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Digitization and profitability

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Abstract
Consider a monopolistic vendor who faces a known demand curve. By setting a price that equates marginal revenue with marginal cost, the vendor will maximize his profit. This logic holds true of both physical and digital goods. But since digitization will lower the variable production cost, it will strictly increase the profit to the vendor. Then, can we conclude that digitization always improves the vendor's profitability? Not necessarily. Now consider what will happen on the *next* day of the sales. Facing deterministic demand, the physical goods vendor must have prepared the exact quantity of the product to sell, and thus all products are sold out. By contrast, the digital goods vendor will have no stock out, thanks to the nature of the *digital* good. Therefore, the rational vendor will try to sell more and achieve a higher profit after the sales date. To this end, the vendor will now lower the price to attract additional customers with lower reservation prices. The process will indefinitely continue. Knowing this would happen, customers will wait for the price reduction. Even the customers who would have purchased on the first day would defer the purchase until price gets lower. The digital goods vendor will anticipate this and accordingly lower the price on the first day and later, thereby compromising his profitability. Note that this downward spiral takes place as a result of digitization. Thus, digitization may not necessarily improve the profitability to the vendor. We develop an economic model to formally analyze the impact of digitization on the profitability to the vendor.

Keywords Strategic customers · Digital goods pricing · Exponentiality

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1 Introduction

Thanks to the Internet, many information goods—such as books, music and movies—are transformed to pure digital goods and distributed over the Internet. Digitization offers numerous benefits on the supply side—zero variable production cost, no stockouts, instant deliveries, zero inventory carrying cost, zero shipping cost, and no damages or losses in handling or delivery. It, therefore, achieves an ideal form of make-to-order production-distribution system with unlimited supply, just-in-time delivery and zero variable cost. Most of these benefits directly or indirectly accrue to the vendor. But does digitization guarantee a higher profit? How about social welfare? This paper investigates the questions by using a simple game-theoretic model.

To start with, consider a monopolistic vendor ("he") who faces a known demand curve. By setting a price that equates marginal revenue with marginal cost, the vendor will maximize his profit. This logic holds true of both physical and digital goods. But since digitization will lower the variable production cost, it will strictly increase the profit to the vendor. Then, can we conclude that digitization always improves the vendor's profitability? Not necessarily. Now consider what will happen on the *next* day of the sales. Since the demand is known, the physical goods vendor must have prepared the exact quantity of the product to sell, and thus all products are sold out. By contrast, the digital goods vendor will have still an infinite supply of the good, thanks to the nature of the *digital* good. Therefore, the rational vendor will try to sell more and achieve a higher profit after the sales date. To this end, the vendor will now lower the price to attract additional customers with lower reservation prices. The process will indefinitely continue. Knowing this would happen, customers will wait for the price reduction. Even the customers ("she" each) who would have purchased on the first day would reconsider buying it now, and instead choose to wait. The digital goods vendor will anticipate this and accordingly lower the price on the first day and later, thereby compromising his profitability. Note that this downward spiral takes place as a result of digitization. Thus, digitization may not necessarily improve the profitability to the vendor. This casual argument requires a more formal investigation, which is the objective of the present paper.

We here offer a brief literature survey. There exist three fields of research related to the present work. First, special characteristics of digital goods generated research with different focuses. A recent work by Goldfarb and Tucker (2019) provide an extensive literature survey on digital economics by starting with the question of "what is different" about digital economics. They emphasize the changes in costs associated with search, replication, transportation, tracking and verification. For example, Bakos and Brynjolfsson (1999) study the effectiveness of bundling a large number of information goods, in the face of diverse tastes and uncertainties. Huang and Sundararajan (2011) study optimal nonlinear pricing for digital goods, when the infrastructure has discontinuous cost. Traditional auction mechanisms designed for physical products need modification when applied to digital goods. In this area Goldberg et al. (2001) analyze the auction mechanisms designed for allocating digital goods. The existing literature do not address the implications of digitization on the multi-period interactions between vendors and customers, highlight the "infinite supply" aspect of digital goods. Our work contributes to the literature by comparing profitability of physical and digital goods from such an angle.

