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Abstract: The etymology of the word "mathematics" can be traced to Greek and Latin roots with meanings such as "to think or have one's mind aroused" or "the art of knowing". The natural philosophers of the Renaissance did not draw an explicit distinction between mathematics, the sciences and to an extent the arts. In this paper we first explore connections forged by the thinkers of the Renaissance between mathematics, the arts and the sciences, with attention to the nature of the underlying questions that call for a particular mode of inquiry. Second, we will examine both the relationship and individual differences between innovative behaviors across domains. Recently Robert Root-Bernstein (2003) introduced the construct of polymathy to suggest that innovative individuals are equally likely to contribute both to the arts and the sciences and either consciously or unconsciously forge links between the two. Several contemporary examples are presented of individuals who pursued multiple fields of research and were able to combine the aesthetic with the scientific. Finally, we will also discuss the possibilities for re-introducing university courses on natural philosophy as a means to integrate mathematics, the arts and the sciences.

A HISTORIC OVERVIEW OF THE INTERPLAY OF THEOLOGY AND PHILOSOPHY IN THE ARTS, MATHEMATICS AND SCIENCES

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The etymology of the word "mathematics" can be traced to Greek and Latin roots with meanings such as "to think or have one's mind aroused" or "the art of knowing". The natural philosophers of the Renaissance did not draw an explicit distinction between mathematics, the sciences and to an extent the arts. In this paper we first explore connections forged by the thinkers of the Renaissance between mathematics, the arts and the sciences, with attention to the nature of the underlying questions that call for a particular mode of inquiry. Second, we will examine both the relationship and individual differences between innovative behaviors across domains. Recently Robert Root-Bernstein (2003) introduced the construct of polymathy to suggest that innovative individuals are equally likely to contribute both to the arts and the sciences and either consciously or unconsciously forge links between the two. Several contemporary examples are presented of individuals who pursued multiple fields of research and were able to combine the aesthetic with the scientific. Finally, we will also discuss the possibilities for re-introducing university courses on natural philosophy as a means to integrate mathematics, the arts and the sciences.

INTRODUCTION

The history and development of mathematics is intricately connected to the rise and fall of ancient and modern civilizations. The progression of humanity from hunter-gather societies onto societies with sophisticated astronomical calendars, visually pleasing architectural forms (temples, mosques, cathedrals etc) reveals our quest to understand the cosmos, our attempts to represent and symbolize it via patterns, symmetries and structure. A common characteristic of many civilizations (both ancient and modern) is the quest to answer three basic philosophical questions:

(1) What is reality? Or what is the nature of the world around us?

This is linked to the general ontological question of distinguishing objects (real versus imagined, concrete versus abstract, existent versus non-existent, independent versus dependent and so forth).

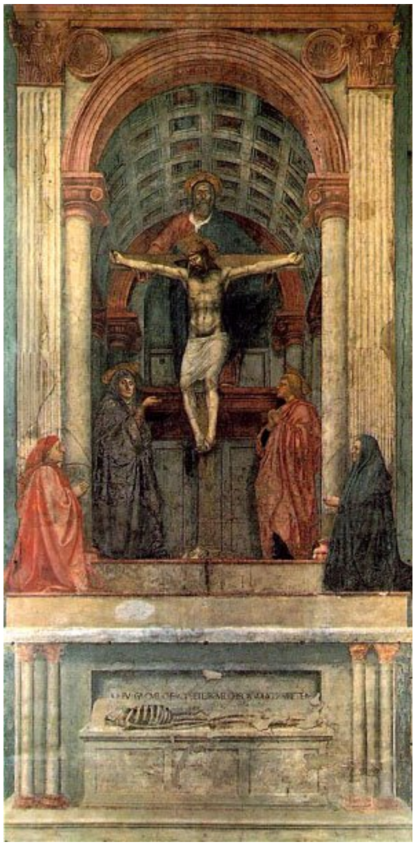
(2) How do we go about knowing the world around us? [the methodological question, which presents possibilities to various disciplines to develop methodological paradigms] and,

(3) How can we be certain in the "truth" of what we know? [the epistemological question].

