

10th Eco-Energy and Materials Science and Engineering

(EMSES2012)

Mathematical Modeling of an Absorption Chiller System Energized by a Hybrid Thermal System: Model Validation

Boonrit Prasartkaew*

*Department of Mechanical Engineering, Faculty of Engineering, Rajamangala University of Technology,
39 Moo 1, Klong 6, Thanyaburi, Pathumthani, 12110, Thailand*

Abstract

Nowadays, global warming and energy crisis problems become serious issues which affect on all creatures on our earth. One of the best ways to simultaneously address or mitigate these problems is more utilizing the renewable energy sources instead of the fossil fuel. A solar-biomass hybrid cooling system is one of the technologies for the climate change and green-house-gas mitigation. The mathematical model of this system was developed and used in the theoretical prediction of its performance and system design. To assess the accuracy of the developed mathematical model, the obtained experimental data is then compared with the simulation results, with the same operating parameters and weather conditions. This paper presents the validation of the developed model. The validation results show that the simulation results are in good agreement with the experimental results from both qualitative and quantitative points of view.

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Selection and peer-review under responsibility of COE of Sustainable Energy System, Rajamangala University of Technology Thanyaburi (RMUTT)

Keywords: Solar-biomass hybrid; Cooling system; Simulation model; Validation

1. Introduction

Nowadays, the global warming and energy crisis problems become the most serious issues. These problems, mainly attributed to the combustion of fossil fuel [1], substantially affect on all life on the earth. To address these problems, the renewable energy based should be encouraged. Among the energy utilization systems, cooling applications demand higher energy for their functioning [2]. Solar cooling technology is environmentally friendly and contributes to a significant decrease of the CO₂ emissions which cause the green house effect [3]. From this point of view, solar powered cooling systems as a green

* Corresponding author. Tel.: +662-549-3430; fax: +662-549-3422

E-mail address: prasartkaew@yahoo.com

cold production technology are the best alternative [4]. As a green energy technology for the climate change and green-house-gas mitigation, a solar-biomass hybrid cooling system was proposed by [5].

Nomenclature

| | |
|----------------|---|
| A | area (m ²) |
| C _p | specific heat capacity, (kJ/kg.K) |
| F _R | heat removal factor |
| G _T | solar insolation on tilted surface (kW/m ²) |
| h | specific enthalpy (kJ/kg) |
| M | mass (kg) |
| \dot{m} | mass flow rate (kg/s) |
| \dot{Q} | energy rate (kW) |
| T | temperature (K) |
| U | heat transfer coefficient (W/m ² K) |
| U _L | overall heat transfer coefficient (W/m ² K) |
| \dot{V} | volumetric flow rate (m ³ /s) |
| v | specific volume (m ³ /kg) |
| x | mass concentration (%) |
| η | efficiency |
| τ | transmittance |
| α | absorptance |
| β | tank load control function |
| γ | temperature differential control function |
| φ | boiler load control function |
| ε | heat exchanger effectiveness |
| ρ | density (kg/m ³) |
| 0 | initial condition |
| a | ambient |
| ab | absorber |
| aux | auxiliary |
| b | boiler |

| | |
|-----|----------------------|
| BM | biomass |
| bl | boiler to load |
| c | collector |
| co | condenser |
| cg | combustion gas |
| ev | evaporator |
| f | liquid state |
| fg | liquid-vapor mixture |
| g | vapor state |
| ge | generator |
| gw | gas to water |
| hi | high |
| i | inlet |
| lo | low |
| o | outlet |
| PG | producer gas |
| set | set point value |
| st | storage tank |
| tl | tank to load |
| u | solar useful energy |
| w | water |
| we | water to environment |

Regarding the use of biomass, which is a CO₂ neutral energy source and fruitfully available in all agricultural countries, gasification of biomass offers advantages over other sources. Using solar and biomass can significantly contribute to the reduction of CO₂ emission. A theoretical study on the performance of a solar-biomass hybrid air conditioning (SBAC) system has been done by [5]. Their results show that the proposed SBAC system for tropical locations is feasible, and can replace conventional vapor compression systems, thus reducing the need for fossil fuel based energy systems for cooling purposes. Subsequently, [6] reported the experimental study on the performance of the SBAC system.