Second, there exists a well-developed literature on sequential bargaining under informational asymmetry, and some are closely related to our paper. Sobel and Takahashi (1983) and Cramton (1984) develop a sequential bargaining model between a buyer and a seller over infinite time. The authors characterize the limiting sequential equilibrium in closed-form expressions. The key aspect of these models is the strategic behavior of customers. Instead of passive or myopic customers who either take it or leave it, strategic and fully rational customers choose the timing of purchase in anticipation of a favorable price. In this regard, Besanko and Winston (1990) compare the performance between strategic and myopic buyers. Hörner and Samuelson (2011) extend the model to N potential buyers in a finite deadline. Board and Skrzypacz (2016) also study revenue management with strategic customers, when new customers continuously arrive to the market. Our model is closest to McAfee (2007) who studies sequential bargaining without information asymmetry. Assuming strategic customers and no information asymmetry, we obtain closed-form trajectories of prices, thresholds and profits. This result is used to compare the profits of the physical and digital goods.

Last, the body of work on the Coase conjecture (Coase 1972) is related to our work. In 1972 Coase conjectured that lack of commitment power and sequential rationality would drive down a durable goods monopolist's profit to zero under certain conditions. The conjecture was formalized by various authors including Stokey (1981), Bulow (1982), Gul et al. (1986), Orbach (2004), McAfee and Te Velde (2006), and McAfee and Wiseman (2008). We show that this conjecture also serves as a distinguishing feature of digital goods, since the conjecture does not apply to physicl goods with a finite supply.

The rest of the paper is organized as follows. In the next section we introduce the model. We analyze the one-period model of physical products in Sect. 3. In Sect. 4 we study the multi-period sales of digital goods and compare the two models. In particular, we encounter the Coase conjecture that arises as a limiting phenomenon for digital goods. In Sect. 5, we investigate the welfare implications of digitization. The last section offers concluding remarks including the strategy of avoiding the Coase conjecture.

2 The model

The main objective of the paper is to analyze the impact of digitization on the profitability to the monopolist vendor. We develop a simple mathematical model to capture the essence of the issue at hand. We consider a market where a monopolistic vendor attempts to sell goods to customers. The goods may be either in the physical or digital form. Below is the detailed description of the model.

- (Demand) We assume that the market consists of a continuum of customers with different valuations. Each customer needs at most one unit of the good. A customer with valuation v is called the 'type-v' customer. The (inverse) demand function is given by $p = \bar{v} - q$ for some positive $\bar{v}$, where p is the price and q the quantity sold. Equivalently, the valuations of the customers are uniformly distributed in $[0, \bar{v}]$ at unit density. Thus, the initial market size is $\bar{v}$, but will shrink over time as more customers buy.
- (Value Depreciation) A customer's valuation depreciates in time at a fixed rate of $\delta(\in (0,1))$ per period. δ is here called the *depreciation factor*. The good valued at v at time 1 becomes worth $\delta^{t-1}v$ in period t. Note that customer 'types' do not depreciate over time, since types are defined at the initial period. A *large* depreciation factor close to 1 means *slow* depreciation. Cash is not discounted, for simplicity.
- (Production Cost) The variable production cost of the physical good is $c(> 0)$ per unit, while that of the digital good is set to zero. We assume that the fixed cost is identical in both goods.
- (Selling Periods) Physical and digital goods operate in different manners. The former has only one selling period, while the latter has an infinite number of selling periods denoted by $t = 1, 2, \ldots$.
- (Strategic Customers) Customers are strategic in the choice of time to buy. Each customer buys in the period that maximizes her (positive) *net value* of the good—the value earned beyond the cost. She would not buy any if the net value is negative.
- (Vendor's Pricing) The vendor of the digital goods would set the price each period to maximize the *residual (gross) profit*—the total profit in the present and future periods before the fixed cost is subtracted. The case of the digital goods is thus modeled as a multi-period game played by two sets of players, who take turns in making a move. We study the decisions of the players and their resulting outcome as the equilibrium of the game and derive managerial insights.

3 Physical and digital goods

3.1 Physical goods

Consider a physical goods vendor who has one period to sell his product. Facing the linear demand we assumed, the vendor will prepare the quantity $Q^o = (\bar{v} - c)/2$ at unit cost of c and set the price at $p^o = (\bar{v} + c)/2$ to sell the entire stock. This will generate the total profit of $\Pi^p = (p^o - c)Q^o = (\bar{v} - c)^2/4$. Consumer surplus (= the sum of consumers' net values) will be $CS^p = (\bar{v} - p^o)Q^o/2 = (\bar{v} - c)^2/8$. Social welfare S^p—the sum of the vendor profit and consumer surplus—will equal $3(\bar{v} - c)^2/8$. Note that both the vendor profit and welfare strictly decrease in c.

