of cultural heritage, also known as Virtual

Heritage, is an established area of research. As Affleck and Kvan [18] observe, the majority of virtual heritage projects attempt to create in the computer a realistic representation of their subject, this is not the primary intent of this project (though it is a possible application). Partially this is an attempt not to fall into the trap of mistaking realism for authenticity, but essentially what is important is not so much to recreate a temple form, but to uncover how its architecture was developed by comparing its formal properties with models from which it may have been derived. Advances in computation provide new ways to explore, analyze and explain the genesis and evolution of these historical artefacts. The application of computational techniques enables us to bring together fragments of evidence, construe "best-fit" strategies and unearth implicit or hidden relationships. For example, in the temple superstructure example described here, the use of the ritual grid, well known in the layout of temple plans [4-6] was projected into the vertical plane to decipher the compositional structure of the superstructure, including the derivation of the curve measure. The use of series mathematics, well known from temple literature [13, 16], was used to develop the tiling models. The complexities of the surface geometry could be explored and repetitive models obtained through generation and parameterization. The example demonstrates the above principles in the context of one type of tenth century superstructure, one that follows the profile of the offset in plan, and a curve in section. The advantage of this process is that changes in any stage due to revision of assumptions or testing of alternatives can be easily propagated between the models. Further, since the models of the surface geometry are based on generic constructions, they can be easily transferred to other, similar forms such as related but different schools of temple building can be easily incorporated in any stage, whether due to revision of assumptions or the testing of alternatives. In placing a specific temple between its possible antecedents, the use of constructive geometry as a generator allows the study of the evolution of temple architecture form over time as a series of related instances of arising out of similar techniques.

Thus, computation of spatial information plays a fundamental role in plotting any links between extant architectural remains and the principles of geometrical and architectural composition as presented in the texts. The representation of the building through the series of computed points is not just a device for aiding visualization but a deep description of its underlying geometry, a reverse analogue to the traits that Evans describes as the "instructional device" for the complex cutting of French renaissance stonework.

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