The interplay of mathematics, arts and sciences is found in the attempts to answer these fundamental philosophical questions. In this sense Philosophy can be viewed as the foundational bridge unifying mathematics, arts and the sciences. In this paper we will first focus on the attempts of the thinkers of the Renaissance, who did not view themselves simply as theologians or mathematicians or inventors or painters or philosophers or political theorists, but who thought of themselves as philosophers in the pursuit of *Knowledge, Truth and Beauty*. Then we will try to pinpoint the thinking characteristics of these natural philosophers, particularly the trait of *polymathy* (Root-Bernstein, 2003) which explains innovative behavior across numerous subject specific domains. Finally we will examine the implications of these findings for present day high school and university education.

REVISITING THE RENAISSANCE

Theology influencing Art and Science (resulting in powerful mathematics as a consequence)



The great works of Art during the Renaissance, particularly those of Italian painters (Massacio, Brunelleschi , Leonardo da Vinci , Michelangelo , Titian, Giotto, Raphael etc)immediately reveal the interplay of church doctrine with art in that these painters inspired devotion in people by painting divine Christian icons. In this process of trying to convey these divine images as realistically and beautifully as possible, these painters moved away from the medieval style of painting (very 2-dimensional) and essentially created the rules of perspectivity that allowed 3-D images to be projected onto a flat surface. This suggests that art was instrumental in initiating the mathematical foundations of the true rules of perspective. Geometrical optics also played a major role in how artists experimentally arrived at the mathematical rules of perspectivity (e.g., *Fig1. Massacio's Holy Trinity*[1] *on previous page*).

Calter (1998) traces a rich lineage of the interaction of optics with art: Euclid's Optica (300 B.C) - Vitruvius' *Ten Books on Architecture*- Ptolemy's *Optica,* (c. 140 A.D) - (the Islamic) Alhazen's *Perspectiva*, (c. 1000 A.D) - Roger Bacon's *Opus Majus,* (c. 1260 A.D.), with sections on optics, "whose geometric laws, he maintained, reflected God's manner of spreading his grace throughout the universe – onto John Pecham's *Perspectiva communis,* (c. 1270 A.D). The amalgamation of mathematical ideas proposed in this lineage was formalized by Desargues (1593-1662) which is today studied in courses on *projective geometry.* Today the visual-artistic side of mathematics is completely lost under the rubble of formalization. However the visual side of mathematics has seen a revival in the 20^{th} century in the area of fractal geometry due to the work of Benoit Mandelbrot. This is explored further at a later junction in the paper.

The scientists-mathematicians-theologians of the Renaissance-Post Renaissance

The relationship between science and theology can be traced back to the pre-Socratic Greeks. Pre-Socratic Greek society evolved from the typical "Sky-God" explanation of creation/reality onto a society that developed a rigorous and systematic philosophy to answer the three aforementioned questions.

The Pythagorean School (c. 500 B.C) developed a mystical numeric system to designate and describe everything in the universe. They even went so far as to claim that "All things of the universe have a numerical attribute that uniquely describes them". For instance numbers were designated abstract attributes. Such as **one**: the number of reason, **two**: the first even or

[1] Retrieved from http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Art.html

female number (the number of opinion), **three**: the first true male number (the number of harmony), **four**: the number of justice or retribution, **five**: marriage, **six**: creation and so on…

> Each number had its own personality - masculine or feminine, perfect or incomplete, beautiful or ugly. This feeling modern mathematics has deliberately eliminated, but we still find overtones of it in fiction and poetry. Ten was the very best number: it contained in itself the first four integers - one, two, three, and four [1 + 2 + 3 + 4 = 10] - and these written in dot notation formed a perfect triangle. (Brumbaugh, 1981)

Further the motion of planets and musical notes were expressed as ratios of numbers. The Pythagorean School developed an elaborate numerical system consisting of even and odd numbers to describe the world around them. However Plato (429-327 B.C.E), and Aristotle (384-322 B.C.E) deviated from the mysticism of the Pythagoreans and instead attempted to understand the universe via reason. Plato suggested that the universe consisted of two realms, the visible realm which was deceptive because of its changing nature and an abstract realm which he believed was eternal and unchanging. Within this "dualistic" ontology of reality, Plato answered the epistemological question by suggesting that knowledge derived empirically from the changing world was fallible whereas knowledge derived from the abstract realm was infallible or absolute. Plato accorded a special place for mathematics in this pursuit of absolute knowledge by claiming that mathematics was derivable independent of the physical senses. Thus the purest form of "thought" was mathematical thought as it was deemed capable of deriving "eternal truths" or absolute knowledge. In spite of the alleged motto of the academy, Plato distinguished "numbers as ideas" from "numbers as mathematical objects". Unfortunately over time this important distinction faded and Platonism approached Pythagoreanism, which in turn influenced Renaissance philosophers, then modern natural science and thereby again modern philosophy.