3.2 Digital goods

By its nature, the digital good is never sold out at the end of each selling period. Knowing this in advance, customers wait to see the price coming down. In response, the vendor adjusts the prices now and for the future. This multi-period game is modeled in the subgame-perfect equilibrium concept. Formally speaking, the equilibrium is the sequence of strategy pairs $\{(p_t, v_t)\}_{t=1,2,\ldots}$, so that taking the history of the game so far as given, in each period t,

1. each type-v customer buys a unit if and only if $v \geq v_t$ for some v_t, meaning that customers in the aggregate choose the 'threshold' value v_t; and
2. the vendor selects the price p_t to maximize his residual profit $\pi_t := \sum_{i=t}^{\infty} p_i(v_{i-1} - v_i)$ in expectation of the customers' strategies.

The strategy in (1) represents the *threshold policy* in which only high-types of customers buy the good now while the rest wait. For now, we assume it is an optimal policy, and later (in Theorem 3) verify this is indeed the case. The vendor's strategy in (2) is to "cream-skim" the customers by steadily lowering the price and selling to the next *interval* of customers each period, where the interval is $[v_t, v_{t-1})$ in period $t = 1, 2, \ldots$, with $v_0 := \bar{v}$. Thus, in equilibrium the thresholds will form a non-increasing sequence $(v_1, v_2, \ldots)$. Now we seek to find the explicit values of $\{p_t\}$, $\{v_t\}$ and $\{\pi_t\}$.

To find the subgame-perfect equilibrium of the game, we work backwards in each period t. That is, given arbitrary values of $(v_1, v_2, \ldots, v_{t-1})$ of the previous-period values, the vendor chooses v_t (for $t = 1, 2, \ldots$) by solving

$$\max_{v_t : v_t \leq v_{t-1}} p_t(v_{t-1} - v_t) + \pi_{t+1}(v_t) \tag{1}$$

s.t.

$$\delta^{t-1} v_t - p_t = \delta^t v_t - p_{t+1}, \tag{2}$$

$$\delta^{t-1} v_t - p_t \geq 0. \tag{3}$$

The objective of (1) tries to maximize the residual profit by selecting the right price p_t that indirectly controls v_t (via the constraints). In order to properly assess the impact of his current decision, he has to look ahead and build a prediction on the future equilibrium values (p_{t+i}, v_{t+i}) for $i = 1, 2, \ldots$. The prediction should be correct in equilibrium. This correct prediction requirement applies to the customers as well.

Under the threshold policy of customers, the incentive compatibility condition states that the threshold customer will find the two options—buy or wait—indifferent. Equation (2) states that for the threshold customer, the option of buying now is as good as the option of buying in the next period. Here, we assume that in every period, some customers buy a good so that the incentive compatibility condition is effective. We later show that indeed there exists an equilibrium satisfying this assumption.

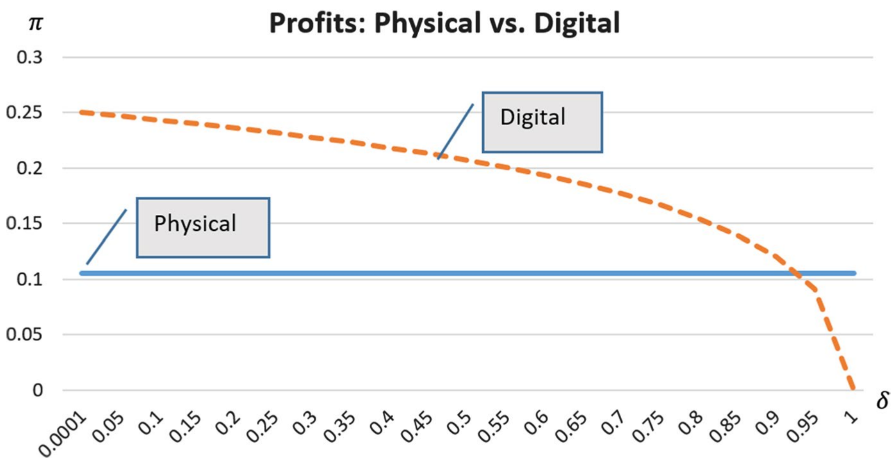

Fig. 1 The profit from the digital good is higher than the physical good when $\delta < 0.93$, where $\bar{v} = 1$ and $T = \infty$

Equation (3) is the individual rationality constraints ensuring that each type of customer gets at least zero net value; otherwise, she would not buy the good. See, for example, Mas-Colell et al. (1995) for detailed discussions of mechanism design.