The main advantage of theoretical study via mathematical model is that the model can be used to predict all output parameters at any conditions. Compared to the experimental study, the simulation results can be obtained with very low expense and less study time. The accuracy of the simulation results, however, depends on the reliability of the model. To know the reliability of developed model, the results of observed experimental results should be compared with the predicted results from the simulation with the same input parameters.

It should be noted that the mathematical model proposed by [5] had already been validated only for some of system components using the experimental results available from a literature (no available SBAC system at that time) and the proposed model has been used for the system design. The complete SBAC system was subsequently constructed and test as presented in [6]. Hence, this study aims at presents the model validation, comparison results, between the simulation results predicted from the developed model [5] and the experimental results obtained from the fabricated experimental system [6]. Section 2 describes the proposed system relevant to the mathematical model and experimental setup. Section 3 presents the uncertainty of measurement analysis. The statistical tool for experiment and model comparison is defined in Section 4. Section 5 reports the validation results using the comparison between the experimental and experimental results. The conclusion of study is presented in section 6.

2. System Description

Developed Mathematical Model and Input Data

Figure 1 shows the schematic of the proposed SBAC system relevant to the developed model. The first part (in the left) is a solar water heating (SWH) system, which consists of a field of flat plate solar collectors, a hot water storage tank and a circulating pump. The second part (in the middle) is a biomass gasifier-boiler (BGB) which consists of an automatic up-draft gasifier and a gas-fired sensible-heat boiler. The part on the right hand side is an absorption chiller (ABC) which consists of an absorption chiller equipped with a fan coil unit, a cooling tower and three aqua-pumps: hot, chilled and cooling water pumps. Where the details of assumptions used in the model development was presented in [5].

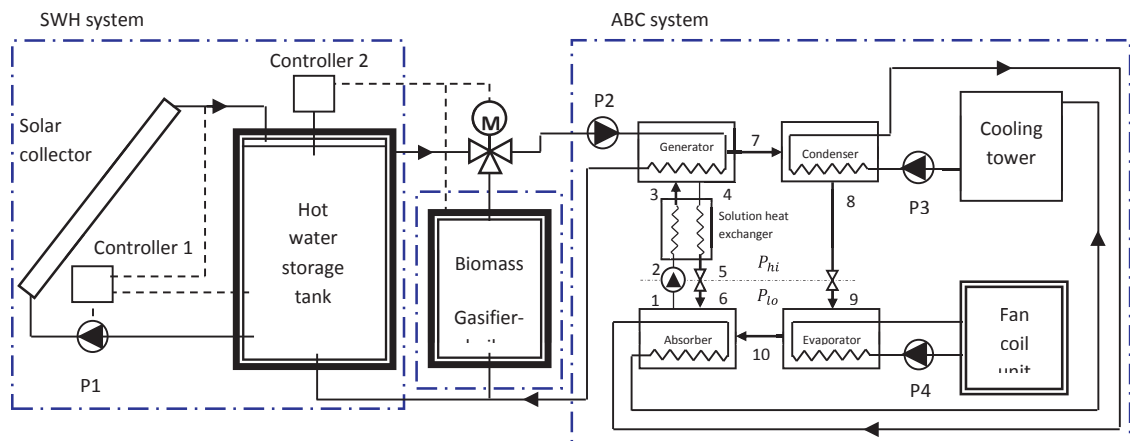


Fig. 1. Schematic diagram of the solar-biomass hybrid absorption cooling system

The model was developed using the following assumptions:

- 1) The model considered the energy and mass balances at each component, and of the overall system.
- 2) The system is considered to be at steady state
- 3) The specific heat and density of the working fluids are constant.
- 4) The loss of the water vapor and moisture (at the hot water storage tank and solar collector vents) is not taken into account.

- 5) There is no pressure loss and no heat loss/gain in the lines (pipes) connecting the system components.
- 6) The fluid temperatures increasing due to the friction in plumbing and valves, blowers and pumps are negligible.
- 7) The energy considered are solar and biomass energy, while the power consumed by other equipment (e.g. pumps, blower, fans and controllers) is excluded.

