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Axion Dissipation Through the Mixing of Goldstone Bosons

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Abstract

By coupling axions strongly to a hidden sector, the energy density in coherent axions may be converted to radiative degrees of freedom, alleviating the “axion energy crisis”. The strong coupling is achieved by mixing the axion and some other Goldstone boson through their kinetic energy terms, in a manner reminiscent of paraphoton models. Even with the strong coupling it proves difficult to relax the axion energy density through particle absorption, due to the derivative nature of Goldstone boson couplings and the effect of back reactions on the evolution of the axion number density. However, the distribution of other particle species in the hidden sector will be driven from equilibrium by the axion field oscillations. Restoration of thermal equilibrium results in energy being transferred from the axions to massless particles, where it can redshift harmlessly without causing any cosmological problems.

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1 Introduction

Goldstone and pseudo-goldstone bosons play a large role in the speculations of particle physicists and cosmologists. Some oft-discussed examples are axions,¹ majorons,² familons,³ and the non-abelian goldstone fields of texture models.⁴ The salient feature of the dynamics of goldstone bosons is that they couple derivatively. If the associated global continuous symmetry is broken at a scale f , then the coupling always involves powers of $f^{-1}\partial_\mu b$, where b is the goldstone boson. The consequence of this is that if f is very large compared to the relevant masses and momenta, the goldstone boson decouples. This fact is exploited in invisible axion models,⁵ where for $f_a \gtrsim 10^{10} \text{ GeV}$ the axion's couplings are small enough for it to have evaded detection and avoided conflict with astrophysical observations.⁶ This fact also lies behind the well-known “axion energy problem”,⁷ which is that if $f_a \gtrsim 10^{12} \text{ GeV}$ the energy in coherent oscillations of the axion field after the QCD phase transition in the early universe is unable to dissipate due to the axion's weak coupling and eventually overcloses the universe.

In this letter we show that different goldstone fields can mix with each other through kinetic terms in the Lagrangian in a way reminiscent of photon-paraphoton mixing,⁸ and that a goldstone boson can thus acquire effective couplings at low energy to certain fields which are much larger than the typical p/f . After demonstrating this point we show how it makes possible scenarios in which the energy in coherent oscillations of the cosmological axion field can be dissipated and the upper limit on f_a removed.

2 Mixing of goldstone bosons

Consider a theory with a global symmetry $U(1)_a \times U(1)_b$. Let Ω and Φ be two scalar fields with charges $(1,0)$ and $(0,1)$ under this symmetry. Suppose that $|\langle\Omega\rangle| \equiv f_a$ and $|\langle\Phi\rangle| \equiv f_b$, where f_b is assumed to be much smaller than f_a , and call the resulting goldstone modes of Ω and Φ , ‘ a ’ and ‘ b ’ respectively. Then it is clear that couplings of a will be suppressed by p/f_a and those of b by p/f_b .

It can happen that by integrating out some fields of mass M an effective higher dimension operator will result of the $U(1)_a \times U(1)_b$ -invariant form

$$\mathcal{L}_{mix} = \frac{c}{M^2} \Omega^\dagger \partial_\mu \Omega \Phi^\dagger \partial^\mu \Phi + h.c. \quad (1)$$

If $\Omega = (f_a + \tilde{\Omega})e^{ia/f_a}$ and $\Phi = (f_b + \tilde{\Phi})e^{ib/f_b}$ are substituted into this expression, the result contains the term

$$\mathcal{L}_{a,b} = \epsilon (\partial_\mu a \partial^\mu b) \quad (2)$$

with

$$\epsilon \equiv -2c \frac{f_a f_b}{M^2}. \quad (3)$$

Through this mixing of a and b , the ‘axion’ a can couple to particles which have $U(1)_b$ charges with effective strength $\epsilon p/f_b$, which can be much greater than p/f_a (see Fig. 1).