The equilibrium of the game is captured by the three variables—the price p_t, the threshold v_t and the residual profit π_t to the vendor. Below we present the equilibrium in two steps—the first-period equilibrium (Theorem 1) and the trajectories of the variables over time (Theorem 3). The equilibrium is given in a surprisingly simple form, even though its derivation is rather complicated.

Theorem 1 *In equilibrium, (P1) of the digital system has the following solution for* $t = 1$:

(i) $v_1^* = \frac{1-\sqrt{1-\delta}}{\delta}\bar{v}$
(ii) $p_1^* = \frac{\sqrt{1-\delta}-1+\delta}{\delta}\bar{v}$
(iii) $\pi_1^* = \frac{\sqrt{1-\delta}-1+\delta}{2\delta}\bar{v}^2$

All proofs are delegated to the "Appendix".

The Theorem completely determines the first-period threshold v_1^* , the opening price p_1^* and total profit π_1^* as a function of the primitive model parameters $\bar{v}$ and δ for the digital system.[1] In particular, note that the total profit π_1^* decreases in the depreciation factor. We are now in a position to compare the profits of the physical and digital systems as follows (see also Fig. 1).

[1] See McAfee (2007) who derives a similar form of total profit in a slightly different setting as a fixed point.

Theorem 2

(i) *The profit* Π^p *of the physical product is given by* $(\bar{v} - c)^2/4$.
(ii) *The profit* $\Pi^d (= \pi_1^*(1))$ *of the digital good is given by* $\frac{\sqrt{1-\delta}-1+\delta}{2\delta}\bar{v}^2$.
(iii) *The digital system is more profitable than the physical system if and only if* $\delta \leq \frac{4c\bar{v}^2(2\bar{v}-c)}{(c^2-\bar{v}^2-2\bar{v}c)^2}$.

Profitability of digitization thus depends on the depreciation factor δ and the variable cost c. At a higher δ, customers do not feel pressured to buy early and instead choose to wait, since most of the original value is retained over time. To counter it, the vendor is forced to lower the price fast to induce them to buy early. Also, if the physical product has a high variable cost, digitization would save much for the digital goods vendor. In summary, digitization will favor the vendor when the physical product has high production cost and the product depreciates fast. Note here that to the digital goods vendor, the infinite supply or the *lack of scarcity* is not a blessing, but a liability.

As a byproduct of the proof of Theorem 1, we can derive the *trajectories* of equilibrium strategies and profits over time. It turns out that the ratio v_t^*/v_{t-1}^* for the digital system is *constant*, independent of time t, so the thresholds take the *exponential* form in the digital goods model. The price p_t^* and the residual profit π_t^* also follow the exponential form. Formally, we have:

Theorem 3 *Let* $R := \frac{1-\sqrt{1-\delta}}{\delta}$ and $K := \frac{\sqrt{1-\delta}-1+\delta}{2\delta}$. *Then, for every* t,

(i) $v_t^* = \bar{v}R^t$
(ii) $p_t^* = 2K\bar{v}(\delta R)^{t-1}$
(iii) $\pi_t^* = K\bar{v}^2(\delta R^2)^{t-1}$

Part (i) confirms that in every period there exist customers who buy the good, so that Eq. (2) is justified as the incentive compatibility condition. This result also shows that the individual rationality condition (Eq. 3) is satisfied in each period t, since $\delta^{t-1}v_t - p_t = \bar{v}\delta^{t-1}R^t - 2K\bar{v}(\delta R)^{t-1} = \bar{v}(\delta R)^{t-1} \cdot \frac{2-\delta-2\sqrt{1-\delta}}{\delta} > 0$ for all t. Thus, the net value to each customer exponentially shrinks to zero as t approaches infinity. See Fig. 1.