Solar Water Heating System

The expression (Eq. (1)) for collector efficiency given by the Hottel-Whillier Bliss equation was used:

$$\dot{Q}_u = \dot{m}c_p(T_{c,o} - T_{c,i}) = \eta_c A_c G_T \quad (1)$$

where,

$$\eta_c = F_R(\tau\alpha) - F_R U_L (T_{c,i} - T_a)/G_T \quad (2)$$

$$T_{c,i} = \beta T_{st} + (1 - \beta)T_a \quad (3)$$

$$T_{c,o} = \beta(T_{c,i} + \dot{Q}_u/(\dot{m}_c C_p)) \quad (4)$$

The temperature distribution in the hot water storage tank is obtained from the energy balance expressed as:

$$(MC_p)_{st} (dT_{st}/dt) = \gamma \dot{Q}_u - \dot{Q}_{tl} - (UA)_{st} (T_{st} - T_a) \quad (5)$$

where, the extracted heat \dot{Q}_{tl} and control functions used for the collector and load energy terms, are defined as:

$$\dot{Q}_{tl} = \beta \dot{m}_{ge} C_p (T_{st} - T_{ge,o}) \quad (6)$$

$$\beta = \begin{cases} 1 & \text{if } T_{st} > T_{set}, \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

$$\gamma = \begin{cases} 1 & \text{if } T_{c,o} > T_{st}, \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

Whenever the temperature of hot water supplied to the chiller machine is lower than SPT, the auxiliary heat (\dot{Q}_{aux}) is needed and this required heat will be supplied by the biomass gasifier-boiler. This required heat can be determined as follows:

$$\dot{Q}_{aux} = (1 - \beta) \dot{m}_{ge} C_p (T_{set} - T_{ge,o}) \quad (9)$$

Biomass Gasifier-Boiler

The gas-fired boiler is modeled as a heat exchanger, where heat is transferred between combustion products and water. The transient temperature of water inside the boiler can be determined by:

$$(MC_p)_b (dT_b/dt) = \dot{Q}_{gW} - \dot{Q}_{bl} - (UA)_{we} C_{p,a} (T_b - T_a) \quad (10)$$

where, the added heat into the water heater boiler \dot{Q}_{gW} and its effectiveness relations for the heat exchangers between combustion gases to water can be calculated from:

$$\dot{Q}_{gW} = \varphi \varepsilon_{gW} (\dot{Q}_{PG} - \dot{m}_{flu} C_{p,flu} T_{flu}) \quad (11)$$

$$\text{where, } \varphi = \begin{cases} 1 & \text{if } \dot{Q}_{aux} > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

$$\varepsilon_{gW} = [1 - \exp(-NTU(1 + R))]/(1 + R) \quad (13)$$

where,

$$R = (\dot{m}C)_{min} / (\dot{m}C)_{max} \quad (14)$$

$$NTU = (UA)_{gW} / (\dot{m}C)_{min} \quad (15)$$

Heat losses from flue gas and from boiler surface to ambient surrounding have been considered in the overall energy supplied by the gasifier, and is given by

$$\dot{Q}_{PG} = \varphi [(\dot{Q}_{aux} / \varepsilon_{gW}) + \dot{m}_{flu} C_{p,flu} T_{flu} + (UA)_{we} C_{p,a} (T_b - T_a)] \quad (16)$$

where, the extracted heat \dot{Q}_{bl} from the boiler to meet the load can be calculated from:

$$\dot{Q}_{bl} = \varphi \dot{m}_{g\epsilon} C_p (T_b - T_{g\epsilon,o}) \quad (17)$$

The consumption rate of biomass feed stock is obtained from:

$$\begin{aligned} \dot{m}_{BM} &= (LHV_{PG} \dot{V}_{PG}) / (\eta_G LHV_{BM}) \\ &= \dot{Q}_{PG} / (\eta_G LHV_{BM}) \end{aligned} \quad (18)$$

Absorption Chiller

The thermodynamic model of absorption chiller was used to simulate its performance with the assumptions presented in [5]. The effectiveness of generator can be calculated as:

$$\varepsilon = 1 - \exp[-(UA) / (\dot{m}_w C_p)] \quad (19)$$

At the generator, knowing the hot water inlet temperature and generator heat flow, the generator temperature can be determined by the following equations:

$$T_{g\epsilon} = T_{g\epsilon,i} - \dot{Q}_{g\epsilon} / (\varepsilon_{g\epsilon} \dot{m}_{w,g\epsilon} C_p) \quad (20)$$

$$T_{g\epsilon,o} = T_{g\epsilon,i} - \dot{Q}_{g\epsilon} / (\dot{m}_{w,g\epsilon} C_p) \quad (21)$$

where,

$$T_{g\epsilon,i} = \beta T_{st} + \varphi T_b \quad (22)$$

$$\dot{Q}_{g\epsilon n} = \dot{Q}_{tl} + \dot{Q}_{bl} \quad (23)$$

For a defined solution mass fraction (range of $45 < X < 70\% \text{LiBr}$) and calculated generator temperature, the saturation pressure, P_4 and h_4 can be calculated from.