Aristotle on the other hand was an empiricist whose prodigious work left a lasting impression until the 13th century (namely the dawn of the Renaissance). The Aristotelian approach to science was empirical and placed a heavy emphasis on perception through the senses. Aristotle rejected the Platonic notion of the mind's capacity to intuit /discern *a priori* reality and instead proposed an *a posteriori* or empirical methodology whereby knowledge is acquired by the mind. Aristotelian science was axiomatic and deductive in nature with the aim of explaining natural phenomenon. The underlying assumption of Aristotelian science was that all natural objects were fulfilling a potential determined by an actual prior natural object. For instance, a seed becomes a plant because it is merely fulfilling its potential of becoming a plant. Science historians today agree that Aristotle was an empiricist, who believed that knowledge is gained via observation, experimentation and experience. The question of whether or not Aristotelian science was the origin of dualism is still a matter of present day debate[2]. Recent scholarship on post Renaissance science and natural philosophy traces a rich intellectual lineage centered on "scholasticized Aristotelianism" from 17th century natural philosophy onto medieval thinkers like Aquinas onto Aristotle[3] (Sriraman & Benesch, 2005, p. 42).

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[2] One could argue that Aristotle drew a distinction in the natural world between the animate and inanimate, whereas Descartes was more focused on the human. Descartes' dualism dominated physics for a substantial period of time. Newtonian physics was one of the consequences of the Cartesian view point. Robinson, H. M. (1983) Aristotelian Dualism. In J. Annas (Ed.), *Oxford studies in ancient philosophy* (Vol. 1, pp. 123-144). Oxford: Clarendon Press.

[3] *The Cambridge History of Seventeenth-Century Philosophy*. Edited by Daniel Garber and Michael Ayers. Vols. 1 and 2,Cambridge: Cambridge. University Press, 1998. p.1616

Thomas Aquinas (1225-1274) synthesized “all that had been argued in Western thought up to his time and he showed it to be compatible with Christian beliefs” (Sharp, 2003, p.346). His argument is superbly summarized by Sharp (2003) as follows:

…Aquinas argued that all our rational knowledge of this world is acquired through sensory experience, on which our minds then reflect. When children are born, their minds are like a clean slate (*tabula rasa*). Aquinas developed a theory of knowledge which is uncompromisingly empirical. The world through which we gain our knowledge is God’s creation, and therefore it is impossible for this gained knowledge to conflict with religious revelation. (p.346)

The Greek philosophers also stumbled upon the idea of the “infinite”, a sophisticated mathematical abstraction, as evidenced in Zeno’s paradoxes[4]. Bertrand Russell observed that scholastic theology was one of the outcomes of mathematical abstraction (Russell, 1945, p. 37). His claim is supported by numerous historical examples, a handful of which are presented here. Sa’id ibn Yusuf (Saadia Gaon), a 10th century theologian and leader of the Babylonian Jews in his theological treatise *Kitab al-Amanat wa-al Itiqadat* (the Book of Beliefs and Opinions) wrote:

> How is it possible to establish this concept (of the Creator) in our minds when none of our senses have ever perceived Him?…It is done in the same way in which our minds recognize the impossibility of things being existent and nonexistent at the same time, although such a situation has never been observed by the senses. (Saadia Gaon,1948)

Saadia Gaon also cleverly reversed Zeno’s paradox of Achilles and the tortoise[5] to prove that Creation occurred.

> If the world were uncreated, <u>then time would be infinite</u>. But infinite time cannot be traversed. Hence, the present moment couldn’t have come about. But since the present moment exists…the world had a beginning. (Saadia Gaon,1948)

The ideas of the Greeks also had a profound influence on post-Renaissance mathematicians like Descartes (1596-1650), Pascal(1623-1662)and Leibniz (1646-1716) among others. One routinely comes across the use of mathematical analogies to prove the existence of God in the theological works of Descartes, Leibniz, and Pascal. For example, Descartes in the *Fifth Meditation* states:

> Certainly the idea of God, or a supremely perfect being is one which I find within me, just as surely the idea of any shape or number. And my understanding that it belongs to his nature that he always exists is no less clear and distinct than is the case when I prove of any shape or number that some property belongs to its nature. Hence, even if it turned out that not everything on which I have meditated in these past days is true, I ought still to regard the existence of God as having at least the same level of certainty as I have hitherto attributed to the truths in mathematics. (Descartes 1996, p.45)

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[4] Zeno (born around 495 B.C.E) was a Greek philosopher and logician, and a student of the philosopher Parmenides. Zeno is remembered for paradoxes that stumped mathematicians for centuries. Zeno’s paradoxes evolved from Parmenides’ ideas about the illusory nature of motion, change and time.