For example, a term such as in Eq. (1) could arise from integrating out a field η , with charges $(-1/2, -1/2)$, mass M , and coupling $[\lambda(\Omega\Phi\eta^2) + h.c.]$. For simplicity let η have positive mass-squared and no VEV. Then the loop shown in Fig. 2 will produce the following effective term (among others) for

momenta less than M :

$$\frac{\lambda^2}{192\pi^2 M^2}(\Omega^\dagger \partial_\mu \Omega \Phi^\dagger \partial^\mu \Phi + h.c.) \quad (4)$$

and therefore, from Eq. (3),

$$\epsilon = -\frac{\lambda^2}{96\pi^2} \frac{f_a f_b}{M^2} . \quad (5)$$

Now, given that the term $\lambda(\Omega\Phi\eta^2)$ exists, it would require an artificial cancellation for M^2 to be much less than $\lambda f_a f_b$, and hence for ϵ to be much larger than $\lambda/(96\pi^2) \sim 10^{-3}$. There is also an absolute limit that $\epsilon < 1$, since the kinetic terms of a and b have the form

$$\frac{1}{2} \begin{pmatrix} \partial_\mu a & \partial_\mu b \end{pmatrix} \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \partial^\mu a \\ \partial^\mu b \end{pmatrix}$$

and a wrong-sign metric would otherwise result. There is no reason, however, why ϵ cannot be of order, though less than, unity.

If the higher dimension term in Eq. (1) were a “hard” term all the way up to momenta of order f_a , then the theory would become strongly coupled and physics uncalculable unless M^2 were $\geq f_a^2$ and hence ϵ were less than about f_b/f_a . The effective coupling of a to particles with $U(1)_b$ charges through mixing then would be of order $\epsilon p/f_b \leq p/f_a$. In other words, the coupling of a would not be enhanced by this mixing. If, on the other hand, the term in Eq. (1) arises through a loop such as that shown in Fig. 2 where the intermediate particles have mass $M^2 \sim f_a f_b$, then it softens for momenta above M and no problem arises in higher order.

3 Alleviating the axion energy crisis

We wish to make use of the mixing of goldstone bosons to let axions couple strongly to some other particles and thus, somehow, allow the primordial energy density in axions to be dissipated.

Let the field a be the axion that solves the strong CP problem. Then $U(1)_a$ has a QCD anomaly and a gets a mass of order Λ_{QCD}^2/f_a from instanton effects. Several astrophysical arguments⁶ suggest that $f_a \gtrsim 10^{10} \text{ GeV}$, otherwise the evolution of stars or their remnants would be drastically different than observed. Thus the coupling of the axion to ordinary matter, which is proportional to $1/f_a$, is highly suppressed.

The axion energy crisis arises as a result of the small coupling and the small mass of the axion. At temperatures above the QCD phase transition the axion mass is small enough that $\dot{a} \approx 0$. As the universe cools, m_a increases until the axion field becomes dynamic, *i.e.* $m_a > H$, where H is the expansion rate. Barring a coincidence of alignment for the Peccei-Quinn angle, the initial value of the axion field is $a \sim f_a$. The axion number density is then $n_a \approx f_a^2 H$. After that point the number of axions in a comoving volume is constant, while the axion mass increases from H to its zero temperature value m_{a0} . If the energy density of axions after the QCD phase transition exceeds a critical value, $\rho_{a,cr} \approx 10^{-7} \Lambda_{QCD}^4$, then the universe would become matter dominated too early, *i.e.* at temperatures $\gtrsim 10 \text{ eV}$. The energy density in axions at $T \sim \Lambda_{QCD}$ is roughly $\rho_a(\Lambda_{QCD}) \sim \Lambda_{QCD}^4 f_a / M_{Pl}$, which results in the constraint $f_a < 10^{12} \text{ GeV}$. In non-inflationary models there are bound to be axionic strings - whose decay may⁹ or may not¹⁰ increase the axion

abundance 100-fold, lowering the cosmological bound to $f_a \lesssim 10^{10} \text{ GeV}$.