$$\text{Log } P = C + D / (T_{ref} + 273) + E / (T_{ref} + 273)^2 \quad (24)$$

$$T_{ref} = \left(-\frac{2E}{D} + [D^2 - 4E(C - \log P)]^{0.5} \right) - 273 \quad (25)$$

$$T_{sol} = \Sigma B + T_{ref} \Sigma A \quad (26)$$

$$h = \Sigma A + T_{sol} \Sigma B + \Sigma C T_{sol}^2 \quad (27)$$

where, all of above cofactors can be calculated as shown in [5].

At point 7, the refrigerant is in superheated state and its enthalpy, h_7 can be determined from

$$h_{sh} = ((H_{SH2} - H_{SH1})/100)T + H_{SH1} \quad (28)$$

where,

$$T = T_{g\epsilon} - T_{ref} \quad (29)$$

$$H_{SH1} = 32.508 \ln P + 2513.2 \quad (30)$$

$$H_{SH2} = 0.00001 P^2 - 0.1193 P + 2689 \quad (31)$$

The mass flow rate of dilute solution in the generator can be determined using the energy and mass balances on the generator, as:

$$\dot{m}_3 = \dot{m}_4 + \dot{m}_7 \quad \text{and} \quad x_3 \dot{m}_3 = x_4 \dot{m}_4 \quad (32)$$

The energy balance on the generator is given by,

$$\dot{Q}_{g\epsilon} = \dot{m}_4 h_4 + \dot{m}_7 h_7 - \dot{m}_3 h_3 \quad (33)$$

$P_8 = P_4$, T_8 can be calculated from Eq. (25) and h_9 can then be obtained by $h_f = h_g - h_{fg}$, where h_g and h_{fg} are given by the curve fit equations. The enthalpy at point 9 can be calculated by considering the throttling process, as $h_9 = h_8$.

$$h_g = -0.00125397T^2 + 1.88060937T + 2500.559 \quad (34)$$

$$h_{fg} = -0.00132635 T^2 - 2.29983657T + 2500.43063 \quad (35)$$

The heat rejected at condenser, where $\dot{m}_7 = \dot{m}_8$, can be determined by writing the heat balance at condenser as:

$$\dot{Q}_{co} = \dot{m}_7(h_7 - h_8) \quad (36)$$

At the evaporator, P_{10} and h_{10} can be calculated from the curve fit of Eqs. (37) and (34).

$$P = 2 \times 10^{-12} T^6 - 3 \times 10^{-9} T^5 + 2 \times 10^{-7} T^4 + 3 \times 10^{-5} T^3 + 0.0014 T^2 + 0.04447 T + 0.6108 \quad (37)$$

From the energy balance at the evaporator, the cooling capacity \dot{Q}_{ev} at the evaporator can be calculated from:

$$\dot{Q}_{ev} = \dot{m}_9(h_{10} - h_9) \quad (38)$$

The enthalpy at point 1 can be calculated by Eq. (27) with absorber temperature (calculated by Eqs. (25) and (26)) at the same pressure as the evaporator.

The solution pump is modeled as an isenthalpic process, then $h_2 = h_1$. At the heat exchanger, the enthalpy h_5 can then be calculated as:

$$\dot{m}_2 h_2 + \dot{m}_4 h_4 = \dot{m}_3 h_3 + \dot{m}_5 h_5 \quad (39)$$

At the throttling process, $h_6 = h_5$. The enthalpy h_2 can be determined at the solution heat exchanger, and the absorber heat rejection can be calculated as:

$$\dot{Q}_{ab} = \dot{m}_{10} h_{10} + \dot{m}_6 h_6 - \dot{m}_1 h_1 \quad (40)$$