[5] Achilles and the tortoise: The running Achilles can never catch a crawling tortoise ahead of him because he must first reach where the tortoise started. However, when he reaches there, the tortoise has moved ahead, and Achilles must now run to the new position, which by the time he reaches the tortoise has moved ahead, etc. Hence the tortoise will always be ahead.

Leibniz in *Theodicy* argued that faith and reason were compatible.

> Theologians of all parties, I believe (fanatics alone excepted), agree that no article of faith must imply contradiction or contravene proofs as exact as those of mathematics, where the opposite of the conclusion can be reached ad absurdum, that is, to contradiction. It follows thence that certain writers have been too ready to grant that the Holy Trinity is contrary to that great principle, which states that two things, which are the same as the third, are also the same as each other. For this principle is a direct consequence of that of contradiction, and forms the basis of all logic; and if it ceases, we can no longer reason with certainty. (Leibniz 1985, 87)

Leibniz's dissertation on the conformity of faith and reason can be interpreted to mean that it is logically contingent and intelligible for a human being to ask why an eternal being exists (Craig 1995).

Blaise Pascal, perhaps the most intriguing mathematical mystic argued for the use of "infinitesimal[6] reasoning" (or reasoning in infinitely small quantities) by proclaiming that the infinitely large and the infinitely small were mysteries of nature that man stumbled on by divine inspiration. Pascal is also remembered for his famous wager, where he argued that if God's existence has a probability of 0.5 (50/50 chance), then it is only rational for us to believe he does exist. Pascal in *Pensees* states:

> If there is a God, He is infinitely incomprehensible, since, having, neither parts nor limits, He has no affinity to us. We are then incapable of knowing either what He is or if He is ... you must wager. It is not optional. You are embarked. Which will you choose then? Let us weigh the gain and the loss in wagering that God is. Let us estimate these two chances. If you gain, you gain all; if you lose, you lose nothing. Wager then without hesitation that he is (Pascal, in Popkin 1989)

The aforementioned historical examples, viz., the number mysticism of the Pythagoreans, the paradoxes of Zeno that brought forth the abstraction of the infinite, the attempts of medieval theologians like Saadia Gaon to systematize theology by constructing uniqueness proofs to theological theorems, and the use of mathematical arguments to prove the existence of a Creator by post-Renaissance mathematicians, illustrate that there has been a rich interplay between mathematics and theology (Sriraman, 2004a).

While these aforementioned thinkers of the Renaissance and post Renaissance who strongly believed in the existence of a creator invoked mathematical arguments to prove their beliefs in their philosophical writings, others such Copernicus and Galileo, who were professed believers of the Catholic Church, found it increasingly difficult to believe in the prescribed view of the world (earth) as the center of the universe. Their model building during this time period reveals the interplay as well as the conflict between theology and science with mathematics replacing Aristotilean logic as the language of description.

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[6] An infinitesimal is a number that is infinitely small but greater than zero. Infinitesimal arguments have historically been viewed as self-contradictory by mathematicians in the area of analysis. The infinitesimal Calculus of Newton and Leibniz was reformulated by Karl Weierstrass in the 19th century for the sole purpose of eliminating the use of infinitesimals. In the 20th century Abraham Robinson revived the notion of infinitesimals and founded the subject of Non-standard analysis to resolve the contradictions posed by infinitesimals within Calculus. Robinson attempted to use logical concepts and methods to provide a suitable framework for differential and integral Calculus.

Modeling the Universe: Copernicus-Galileo-Kepler

The Ptolemaic model (c. 87-150 A.D) of astronomy was based on the assumption that the earth was the center of the universe which was accepted by the Catholic Church as being compatible with its teachings. However this geocentric view of the world could not explain the curious planetary phenomenon observed by Nicholaus Copernicus (1473-1543). That is the retrograde motion (moving backwards and then forwards) of Mars, Jupiter and Saturn, in addition to nearly invariant times that Venus and Mercury appeared in the sky which is shortly before sunrise and after sunset.