There have been several approaches to solving the axion energy crisis. For example, entropy production¹¹ may dilute the energy density in axions relative to ordinary matter, or inflation may fix the initial value of the Peccei-Quinn angle to a suitably small value¹². However, attempts to dissipate the axion energy typically fail. Direct axion absorption presents several challenges. First, the goldstone boson nature of axions demands a factor of $1/f_a$ at each vertex. Second, since the axions to be absorbed are non-relativistic the derivative couplings demand an additional factor of m_a at each vertex. Third, any process for single axion absorption must admit axion emission in the time reversed channel. This leads to a reduction in the dissipation rate by a factor of m_a/T from a naive approach that would ignore the back reactions. Flynn and Randall¹³ considered the non-derivative coupling of axions to mesons¹⁴, but the first and third arguments above were sufficient to leave axions undamped by several orders of magnitude.

In the present case, the mixing of axions with some other goldstone boson allows the possibility of coupling axions strongly to some other non-standard matter. However, without some explicit symmetry breaking of the $U(1)_b$, the axions must couple derivatively. The combination of powers of m_a from the couplings and from consideration of back reactions has caused all our attempts at axion absorption to fail.

Nonetheless, there is a mechanism whereby axion oscillations can be damped, allowing for f_a of order the grand unified scale M_{GUT} , or even the Planck scale M_{Pl} . Let the axion be mixed, as described above, with strength ϵ with a goldstone boson b arising from the breaking of $U(1)_b$. The

axion coupling to ‘ b -matter’, as we will call it, will be vastly stronger than its coupling to ordinary matter. Our basic idea is then that the oscillating axion field causes the energies of b -particles to change in energy. If a thermal bath of such b -particles were present in the early universe when the axion field started to oscillate, then those oscillations would drive the b -distribution out of thermal equilibrium. By adjusting the interaction rates of the b -particles to a suitable value the axion energy can be dissipated into thermal energy of b particles. The final step is to note that the b goldstone boson is massless, so that all b energy can in principle redshift away as radiation. The axion energy is thus put into a harmless form which never dominates the universe.

4 An explicit model

To illustrate how this may happen let us calculate in a simple model in which the ‘ b -matter’ consists of the field Φ whose vacuum expectation value breaks $U(1)_b$ and whose phase is b , and a scalar field χ which has no VEV. Let the $U(1)_b$ charges of Φ and χ both be $+1$ and let them have a coupling ($\frac{1}{4}g\Phi^{*2}\chi^2 + h.c.$).

The terms of the Lagrangian that will be relevant for our analysis are

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{1}{2}(\partial_\mu b)^2 + \epsilon(\partial_\mu a \partial^\mu b) + \\ & \frac{1}{2}|\partial_\mu \chi|^2 - \frac{1}{2}m^2 |\chi|^2 - \frac{1}{4}\mu^2 \left(e^{-2ib/f_b} \chi^2 + h.c. \right). \end{aligned} \quad (6)$$

The last term arises from expanding Φ as $(f_b + \tilde{\Phi})e^{ib/f_b}$ in the $\Phi^{*2}\chi^2$ coupling and defining $\mu^2 \equiv gf_b^2$. At this point we are free to choose the parameters of \mathcal{L} . For purposes of illustration it is convenient to choose $f_b \sim m \sim \Lambda_{QCD}$

and to choose μ somewhat less than m . These values are not absolutely necessary, but they clearly show the damping mechanism to work.

The first task is to resolve the mixing of a and b . To bring the kinetic terms of a and b to canonical form we perform a non-orthogonal transformation

$$\begin{aligned} a' &= a\sqrt{1-\epsilon^2} \\ b' &= b + \epsilon a . \end{aligned} \quad (7)$$

This gives

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(\partial_\mu a')^2 - \frac{1}{2}m_a^2(1-\epsilon^2)^{-1}a'^2 + \frac{1}{2}(\partial_\mu b')^2 \\ &+ \frac{1}{2}|\partial_\mu \chi|^2 - \frac{1}{2}m^2|\chi|^2 - \frac{1}{4}\mu^2 \left(e^{2i/f_b(-b' + \frac{\epsilon a'}{\sqrt{1-\epsilon^2}})} \chi^2 + h.c. \right) . \end{aligned} \quad (8)$$