The minimum pump power can be determined by

$$w = \dot{m}_1 v_1 (P_2 - P_1) \quad (41)$$

$$\rho_x = 1145.36 + 470.84 X_0 + 1374.79 X_0^2 - (0.333393 + 0.571749 X_0)(273 + T) \quad (42)$$

where,

$$X_0 = X_1 / 100 \quad (43)$$

All four heat quantities must be satisfy the chiller energy balance equation, expressed as

$$\dot{Q}_{ge} + \dot{Q}_{ev} - \dot{Q}_{co} - \dot{Q}_{ab} = 0 \quad (44)$$

Experimental Setup

Figure 2 shows the experimental setup of SBAC system relevant to the model used in [6]. The collector field set for the experimental study had 26 collectors with total area about 49 m² (The specification of solar collector is given in [5]) connected in series and parallel (as shown in Figure 3).



Fig. 2. Fabricated experimental setup of the solar-biomass hybrid absorption cooling system [6]

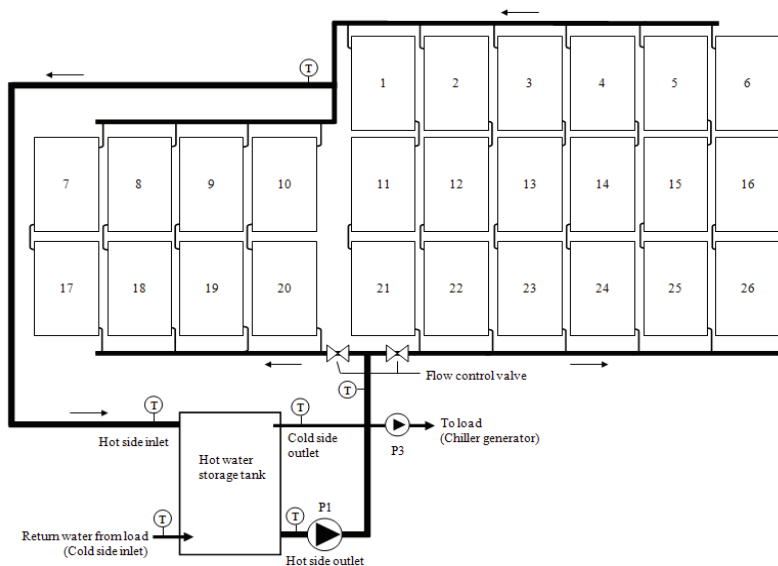


Fig. 3. Schematic diagram of the collector field set for the experimental study [6]

3. Uncertainty of Measurement

The errors or uncertainties of the measurement are determined and used for assessing the reliability of the experiments. Generally, it has two terms: relative uncertainty (REU) and absolute uncertainty (ABU). For the direct measurement values, the accuracy of devices used for the measurement can be adopted as the uncertainty of measurement. For the quantities which cannot be measured directly, their uncertainties have to be calculated based on the measured parameters [7].

Uncertainties of heat rate

The uncertainties of all heat rates, e.g.: heat removal rate, depend on the uncertainties of the water flow rate, and the water temperatures at the inlet and outlet of each component. The water flow rates and temperatures are measured using flow cells and thermocouples with the same accuracy of $\pm 0.1 \text{ m}^3/\text{hr}$ (or 0.028 kg/s) and $\pm 0.8 \text{ }^\circ\text{C}$, respectively. The relative uncertainties of heat rate at all components should be about the same level which can be calculated from Eq. (45):

$$\frac{\delta Q}{Q} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta T_o + \delta T_i}{(T_o - T_i)}\right)^2} \quad (45)$$

Uncertainties of COP

The COP can be calculated as [2,3]. Therefore, the REU of this output variable is then be calculated as Eq. (46):

$$\frac{\delta(\text{COP})}{\text{COP}} = \sqrt{\left(\frac{\delta Q_{ev}}{Q_{ev}}\right)^2 + \left(\frac{\delta Q_{gs}}{Q_{gs}}\right)^2} = 9.6\% \quad (46)$$

Uncertainties of COP_{sys}

The REU of the COP_{sys} can be calculated from Eq. (47):

$$\frac{\delta(\text{COP}_{sys})}{\text{COP}_{sys}} = \sqrt{\left(\frac{\delta Q_{ev}}{Q_{ev}}\right)^2 + \left(\frac{\delta G_T}{G_T}\right)^2 + \left(\frac{\delta A_c}{A_c}\right)^2 + \left(\frac{\delta m_{BM}}{m_{BM}}\right)^2} \quad (47)$$