3.3 The coase conjecture

We have shown that the digital goods vendor will be *worse off* when the good depreciates slowly. Even worse, Theorem 3 (ii) and Fig. 1 show a prominent feature of the digital good—both the opening price and total profit continuously decline to *zero* as δ approaches 1. That is, $\lim_{\delta\to1} p_1^* = 0$ and $\lim_{\delta\to1} \pi_1^* = 0$. In the limit, the digital goods vendor is trapped in the zero-revenue paradox Coase conjectured in his 1972 paper. Coase (1972) considered a monopolistic vendor who owns an ample stock of a durable good. Further it is assumed that the vendor sells the stock to patient customers with different valuations over an infinite number of periods and cannot commit to the price in advance. In this setting Coase conjectured that a monopolist would make zero profit. In our setting the downward spiral between customers' deferred purchases and the vendor's price reduction drives the result in the limit. A large literature on the Coase conjecture identified and studied the conditions under which the conjecture holds true. The Coase conditions include: (1) the product is durable (not consumable) (Coase 1972; Bulow 1982; Orbach 2004; 2) the cost of transacting each deal is zero (Coase 1972; 3) the price can be instantly changed (Coase 1972; 4) the depreciation factor is close to 1 (Gul et al. 1986); and (5) the vendor has no power to commit to the future prices (McAfee 2007). One implicit condition only lightly covered in the literature is that the vendor has an infinite (or sufficiently large) supply. Coase (1972) seems to have used "all the land of the United States" as a proxy for an infinite supply, but did not fully utilize the property in his analysis. Our model highlights the fact that the Coase conjecture may apply to the digital good market with its infinite supply, but not to the physical product market where there is a potential shortage due to its finite supply.

What is the intuition behind the Coase conjecture in the digitization context? One way of interpreting it is as follows. A digital good is infinitely available and incurs zero variable cost. For such a product, one might anticipate zero price. Indeed, the price will "ultimately" decline to zero. However, in the early transient periods, the digital good will have strictly positive prices. In this sense, the seller is selling the "freshness" of the product, charging the buyer for early access. In the absence of depreciation (with depreciation factor close to 1), however, freshness will be maintained for a long time, so that the value of early access is close to zero. The vendor has nothing to sell.

The relationship between digitization and profitability may apply to various situations. The value of the newspaper depreciates very quickly. Thus, the newspaper publisher will not fall into the trap of the Coase condition. As another example, consider the textbooks. Since students need a textbook as soon as the course starts, the value of the book decreases quickly. While the publisher can control the number of physical copies, he cannot do the same with its digital version, so he cannot utilize the scarcity effect. However, depreciation will be fast at the beginning of the new semester, so it will avoid the Coase conjecture or any dramatic loss of profit as a side effect of digitization.

Lastly, consider the case of movies. The value of movies depreciates over time—but slower than newspapers or textbooks. A new film is first shown to the audience in a movie theater, after which it is released to the video-on-demand (VOD) service, sold in a DVD and finally, broadcast free of charge. It is reported that the number of moviegoers is decreasing. When asked why they were going to the movies less often, 24 percent said they wait for VOD (Cunningham 2015). Nevertheless, movie studios do not skip any other stage of offering movies, partly due to market competition and availability of new technologies, and partly because customers anticipate so. Given the belief of the customers, the movie studio provides the product in multiple stages and stepwise lowers the price. As a whole, the movie market can be depicted as a game between the studio and customers where the studio selects the price and customers select the time of purchase. Digitization plays a critical part of the game, and digitization may not necessarily have increased the profitability for the studios.

In sum, the Coase conjecture suggests a clear and present risk to digitization. The digital goods vendor should make sure to avoid the Coase conditions. We discuss it further in the final section.

4 Social welfare

We turn to the welfare implications of digitization. For the physical product case, social welfare is $S^p = 3(\bar{v} - c)^2/8$, as discussed in Sect. 3.

Now consider the welfare in the digital system. Social welfare S^d can be expressed as the total value generated by the purchases. In period t the group of customers belonging to $[v_t, v_{t-1})$ will purchase the product. The size of this group is $v_{t-1} - v_t$ with the average gross value $\delta^{t-1}(v_{t-1} + v_t)/2$ per customer.

The total welfare across all groups is thus given by

$$S^d = \sum_{t=1}^{\infty} \delta^{t-1}(v_{t-1} - v_t)(v_{t-1} + v_t)/2 = \sum_{t=1}^{\infty} \delta^{t-1}(v_{t-1}^2 - v_t^2)/2,$$

where v_t is given in Theorem 4.3. Then, it can be shown that

$$S^d := \frac{1 + \delta - \sqrt{1 - \delta}}{4\delta}\bar{v}^2.$$

Thus, we have:

Theorem 4

(i) *Social welfare of the physical and digital systems S^p and S^d are respectively given by $S^p = 3(\bar{v} - c)^2/8$ and $S^d = \bar{v}^2(1 + \delta - \sqrt{1 - \delta})/(4\delta)$.*
(ii) *Digitization always improves social welfare.*

Why does digitization improve social welfare? Note that there are three factors that determine social welfare—cost of production, market penetration and the timing of sales. The digital system improves welfare through lower production costs. Also, digitization increases the number of customers served. The physical goods vendor deliberately limits the quantity and only sell to high types of customers to the detriment of social welfare, while the digital goods vendor will ultimately sell to all customers over the infinite horizon. However, it does not prove that the digital system is better in terms of social welfare, because welfare also depends on how fast sales happen before the product loses its value. In this respect the physical product performs better, since all sales take place in the first period. The Theorem, however, reports that the second effect strictly dominates the third, while the first separately re-enforces the benefit of the digitization.