At this point it is convenient to drop the primes and work with the canonically normalized fields. For further convenience we represent the coherent oscillation of the axion field by

$$a(t) = a_0\sqrt{1-\epsilon^2}\sin(m_a t) . \quad (9)$$

Then the effective Lagrangian for χ in the presence of axion oscillations becomes

$$\mathcal{L}_{eff} = \frac{1}{2}|\partial_\mu \chi|^2 - \frac{1}{2}m^2|\chi|^2 - \frac{1}{4}\mu^2 \left(e^{2i\theta(t)} \chi^2 + h.c. \right) \quad (10)$$

with $\theta(t) \equiv (\epsilon a_0/f_b)\sin(m_a t)$.

It is important to note that even though $a(t)$ is slowly varying (with frequency m_a), $\theta(t)$ is rapidly varying: $q \equiv \dot{\theta}(t) = \epsilon \frac{a_0 m_a}{f_b} \cos(m_a t)$. Since $a_0 m_a = \rho_{axion}^{1/2} \sim \Lambda_{QCD}^2 (f_a/M_{pl})^{1/2}$ (initially), $q \sim \epsilon \Lambda_{QCD}$. For time scales short compared to m_a^{-1} we may treat q as being constant and write \mathcal{L} as

$$\mathcal{L}_{eff} = \frac{1}{2}|\partial_\mu \chi|^2 - \frac{1}{2}m^2|\chi|^2 - \frac{1}{4}\mu^2 \left[e^{2iqt} \chi^2 + h.c. \right] . \quad (11)$$

The problem now is to find the evolution of the χ field, a non-trivial task because of the explicit time dependence. This is done in the Appendix, but we give a simplified version here. First, if $\mu^2 = 0$ then for each momentum \vec{k} there would be two degenerate eigenstates. One could choose to work in a basis with linearly polarized or with circularly polarized states. With degenerate states it is easy to show that the Hamiltonian is constant for any linear combination. Next, consider the case $\mu^2 \neq 0$, but $q = 0$. Then the two eigenstates have masses $(m^2 \pm \mu^2)^{1/2}$, and correspond to the linearly polarized states along the real and imaginary components of χ ; *i.e.* they are $A(x^\mu)$ and $B(x^\mu)$, where $\chi(x^\mu) = A(x^\mu) + iB(x^\mu)$. Now, there is no explicit time dependence so the Hamiltonian is still constant. This can be seen explicitly by writing the Hamiltonian in terms of the linearly polarized states and observing that there is no mixing of different frequency states in the Hamiltonian. Finally, for non-zero q , the Hamiltonian is time dependent and the energy of the χ field varies with time. For μ^2 small the evolution is well approximated by the $\mu^2 = 0$ eigenstates, but there is a qualitative difference between the linearly polarized and circularly polarized solutions.

For the linearly polarized case, the potential in the A (or B) direction has a curvature that varies sinusoidally in time: $m_{eff}^2 = m^2 \pm \mu^2 \cos(2qt)$. Since we will assume $q \ll m$, the variation in m_{eff}^2 is adiabatic. In that case one expects each quantum to have energy $\sqrt{m^2 + \vec{k}^2 + \mu^2 \cos(2qt)} \simeq \omega_0 + \frac{1}{2} \frac{\mu^2}{\omega_0} \cos(2qt)$, where $\omega_0 \equiv \sqrt{m^2 + \vec{k}^2}$. The number of quanta of oscillation is an adiabatic invariant.

On the other hand, for the circularly polarized case, the state samples the potential at all phases of χ . Since $m \gg q$, this averaging takes place in

a time small compared to the evolution of the potential and there is little time dependence in the energy. In the Appendix we show that, in fact, the true eigenmodes are approximately circularly polarized. There is no time dependence for pure states; however, in a thermal bath we do not expect pure states unless there is some conserved quantum number. The corresponding symmetry in our case is the b symmetry, which is broken, so that b charge is not conserved. We therefore expect all polarizations to play a role, and having identified the linearly polarized states as having an energy dependence driven by the axion field we look for dissipation of the axion energy through those modes.