Fig.2 Claudius Ptolemy[7]

However these queer motions were perfectly reasonable if one viewed the sun as the center of the "universe" as opposed to the earth. In such a model the peculiarities of the inner planets (Mercury and Venus) as well as the outer planets (Mars, Jupiter and Saturn) in relation to the earth make perfect sense. The retrograde motion of outer planets is due to the fact that they get overtaken by the earth in its orbitary motion. Similarly Venus and Mercury appear static and only before sunrise and after sunset because their orbitary motions do not allow them to get behind the earth and manifest in the night sky. It is amazing what a little change in perspective does to our perceptions! However the conflicts of Copernicus' findings with Church dictum prevented a wider dissemination of his simpler planetary model until his death. Centuries later, the great German philosopher, writer, scientist Goethe (1749-1832) reflected[8] on Copernicus' new perspective of our reality:

> "Of all discoveries and opinions, none may have exerted a greater effect on the human spirit than the doctrine of Copernicus. The world had scarcely become known as round and complete in itself when it was asked to waive the tremendous privilege of being the center of the universe. Never, perhaps, was a greater demand made on mankind - for by this admission so many things vanished in mist and smoke! What became of our Eden, our world of innocence, piety and poetry; the testimony of the senses; the conviction of a poetic - religious faith? No wonder his contemporaries did not wish to let all this go and offered every possible resistance to a doctrine which in its converts authorized and demanded a freedom of view and greatness of thought so far unknown, indeed not even dreamed of."

Galileo Galilei (1564-1642) pushed things further by using mathematics to explain interplanetary motion. In fact many science historians claim that Galileo was the first person to systematically use mathematics as the language of science instead of Aristotilean logic. Aristotle's conceptions of motion had several flaws which were rectified by Galileo by determining that velocity and acceleration were distinct. More importantly the question that vexed Copernicus of why the motion of the Earth was unfelt (if in fact it was moving) was answered by Galileo by suggesting that (1) only acceleration is felt whereas velocity is unfelt

[7] Retrieved from http://obs.nineplanets.org/psc/img/ptolemybig.gif
[8] Retrieved from http://www.blupete.com/Literature/Biographies/Science/Copernicus.htm

and invariant except when acted on by an external force (the notion of inertia). Thus, Galileo suggested that the Earth in addition to orbiting around the sun was also rotating on its own axis. Needless to say, his attempt to make his model public met with fierce resistance from the Church and led to his condemnation by the Inquisition.

During this same time period the German astronomer Johannes Kepler (1571- 1630) confirmed and supported many of Galileo's well formulated theories. Johannes Kepler was born in Weil der Stadt, Württemburg, (and not very far from here). While studying for Lutheran ministry at the University of Tübingen, he became familiar with the Copernican model, which he defended explicitly in the *Mysterium Cosmographicum*. The political forces of that time period with his unique personal circumstances, namely his strong adherence to the *Augsburg Confession* but rejection of several key Lutheran tenets, the use of the calendar introduced by Pope Gregory XIII, his rejection of the *Formula of Concord* and finally his snub to Catholicism led to him to exile in Prague where he worked for the Danish astronomer Tycho Brahe. With the help of Brahe's data, Kepler made several seminal discoveries published in *Astronomia Nova*. The beauty of this work lies in the fact that Kepler arrived at the first two laws of planetary motion by working with incomplete/imperfect data (we must remember that this data was obtained before the invention of the telescope!). The first two laws were (1) Planets move in ellipses with the Sun at one focus, and (2) The radius vector describes equal areas in equal times. Finally the third law was published *Harmonices Mundi* in 1619. The third law states that: the squares of the periodic times are to each other as the cubes of the mean distances. Incidentally Newton's theory of gravitation grew out of Kepler's third law (and not a fallen apple as suggested by myth).

Szpiro (2003) recently suggested that among the forces driving Kepler's work during his turbulent Tübingen years was to seek a theological explanation to his questions:

> Since God had created a perfect world, he thought it should be possible to discover and understand the geometric principles that govern the universe. After much deliberation Kepler believed he had found God's principles in the regular solids….His explanations of the universe were based on an imaginary system of cubes, spheres and other solids that he thought were fitted between the sun and other planets…published in Mysterium Cosmographicum. This tome did not unveil any mysteries of the planetary system…since no solids exist that are suspended in the universe. But the book came to the attention of Tycho Brahe. (Szpiro, 2003, p.13)