From a broader perspective on the comparison, competition exists between the digital system's *efficiency* and the physical system's *scarcity* . To the profit-maximizing digital goods vendor, efficiency boosts his profit, but lack of scarcity hurts it. On the other hand, both efficiency and lack of scarcity contribute to social welfare. Therefore, digitization may improve or hurt the vendor profit, but it always improves social welfare.

5 Concluding remarks

We have investigated if digitization would lead to higher profitability. The answer depends on the variable cost of the physical product and the depreciation factor of the product. Digitization favors the vendor when the variable cost is high and depreciation is fast. In the extreme case of infinite time horizon and zero depreciation, the digital goods vendor will lower the price to zero "in the twinkle of an eye,"[2] and face zero profit.

The digital goods vendor should take actions to mitigate the liability of digitization driven by the infinite supply, while maintaining its advantage of low variable cost. In particular, he should by all means avoid the Coase conditions. We consider three approaches to it—a finite time horizon, a small depreciation factor and a limited number of copies available.

The first approach is to develop a mechanism to artificially end the sales in a finite time, say, T. A finite T, preferably a small T, will benefit the vendor. It may sound counter-intuitive, since a shorter time horizon may incur loss of sales on the tail end of the time horizon. But it changes the game dynamics as it pressures customers to buy before it is too late. In fact, the finite horizon game shows a completely different pattern of profitability from our infinite horizon case. The profit in the former changes in a U-shaped curve with respect to the depreciation factor δ (see Fig. 2 for $T = 2$), instead of a monotone-decreasing curve in the latter (see Fig. 1 for $T = \infty$). The largest depreciation factors offer as high profits as the lowest depreciation factors. The vendor can achieve the maximum profits at the highest depreciation factors when the time horizon is finite. Moreover, this case will prevent the rise of the Coase conjecture. An example of this approach is to build a reputation of "no markdown" policy. This is equivalent to choosing $T = 1$. Building such a reputation requires multiple rounds of long-term efforts, sometimes risking short-term losses. It will achieve the same profit as a physical product with zero variable cost.

The second approach is to choose a smaller depreciation factor. A small depreciation factor (i.e., smaller δ) has a positive effect on profitability similar to a short selling season (smaller T). For example, the condition $\delta = 0$ is equal to $T = 1$. In both, the downward spiral will stop before reaching the zero profit level. To execute this strategy, the vendor may shorten the effective product life cycle and introduce a new version of the product at regular intervals. He may continue to offer the current version of product (i.e., keep T at infinity) even after the new product is introduced, but

[2] See Coase (1972).

the value of the product will significantly erode, thereby achieving a small depreciation factor. This way, scheduled obsolescence would keep the depreciation factor away from 1 and block the Coase conditions.

Lastly, the vendor may create "limited editions" of the digital good. By limiting the supply and creating scarcity, the vendor would overcome the main disadvantage of digital goods such as digital arts and photography. One way is to utilize Blockchain technology [see, for example, Swan (2015)]. For example, CryptoKitties, Kodak (www.kodak.com/kodakone) and Ascribe (www.ascribe.io) offer a Blockchain-based service that enables a digital goods vendor to issue a limited number of copies. They assign a unique pair of public and private keys to each copy and track its movement on the Blockchain (Deforge 2018; Zheng 2018).