Time variation of the energy levels of the χ field can lead to dissipation. Suppose the rate at which a linearly polarized χ particle of momentum \vec{k} scatters is given by $\gamma(\vec{k})$. Then the occupation number $f_{\vec{k}}$, of that state will obey the differential equation

$$\dot{f}_{\vec{k}} = \gamma(\vec{k}) \left[f_0 \left(\frac{E(\vec{k}, t)}{T} \right) - f_{\vec{k}}(t) \right] \quad (12)$$

where $E(\vec{k}, t) = \omega_0 + \frac{1}{2} \frac{\mu^2}{\omega_0} \cos(2qt)$ and f_0 is the equilibrium occupation number (Bose–Einstein). Treating $\frac{\Delta E}{T} = \frac{1}{2} \frac{\mu^2}{T\omega_0}$ as small, this has solution

$$f_{\vec{k}}(t) = f_0 \left(\frac{\omega_0}{T} \right) + f_0' \left(\frac{\omega_0}{T} \right) \frac{\gamma}{\sqrt{\gamma^2 + 4q^2}} \frac{\mu^2}{2T\omega_0} \cos(2qt + \beta) \quad (13)$$

where $f_0' \equiv \partial f_0 / \partial x$ and $\tan \beta = -(2q)/\gamma$. The occupation numbers thus ‘lag’ by a phase β behind the equilibrium value as that oscillates in time. As $\gamma \rightarrow \infty$, this lag goes to zero and $f_{\vec{k}}(t) \rightarrow f_0 \left(\frac{E(\vec{k}, t)}{T} \right)$; the system remains in equilibrium. As $\gamma \rightarrow 0$, the interactions turn off and there is no time variation of $f_{\vec{k}}(t)$. Maximum dissipation occurs for $\gamma = 2q$. The average

power dissipated over a period $(2q)^{-1}$ by the mode with momentum \vec{k} is given by

$$\begin{aligned}
P &= -(2q) \int_0^{(2q)^{-1}} dt \dot{f}_{\vec{k}}(t) E(\vec{k}, t) \\
&= f'_0 \left(\frac{\omega_0}{T} \right) \frac{\gamma(2q)^2}{\sqrt{\gamma^2 + (2q)^2}} \frac{\mu^4}{4\omega_0^2 T} \times \\
&\quad \int_0^{(2q)^{-1}} dt \sin(2qt + \beta) \cos(2qt)
\end{aligned} \tag{14}$$

or

$$P = -f'_0 \left(\frac{\omega_0}{T} \right) \frac{\gamma(2q)^2}{\gamma^2 + (2q)^2} \frac{\mu^4}{8\omega_0^2 T} . \tag{15}$$

The total power dissipated per unit volume is then given by

$$P = -\frac{1}{3} \int \frac{d^3 \vec{k}}{(2\pi)^3} f'_0 \left(\frac{\omega_0}{T} \right) \left[\frac{\gamma(2q)^2}{\gamma^2 + (2q)^2} \right] \frac{\mu^4}{8\omega_0^2 T} . \tag{16}$$

The factor 1/3 comes from averaging over all possible polarizations of the χ particles.

Equating the dissipation power with the change in axion energy density, $P = \dot{\rho}_a$, and using $\langle q^2 \rangle = \langle (\epsilon \rho_a^{1/2} / f_b \cos(m_a t))^2 \rangle = \frac{1}{2} \epsilon^2 \frac{1}{f_b^2} \rho_a$, the energy in coherent axion oscillations evolves as

$$\dot{\rho}_a \simeq \frac{1}{4\pi^2} C(m, T) \frac{2\epsilon^2 \rho_a \gamma \mu^4}{\gamma^2 f_b^2 + 2\epsilon^2 \rho_a} , \tag{17}$$

where $C(m, T) = \int dx \frac{x^2}{c^2 + x^2} f'_0(\sqrt{c^2 + x^2})$, with $c \equiv m/T$ and $x \equiv k/T$. Let us now make the approximation that whatever dissipation occurs takes place in less than an expansion period - so that the functions γ and $C(m, T)$ may be approximated as constant. Then, taking the integral $\int dt \approx 0.1 M_{pl}/T^2$, the final axion energy density is

$$\rho_{a,f} e^{\frac{2\epsilon^2 \rho_{a,f}}{f_b^2 \gamma^2}} = \rho_{a,i} e^{\frac{2\epsilon^2 \rho_{a,i}}{f_b^2 \gamma^2}} e^{-\frac{10^{-3} \epsilon^2 \mu^4 M_{pl}}{f_b^2 \gamma T^2}} \tag{18}$$