And the rest is history…

Isaac Newton's (1642-1727) prodigious work included a mathematical model of the planetary system, in a sense suggesting that the universe was governed by certain laws, expressible via mathematics and discernible by humans (Sharp, 2003). This led to the development of natural philosophy as an answer to the ontological, methodological and epistemological questions with mathematics becoming the medium of establishing truth. One consequence of Aristotle's empiricist tradition was the acceptance of the notion that knowledge of the external world was derived by an active soul[9], which was in essence separate from that world. Salmon (1990) comments[10]:

> " It is illuminating to recognize that Cartesian dualism offered a way of resolving the conflict between science and religion-which had brought such great troubles to Galileo-by providing each with its own separate domain. Physical science could deal with matter, while religion could handle whatever pertains to the soul. " (p.236).

This led to a growing acceptance among 17th century natural philosophers in the notion of duality (or dualism). René Descartes (1596-1650) is considered the founder of this belief system since he initiated the mind-body problem. Cartesian dualism essentially proclaims that we are composed of two distinct and basic substances, namely the mind (soul) and matter. Matter was the material substance that extended into the world and took up space, whereas the mind (soul) was a thinking substance, which was not "localizable" in space. "If these two aspects (mind-matter) are to be held in equal balance, it seems that it will have to be in some way more subtle than mere juxtaposition[11]". The problem of dualism can be reformulated as follows: "One can think of subject and object as two unique and separate natures, neither of which is reducible to each other. The question of course in such a dualistic assumption is 'how do these two natures relate to each other?'

THE MODERN DAY RENAISSANCE

Scientists in the 20th and 21st centuries have developed the technical tools and the analytic and theoretical maturity necessary for analyzing nature at unprecedented micro and macrocosmic levels. By doing so, they have reaffirmed the dynamic nature of the whole that was reflected in the paradoxes of the ancients. The result is a view of nature in which processes have supplanted 'things' in descriptions and explanations.

By the end of the 19th Century, limitations of the classical Newtonian/Euclidean world-view had become increasingly problematic as physicists began exploring nature at the sub-atomic level. The paradoxes posed by uncertainty, incompleteness, non-locality, and wavicles, etc. let it seem apparent that in the sub-atomic world observations and observers are aspects of a whole. The physicist John Wheeler comments:

> "...In the quantum principle we're instructed that the actual act of making an observation changes what it is that one looks at. To me, this is a perfectly marvelous feature of nature…. So the old word observer simply has to be crossed off the books, and we must put in the new word participator. In this way we've come to realize that the universe is a participatory universe." [12]

Biologists have found that methodological reductionism, i.e. going to the parts to understand the whole, which was central to the classical physical sciences, is less applicable when dealing with living systems. Such an approach may lead to a study not of the 'living' but of

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[9] Polkinghorne comments that Aristotle's view of the soul as the underlying "form" or pattern of the body was "taken up by Thomas Aquinas who rejected Platonic dualism that had dominated Western Christian thinking since Augustine." Sir John Polkinghorne, *Science & Theology*, Fortess Press, 1998, p.63.

[10] Salmon, W.C.(1990). Philosophy and the Rise of Modern Science" Teaching Philosophy, p.236.

[11] Sir Polkinghorne, *Ibid*, p. 54.

[12] Comment made by the physicist John Wheeler in Paul Buckley and F. David Peat: *Conversations in physics and Biology*, University of Toronto Press, 1979, p. 53-4

the ‘dead’, because in the examination of highly complex living systems “only by ripping apart the network at some point can we analyze life. We are therefore limited to the study of ‘dead’ things.”[13]

One of the most important shifts in the natural sciences in the modern period has been away from the view of a simple and complete separation between observer and observed to an awareness that an observer also represents a living aspect of that which is being observed--both as a product of nature and as the mental possibility in nature of observing, as in the notion of the ‘participator universe[14]’. A synthesis of ‘product’ and ‘process’ are at the heart of the puzzles and paradoxes that we associate with ideas of ‘indeterminacy’ in physics, and with genes in biology. The very concept of ‘objectivity’ maintains that the observed and observer are separate does not hold in the study of “highly complex biological processes such as evolution or the functioning of the central nervous system ... we cannot distance ourselves from the object being considered; indeed, this is so at the very moment we start to think.”[15]
It is amazing how close in understanding, and that across six centuries, modern physics and biology are to the Neo-Confucian Philosopher, Wang Yang-Ming’s continuum view of ‘innate knowledge’:

> The innate knowledge of man is the same as that of plants and trees, tiles and stones…Heaven, Earth, the myriad things, and man form one body. The point at which this unity is manifested in its most refined and excellent form is the clear intelligence of the human mind.[16]

The very process of generalizing implies a belief in the unity of the world: “if the different parts of the universe were not like the members of one body, they would not act on one another…know nothing of one another, and we…would know only one of these parts. We do not ask if nature is one, but how it is one.”[17] The position on mind and nature of theoretical physicists seems consistent with that of the neo-Confucian philosopher. Another physicist suggest that the heliocentric universe is again becoming geo or human centered in that it is “*... formless potentia ... and becomes manifest only when observed by conscious beings*...Of course, we are not the geographical center, but that is not the issue. *We are the center of the universe because we are its meaning.”*[18] This is yet another amazing shift in perspective.

Benoit Mandelbrot (1924-), a very recognizable name in 20th century mathematics because of his seminal contributions to the development of fractal geometry has repeatedly emphasized the need to re-orient our perspectives to better understand the world around us. He has often very humbly characterized himself as an “accidental” mathematician. In spite of his early interest and precocity in the study of geometry he was “encouraged’ by the French university establishment to embrace formalization which led him to leave the *École Normale Supériere*. He writes

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[13] Cramer, *Ibid*, 214
[14] John Wheeler’s participatory universe as quoted in footnote 12.
[15] Friedrich Cramer: *Chaos and Order,* VCH Publishers, New York, 1993, p. 212
[16] Wing-tsit Chan, *Wang Yang-Ming* p.221
[17] Henri Poincaré, *The Foundations of Science,* George Bruce Halsted trans., The Science Press, Lancaster, PA., 1946,
[18] Amit Goswami, *The Self-Aware Universe*, G. P. Putnam Sons, N.Y. 1993, p. 141

> *"*I spent several years doing all kinds of things and became, in a certain sense, a specialist of odd and isolated phenomena…I did not know or care in which field I was playing. I wanted to find a place, a new field, where I could be the first person to introduce mathematics. Formalization had gone too far for my taste, in the mathematics favored by the establishment... *"* (Mandelbrot, 2001, p.192)

Mandelbrot made his astonishing mathematical discovery when working on an economics problem accidentally handed to him by a friend. Economists had long attempted to make sense of (and predict) stock market fluctuations and had proposed theories based on existing data which did not hold up when tested with primitive computers. Mandelbrot viewed fluctuations from the perspective of changing scales. That is the time scale can be in days or months or years. He suggested that the interchangeable nature of the time scales was the key to understanding the fluctuations.

> I cooked up the simplest mathematical formula I thought could explain this phenomenon…[making] no assumptions about people, markets or anything in the real world. It was based on a 'principle of invariance', -the hypothesis that, somehow economics is a world in which things are the same in the small as they are in the large except, of course for a suitable change of scale (Mandelbrot, 2001).

In 1960, Edward Lorenz, who was modeling the earth's atmosphere with nonlinear equations at MIT, switched from rounding his equations to the sixth decimal point to doing so to the third. What emerged was a totally different system! He attributed the difference to a combination of the iteration of his equations plus the sensitivity of the system to initial conditions---in this case, the changes in the terminal decimal points. Lorenz named this randomness within his non-random weather models the 'butterfly effect' in a paper he wrote entitled "Can the flap of a butterfly's wing stir up a tornado in Texas?" The discovery of 'sensitive dependency on initial conditions' coupled with the 'iteration of patterns or data' which produce random irregularities in deterministic systems is the beginning of the contemporary science of "deterministic chaos."[19]

The term 'fractal' was coined from Latin *fractua* 'irregular,' to refer to the results of this combination of iteration and sensitivity. And it was Mandelbrot who provided the pictures of this deterministic chaos in his computer generated fractal images---what is described as "…a way of seeing infinity."[20] We discover these irregular nonlinear fractal structures and patterns throughout nature, in the iterations of buds in Romanesco broccoli, the arterial and venous systems of kidneys, lungs, brains, coast lines, mountain ranges, root systems, turbulences in fluids. For example, one might ask the length of a head of cauliflower or a coastline. At one level, the answer might be eight inches or 580 miles. However, at the fractal level of iteration of growth patterns and/or ocean forces, both can be seen as infinite.