4. Statistical Parameters

The statistical indicators are used for the comparison between the values obtained from model and experiments results. Two indicators: root mean square error (RMSE) and mean bias difference (MBD) are used in the validation. If the values of MBD and RMSE are close or lower than its uncertainties, it shown that the model and experimental results are in good agreement with each other. RMSE is defined as Eq. (48):

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^N (G_i - M_i)^2}{N}} \quad (48)$$

MBD is defined as Eq. (49):

$$MBD = \frac{1}{N} \sum_{i=1}^N (C_i - M_i) \quad (49)$$

where C_i corresponds to the model simulation values,

M_i corresponds to the measurements values

N is the number of data values.

5. Validation Results

All parameters of the experimental system were used as input parameters of the model as shown in Table 1. The measured solar insolation and ambient temperature of this day were also used as weather data input. The between experimental and simulation results were compared. The comparison results of each component are compared as demonstrated in Figure 4 to 7.

The temperature profile of water at collector inlet and outlet, and the useful energy rate at the SWH from both the experimental observations and model predictions are illustrated in Figure 4. The comparison results show that the RMSE of collector inlet/outlet temperatures and useful energy were 0.6, 0.73 and 1.76, respectively. With the same trend, the MBD of these values were -0.18, 0.4 and 1.14, respectively. This indicates the predicted values by the model are in agreement with the experimental results.

Table 1. The input parameters used in the validation

| Component | Inputs | Value | Unit |
|---|---|-------|--------------------|
| Weather data | | | |
| | Global rad. on tilted surf., G_T | | W/m ² |
| | Ambient temperature, T_a | | °C |
| Solar Water Heating System (SWH) | | | |
| Collector | Area, A_c | 50 | m ² |
| | Intercept efficiency, $F_R (\tau\alpha)$ | 0.789 | - |
| | Loss efficiency coefficient, $F_R U_L$ | 5.829 | W/m ² K |
| | Specific heat of water, C_p | 4.19 | kJ/kg.K |
| | Collector Flow rate, \dot{m}_c | 1200 | kg/hr |
| | Time step, dt | 300 | sec |
| Tank | Volume, V_{st} | 0.4 | m ³ |
| | Mass of fluid in tank, M_{st} | 400 | kg |
| | Tank heat loss, $(UA)_{st}$ | 4.068 | W/m ² K |
| | Hot water set temperature, T_{set} | 84 | °C |
| Biomass Gasifier-Boiler (BGB) | | | |
| | Maximum heating power | 29 | kW |
| | Conversion efficiency, η_c | 75 | % |
| Absorption Chiller (ABC) | | | |
| | Absorption chiller size, \dot{Q}_{ev} | 7 | kW |
| | Generator water flow rate, \dot{m}_{ge} | 1500 | kg/hr |

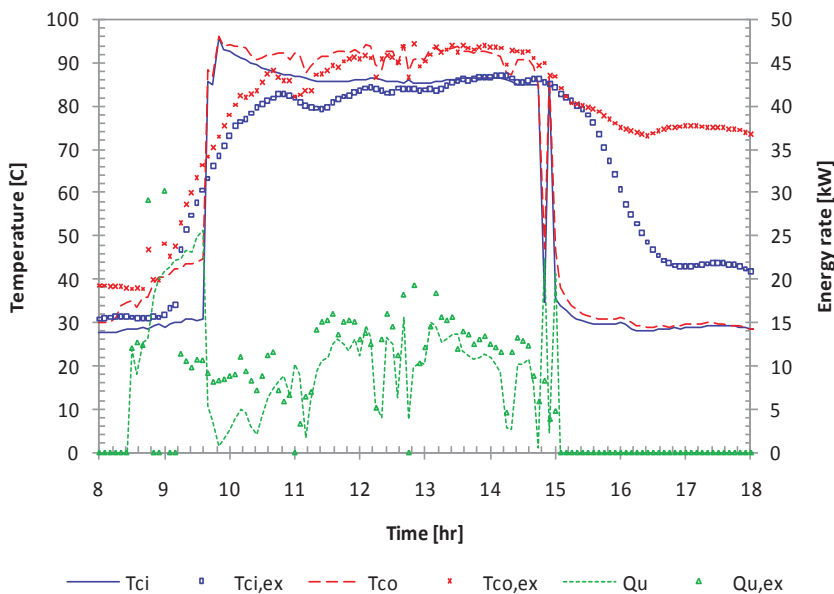


Fig. 4. Measured and predicted parameters of SWH system

Figure 5 shows the measured and predicted boiler temperature and its energy rate supplied to the chiller generator. The 17-data of each parameter during the steady state condition were compared. The comparison results show that the RMSE of boiler temperature and energy rate supplied from boiler to load were 0.71 and 1.12, respectively. The MBD of these values was 0.59 and 0.81, respectively. This indicates the close prediction of developed model with the experimental results provides reliable results.