This solution embodies a number of special cases, but we are mostly interested in the cases where we can treat $(2\epsilon^2\rho_a)/(f_b^2\gamma^2)$ as small. Then, the axion density decreases exponentially during one expansion time. Given that the maximum initial value of ρ_a is approximately Λ_{QCD}^4 , as long as $(10^{-3}\epsilon^2\mu^4 M_{pl})/(f_b^2\gamma T^2) > 20$, the axion density will be reduced to acceptable levels.

We take the temperature to be about Λ_{QCD} . Dissipation is fastest if f_b is small. On the other hand, if $f_b < T$, the $U(1)_b$ symmetry will be unbroken and the axion will have no goldstone boson, b , to mix with, and this dissipation mechanism will not operate. So the greatest dissipation should exist for $f_b \sim T \sim \Lambda_{QCD}$, and

$$\epsilon^2 \left(\frac{\mu}{\Lambda_{QCD}} \right)^4 \left(\frac{\gamma}{\Lambda_{QCD}} \right)^{-1} \geq 10^{-16} . \quad (19)$$

On the other hand, the dissipation cuts off for $q \ll \mu^2/m$. So the energy in the axion field cannot be reduced below

$$\rho_{f,min} \approx \frac{\mu^4 f_b^2}{\epsilon^2 m^2} \quad (20)$$

by this mechanism. Demanding that this be less than $\sim 10^{-8}\Lambda_{QCD}^4$ in order to comfortably solve the axion energy problem gives $(\mu/\Lambda_{QCD})^4 \leq \epsilon^2 10^{-8}$, and from Eq. (19), $\epsilon^4(\gamma/\Lambda_{QCD})^{-1} \geq 10^{-8}$. These two constraints can easily be satisfied, for example by $\epsilon \sim 10^{-3}$, $\gamma < 10^{-4}\Lambda_{QCD}$, and $\mu^2 \approx 10^{-7}\Lambda_{QCD}^2$.

5 Discussion

We have established that it is possible to dissipate the primordial energy density in axions through their interactions with other particles. A key fea-

ture of the process we envision is to mix the axion with some other goldstone boson b , whose decay constant f_b is much smaller than f_a . The mixing may be large, and this allow axions to couple strongly to ‘ b matter’. By adjusting the matter content of the b -sector we can arrange for the axion oscillations to drive oscillations in the energy of b matter. This pushes the b matter out of thermal equilibrium, and ultimately allows the axion energy density to dissipate in the b -sector.

Although, we have constructed a model whereby the axions dissipate, the addition of new particles may have other troublesome consequences, chief of which is that b matter may itself come to dominate the universe. The easiest way to avoid this is to adjust the masses and couplings of the b -sector so the real part of the Φ field is the lightest b -particle other than the b goldstone boson itself. Then as the universe cools, first all b matter will end up as ρ_b ’s which will then decay with a rate of order f_b into b goldstone bosons.