The model has several limitations. First, it is a simple stylistic model compromised for tractable analysis. We assumed a monopolistic vendor, linear demand, a simplistic market mechanism for physical products and a simple game structure with no uncertainties in the digital system. These simplifying assumptions may admittedly limit the power of the model's conclusions and applicability. The Coase conjecture, in particular, may be viewed as an artifact of theoretical exercise. However, managers may draw qualitative insights from such extreme results. Second, we have not allowed the physical product vendor to produce extra units to sell over multiple periods and apply dynamic pricing. But it will not generate a higher profit to the vendor. Figure 2 illustrates this point, when we repurpose it for a physical product with two periods of sales and $c = 0$. The profit is maximal when δ is 0, or equivalently, when $T = 1$. That is, it is not optimal for the physical product vendor to sell in multiple periods, even if the vendor is allowed to. Thus, we have not lost optimality with this limitation. Another limitation is, the model captures only two key differentiating features—low production cost and infinite supply—of digital goods. It left out of consideration other benefits of digitization such as instant gratification

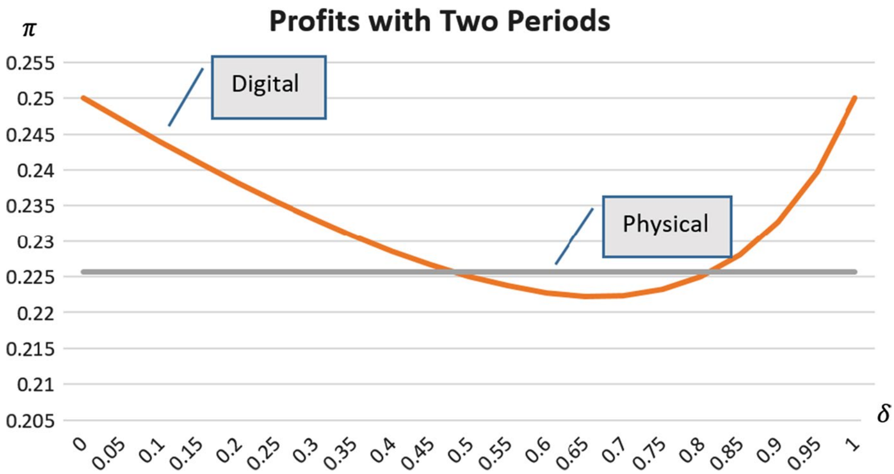

Fig. 2 The profit from the digital good is higher than the physical when δ is either smaller than .45 or larger than .8, where $\bar{v} = 1$ and $T = 2$

and ease of updating and upgrades. Further, digitization is often associated with network externality, which has a significant implication in pricing. However, network externality is a phenomenon associated with the demand side of the good, often in the context of multi-period market competition. This is well beyond the scope of our model and require a new setup for analysis. We hope to see more work to complement our deficiencies with more sophisticated models and empirical research.

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Appendix: Proofs

Proof (Theorem 1) To analyze the problem, we consider a slightly more general problem as follows.

- A customer is identified by type θ, where θ is uniformly distributed over [0, 1]. Note we are deviating from the main text where we use v as the customer type. These are equivalent, but the new index will simplify the presentation of the proof.
- The reservation value of type-θ customer is given by $\bar{v}\theta$. Thus, the highest reservation value among all customers is given by $\bar{v}$. Note that v and θ are related through $v = \theta\bar{v}$.
- The market size is given by $\bar{q}$.

Thus, the inverse demand function is given by $p = \bar{v}(1 - q/\bar{q})$. Denote by $\bar{\pi}(\bar{q}, \bar{v})$ the total profit the vender earns from period 1 onwards. Since $\bar{v}$ and $\bar{q}$ are mere scaling factors of the reservation value and the market size, the equilibrium price p_t^* in period t is independent of $\bar{q}$ and is linear in $\bar{v}$. Thus, for every $\alpha > 0$,

$$p_t^*(\alpha\bar{v}) = \alpha p_t^*(\bar{v}) \tag{4}$$

where $p_t^*(\bar{v})$ is the price in period t when the highest valuation in period 1 is $\bar{v}$. Also, it is clear that the profit $\bar{\pi}(\bar{q}, \bar{v})$ is linear in both $\bar{q}$ and $\bar{v}$, i.e.,

$$\bar{\pi}(\bar{q}, \bar{v}) = \bar{q}\bar{v}\bar{\pi}(1, 1). \tag{5}$$

Note that the problem in period 2 has the market size $\theta_1\bar{q}$ and the highest valuation $\delta\theta_1\bar{v}$. Thus, problem $\bar{\pi}(\bar{q}, \bar{v})$ is expressed as

$$\bar{\pi}(\bar{q}, \bar{v}) = \max_{\theta_1, p_1} p_1\bar{q}(1 - \theta_1) + \bar{\pi}(\theta_1\bar{q}, \delta\theta_1\bar{v}) \tag{6}$$

s.t.