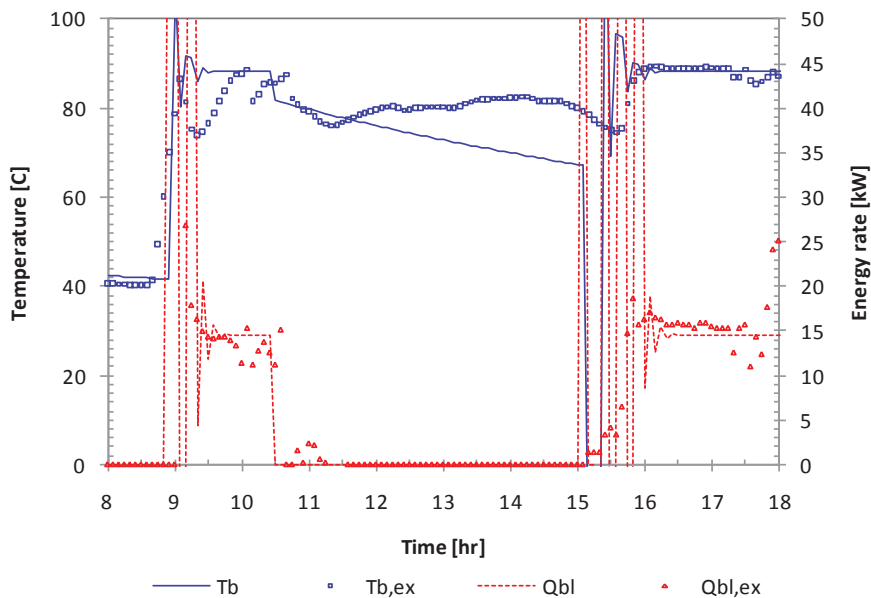


Fig. 5. Measured and predicted parameters of BGB system

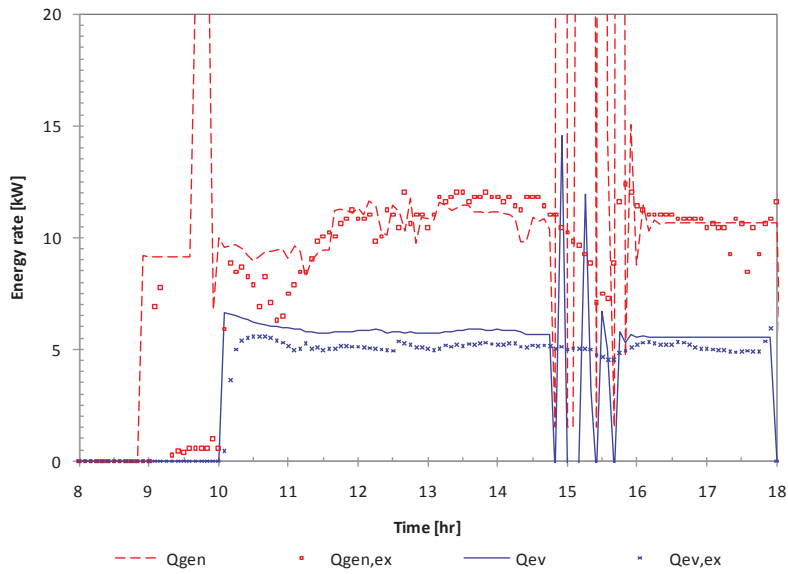


Fig. 6. Measured and predicted parameters of ABC system

The heat power supplied to its generator and absorbed at evaporator during the steady state condition were compared and shown in Figure 6. The comparison results show that the RMSE of generator and evaporator heat rates were 0.3 and 0.41, respectively. The MBD of these values was 0.57 and -0.39, respectively. This also indicates that the predicted values by the model are in good agreement with the experimental results