We argued earlier that it is difficult to dissipate axion energy through the process of single axion absorption, yet, in the present scenario axion energy is indeed absorbed. It is natural to ask if there is a ‘‘Feynman diagram’’ explanation of our process. The answer is yes. Imagine a process whereby a single axion is absorbed from the coherent state, as in Fig. (3a) and a second process whereby two axions are absorbed, as in Fig. (3b). The ratio of the rates for these two processes is

$$R = \Gamma_2/\Gamma_1 \sim n_a \frac{\epsilon^2 m_a^2}{f_b^2}, \quad (21)$$

where n_a is the occupation number for the state. As stated earlier the effective single axion absorption rate is reduced by a factor of m_a/T when back

reactions are included. However, this is also true of the two axion absorption rate, and so the ratio in Eq. (21) is correct even in the presence of backreactions. Now, for the coherent state, n_a is approximated by $n_a \sim f_a^2/H^2$, so the ratio becomes $R \sim \epsilon^2 M_{pl}^2/\Lambda_{QCD}^2$, which is quite large. This leads to the situation that two axion absorption formally exceeds one axion absorption. The three axion rate would be even larger... In this situation what one must do is sum over diagrams with any number of axions attached to the particles participating in the scattering process. This is equivalent to solving for the propagator of these particles in the presence of the coherent oscillating axion field, as we have done in this paper.

Along these lines, even in a normal axion model with no mixing each additional axion brings a factor of M_{pl}/f_a to the amplitude, so here also one should not calculate individual absorption rates. Rather, one should calculate the propagators for the other particles, including the time dependent axion field, to arrive at the axion dissipation rate. We are presently considering the question of what dissipation may result from the non-derivative couplings to mesons, and plan to present results in a separate paper. Although derivative couplings may seem important too, one must remember that such couplings are less effective at damping the non-relativistic coherent axion oscillations due to the factor of m_a in the coupling.

Besides damping the coherent oscillations in the early universe, the kind of mixing of axions with other goldstone bosons we have been discussing could lead to other interesting effects. In particular, it is possible now to contemplate that even invisible axions with $f_a \sim M_{GUT}$ or M_{Pl} can have sizable couplings to some kinds of matter, perhaps even to particles that

carry standard model gauge charges. This would mean that the “invisible axion” may not be quite as invisible as was thought.

Also worth further investigation is whether similar mixing in familiar majoron, familon or texture models could lead to interesting phenomena.

Appendix

The easiest way to find the eigenstates of the Lagrangian in Eq. (11) is to make the field redefinition $\psi = e^{iqt}\chi$, after which the Lagrangian becomes

$$\mathcal{L}' = \frac{1}{2} \left(|\partial_\mu \psi|^2 - (m^2 - q^2) |\psi|^2 + iq(\dot{\psi}\psi^* - \psi\dot{\psi}^* - \frac{1}{2}\mu^2(\psi^2 + \psi^{*2})) \right). \quad (\text{A1})$$

The equations of motion for \mathcal{L}' are

$$\begin{aligned} \ddot{\psi} - \nabla^2 \psi + (m^2 - q^2)\psi - 2iq\dot{\psi} + \mu^2\psi^* &= 0 \\ \ddot{\psi}^* - \nabla^2 \psi^* + (m^2 - q^2)\psi^* + 2iq\dot{\psi}^* + \mu^2\psi &= 0. \end{aligned} \quad (\text{A2})$$

The solutions to these equations are

$$\begin{aligned} \psi_i &= \alpha_i \left(e^{i(\omega_+ t - k \cdot x)} + \delta_+ e^{-i(\omega_+ t - k \cdot x)} \right) \\ &\text{or} \\ &= \alpha_i \left(e^{-i(\omega_- t - k \cdot x)} + \delta_- e^{i(\omega_- t - k \cdot x)} \right), \end{aligned} \quad (\text{A3})$$

where the index i denotes one of four solutions for a given wave vector \vec{k} . Due to the mixing of ψ and ψ^* , to simultaneously solve both equations of motion requires that α_i be either pure real or pure imaginary, but otherwise the normalization amplitude is arbitrary. The positive eigenfrequencies are given by

$$\omega_\pm \equiv \left[\omega_0^2 + q^2 \pm \left[4\omega_0^2 q^2 + \mu^4 \right]^{1/2} \right]^{1/2}. \quad (\text{A4})$$

Negative frequency solutions for momentum \vec{k} are equivalent to positive frequency solutions with momentum $-\vec{k}$, and so are not distinct. For the case when α_i is real, the mixings are given by

$$\delta_{\pm} = \frac{-\mu^2}{\omega_0^2 - (\omega_{\pm} \pm q)^2} . \quad (\text{A5})$$

For the case where α_i is imaginary, the sign of δ is changed.