$$\theta_1 \leq 1, \tag{7}$$

$$\theta_1 \bar{v} - p_1 \geq 0, \tag{8}$$

$$\theta_1 \bar{v} - p_1 = \delta\theta_1 \bar{v} - p_2^*. \tag{9}$$

where p_2^* is the price in period 2, which is the first period price of the problem $\bar{\pi}(\theta_1 \bar{q}, \delta\theta_1 \bar{v})$, so that [from (4)]

$$p_2^* = p_1^*(\delta\theta_1 \bar{v}) = \delta\theta_1 p_1^*(\bar{v}). \tag{10}$$

We now evaluate $\bar{\pi}(1,1)$. From the incentive compatibility condition (9) and (6) with $\bar{v} = 1$,

$$p_1 = \theta_1 - \delta\theta_1 + \delta\theta_1 p_1^*$$

where $p_1^* = p_1^*(1)$. For now, we ignore (7) and the individual rationality condition (8). Using (4) and (5) and setting $\bar{q} = \bar{v} = 1$ in (6), we have

$$\bar{\pi} = \max_{\theta_1}(\theta_1 - \delta\theta_1 + \delta\theta_1 p_1^*)(1-\theta_1) + \delta\theta_1^2 \bar{\pi},$$

where $\bar{\pi} = \bar{\pi}(1,1)$. Thus,

$$\begin{aligned} \theta_1^* &= \frac{1-\delta+\delta p_1^*}{2-2\delta(1+\bar{\pi}-p_1^*)} \\ \bar{\pi} &= p_1^*(1-\theta_1^*) + \delta\theta_1^{*2}\bar{\pi} \\ p_1^* &= \theta_1^* - \delta\theta_1^* + \delta\theta_1^* p_1^*. \end{aligned}$$

Solving these three equations w.r.t. θ_1^*, $\bar{\pi}$ and p_1^*, we obtain

$$\theta_1^* = (1-\sqrt{1-\delta})/\delta \tag{11}$$

$$\bar{\pi} = \left(\sqrt{1-\delta} - 1 + \delta\right)/(2\delta) \tag{12}$$

$$p_1^* = (\sqrt{1-\delta} - 1 + \delta)/\delta. \tag{13}$$

We see that these formulas indeed satisfy (7) and (8). Thus, Theorem 1 holds true.

□

Proof (Theorem 3) Consider $\bar{\pi}(\bar{q}, \bar{v})$. Clearly, its solution $\theta_1^*(\bar{q}, \bar{v})$ is independent of the scaling factors $\bar{q}$ and $\bar{v}$. Since the original problem in period t is given by $\bar{\pi}(v_{t-1}^*, \delta^{t-1} v_{t-1}^*)$, $\theta_t^*(1, \bar{v}) = \theta_1^*(v_{t-1}^*, \delta^{t-1} v_{t-1}^*)$ is independent of t. Thus, the ratio v_t^*/v_{t-1}^* is given by $\theta_1^* = \theta_1^*(1)$ in (11) for all t, proving Part (i). From (4), (13) and Part (i), $p_t^*(\bar{v}) = p_1^*(\delta^{t-1} v_{t-1}^*) = \delta^{t-1} v_{t-1}^* p_1^*(1) = 2K\bar{v}(\delta R)^{t-1}$, proving Part (ii). From

(5), (12) and Part (i), $\pi_t^*(\bar{v}) = \bar{\pi}(v_{t-1}^*, \delta^{t-1} v_{t-1}^*) = \delta^{t-1} v_{t-1}^{*2} \bar{\pi}(1,1) = K\bar{v}^2(\delta R^2)^{t-1}$, proving Part (iii). We also see that these formulas indeed satisfy the threshold policy and the individual rationality. □

Proof (Theorem 4 (ii)) Note first that S^d is monotone increasing in δ. Also, note that

$$\begin{aligned}
\lim_{\delta \to 0} S^d &= \lim_{\delta \to 0} \frac{\bar{v}^2(1 + \delta - \sqrt{1-\delta})}{4\delta} \\
&= \lim_{\delta \to 0} \frac{\bar{v}^2(1 + \delta - \sqrt{1-\delta})(1 + \delta + \sqrt{1-\delta})}{(4\delta)(1 + \delta + \sqrt{1-\delta})} \\
&= \lim_{\delta \to 0} \frac{\bar{v}^2(3 + \delta)}{4(1 + \delta + \sqrt{1-\delta})} \\
&= 3\bar{v}^2/8 \\
&\geq 3(\bar{v} - c)^2/8 \\
&= S^p,
\end{aligned}$$

which completes the proof. □

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