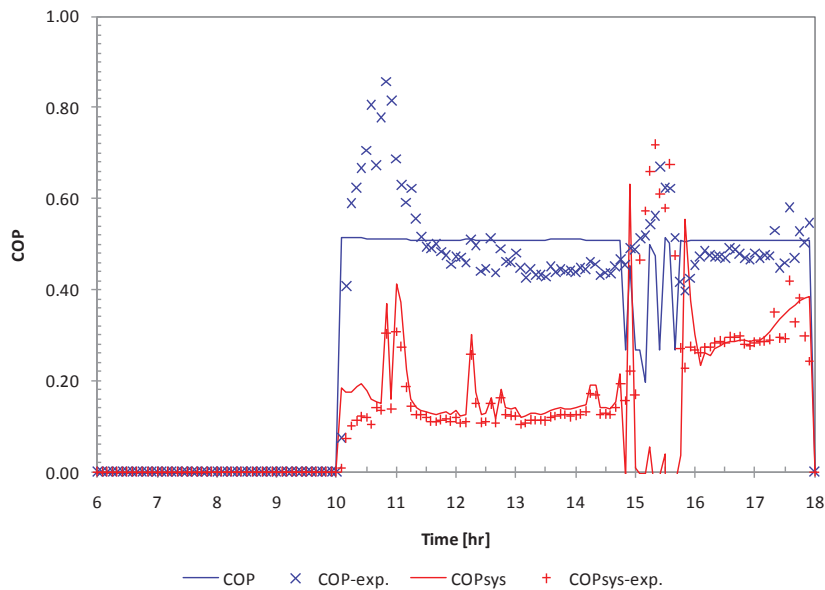


Fig. 7. Measured and predicted COP and COP_{sys}

Finally, the measured and predicted values of COP and COP_{sys} were compared as presented in Figure 7. The comparison result shows that the RMSE of COP and COP_{sys} were 0.05 and 0.08, respectively. The MBD of these values was -0.04 and -0.02, respectively. This indicates the close prediction of developed model with the experimental results.

6. Conclusion

To know the reliability of developed model, the results of observed experimental results were compared with the predicted results from the simulation with the same input parameters. The calculated statistical parameters, RMSE and MBD, of all temperature are very close to the uncertainty of the temperature measurements. The average value of RMSE for temperature, energy rate and COP are 0.96, 0.9 and 0.07, respectively. The average value of MBD for these parameters are 0.25, 0.53 and -0.03, respectively. The validation results show that the simulation results predicted by the developed model are in good agreement with the experimental results from both qualitative and quantitative points of view.

Acknowledgment

The author wishes to thank Rajamangala University of Technology Thunyaburi (RMUTT) for providing financial support for his research.

References

- [1] Saidura, R.; Abdelaziza, E.A.; Demirbasb, A.; Hossaina, M.S.; and Mekhilef, S. 2011. A review on biomass as a fuel for boilers, *Renewable and Sustainable Energy Reviews* 15: 2262–2289.
- [2] Ramana, A.S.; Chidambaram, L.; Kamaraj, G.; and Velraj, R. 2012. Evaluation of renewable energy options for cooling applications, *International Journal of Energy Sector Management* 6: 65-74.
- [3] Zhaia, X.Q.; Qub, M.; Li, Y.; and Wang, R.Z. 2011. A review for research and new design options of solar absorption cooling systems, *Renewable and Sustainable Energy Reviews* 15: 4416– 23.
- [4] Hassan, H.Z.; and Mohamad, A.A. 2012. A review on solar cold production through absorption technology, *Renewable and Sustainable Energy Reviews* 16: 5331–48.
- [5] Prasartkaew, B.; and Kumar, S. 2010. A low carbon cooling system using renewable energy resources and technologies, *Energy and Buildings* 42: 1453–1462.
- [6] Prasartkaew, B., and Kumar, S. 2011. The Quasi-steady State Performance of a Solar-Biomass Hybrid Cooling System, In *Proceedings of the Second TSME International Conference on Mechanical Engineering (The 2nd TSME-ICoME 2011)*, Krabi, Thailand, 19-21 October.
- [7] Taylor, J. R. 1997. *An Introduction to Error Analysis* (2nd Ed.), Sausalito: University Science Books.