Counting ω_{\pm} , and the real and imaginary α_i , there are eight eigenfunctions combining the wavenumbers k and $-k$, which is the right number to determine the field and its first time derivative at each point; so this is a complete set of states for the ψ field.

For the χ field, the states are given by $\chi_i = e^{-iqt}\psi_i$. We can choose any four of the eight eigenstates to characterize the momentum k (using the other four for $-k$). It seems natural to choose a set where $\omega - q \rightarrow \omega_0$ as $\mu^2 \rightarrow 0$, since in that limit we must get back the free field theory for χ . With this in mind, the general solution that we will associate with momentum k is

$$\chi = \sum_{s=\pm} (\alpha_{s+}\chi_{s+} + i\alpha_{s-}\chi_{s-}) \quad (\text{A6})$$

where the $\alpha_{ss'}$ are real numbers and where

$$\chi_{ss'} \equiv N_{ss'}^{-1/2} \left[(1 - \delta_{ss'}) e^{i(\omega_s - q)t - ik \cdot x} + \delta_{ss'} e^{-i(\omega_s + q)t + ik \cdot x} \right] . \quad (\text{A7})$$

Here

$$\begin{aligned} N_{ss'} &\equiv (1 - \delta_{ss'})^2 + (\delta_{ss'})^2 \\ \omega_s &\equiv s \left[\omega_0^2 + q^2 + s \left[4\omega_0^2 q^2 + \mu^4 \right]^{1/2} \right]^{1/2} \\ \delta_{ss'} &\equiv \frac{\omega_0^2 + s'\mu^2 - (q - \omega_s)^2}{4q\omega_s} . \end{aligned} \quad (\text{A8})$$

For $\mu^2 < mq$, which is the limit taken in the text, Eqs. (A8) reduce to

$$\omega_s \simeq sm + q \quad (\text{A9})$$

and

$$\delta_{ss'} \simeq \frac{ss'\mu^2}{4q(m+sq)} - \frac{\mu^4}{16q^2(m+sq)^2} . \quad (\text{A10})$$

The energy density computed by just substituting Eq. (A6) into the Hamiltonian density is

$$\rho = \rho_0 + Re \left(\rho_{2q} e^{2iqt} \right) + \dots \quad (\text{A11})$$

where the ellipses represent more rapidly varying terms (eg. terms varying with frequency $\omega_+ - \omega_-$ or $2\omega_s$) and

$$\begin{aligned} \rho_0 &= \left[\sum_{s,s'} (\alpha_{ss'})^2 \right] \omega_0^2 , \\ \rho_{2q} &= \mu^2 (\alpha_{++} + i\alpha_{+-}) (\alpha_{-+} + i\alpha_{--}) . \end{aligned} \quad (\text{A12})$$

The approximately ‘linearly polarized’ modes with $\arg(\chi) \simeq \theta/2$ correspond to $(\alpha_{-+} + i\alpha_{--})/(\alpha_{++} + i\alpha_{+-}) = e^{i\theta}$, whereas the approximately ‘circularly polarized’ modes correspond to the cases $(\alpha_{++} + i\alpha_{+-}) = 0$ and $(\alpha_{-+} + i\alpha_{--}) = 0$.

For linearly polarized modes

$$\rho \propto \left(\omega_0^2 + \frac{1}{2}\mu^2 \cos(2qt) \right) \quad (\text{A13})$$

and the energy per quantum is

$$E = \omega_0 + \frac{1}{2} \frac{\mu^2}{\omega_0} \cos(2qt) \quad (\text{A14})$$

as given in the text.

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Figure Captions

Figure 1. Effective coupling to ‘ b -matter’ through goldstone boson mixing.

Figure 2. Mixing of the ϕ and Ω scalars through a loop. The η particle carries both a and b charges.

Figure 3. Similar processes involving the absorption of a) a single axion and b) two axions. If the axions come from the coherent state describing the classical field oscillation, then the rate for b) exceeds that for a).