In most, perhaps all of nature, we encounter a kind of deterministic chaos in a world described by 'fractal geometries' which have "…become a way of measuring qualities that otherwise have no clear definition: the degree of roughness or brokenness or irregularity in an object."[21] This is the heterogeneous and nonlinear world of the branching of buds in the

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19 Heinz-Otto Peitgen, Harmut Juergens, Dietmar Saupe: Chaos and Fractals, NewFrontiers of Science, Springer Verlag, New York, 1992, p. 48

20 James Gleick, , Chaos, Making a New Science, Penguin Books, N.Y. 1987, p. 98

21 Gleick, Ibid.

cauliflower head, the spongy tissue of the lungs, the indentation on the beach. "Chaos is more like the rule in nature, while order (=predictability) is more like the exception."[22]

Fig3. Cubism[23] 2 variations[24]

An unpredictable consequence of fractal geometry coupled with advances in computer graphics was that it was now possible for machines to produce geometric "art" based on very simple formulae which "shows surprising kinship to Old Masters paintings, Cubist paintings or Beaux Arts architecture. An obvious reason is that classical visual arts, like fractals, involve very many scales of length and favor self-similarity (Mandelbrot, 1981, 1989).

The discovery that self-similarity was an inherent property of nature as mathematically conceptualized by Mandelbrot was long written about and expressed by poets, satirists, writers, philosophers and numerous religious traditions. For instance, in Southern India, Kolam is an art form used by women to decorate the entrance to homes and courtyards (see Fig.4). These art forms go back over 6000 years and consist of self-similar patterns repeated in different scales in very sophisticated fashion.

22 Heinz-Otto Peitgen, Harmut Juergens, Dietmar Saupe: p. 48

[23] "Cubism is a more modern art movement in which forms are abstracted by using an analytical approach to the object and painting the basic geometric solid of the subject. Cubism is a backlash to the impressionist period in which there is more of an emphasis of light and color. Cubism itself follows Paul Cezanne's statement that "Everything in nature takes its form from the sphere, the cone, and the cylinder." in which these 3 shapes are used to depict the object of the painting. Another way that the cubist expressed their painting was by showing different views of an object put together in a way that you can not actually see in real life. The Cubism period stated in Paris in 1908, reached its peak in 1914, and continued into the 20's. Major cubists were Pablo Picasso and Georges Braque"
Retrieved from http://abstractart.20m.com/cubism.htm

[24] Retrieved from http://www.fractal-art.com/htmle/men-gallery01.html

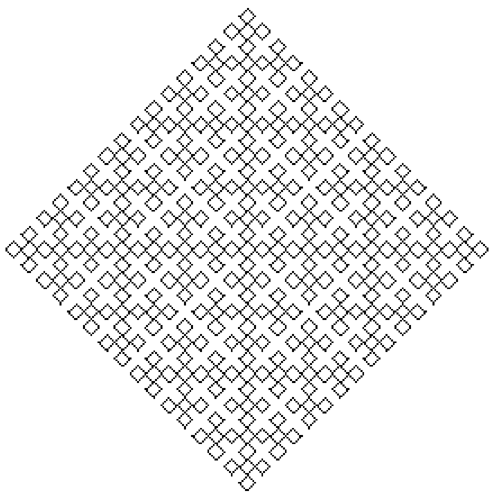

Fig.4 Anklets of Krishna

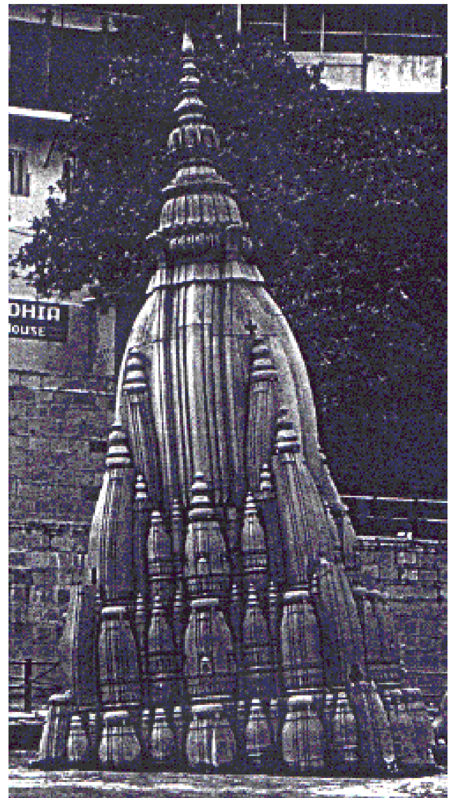

Fig.5. Spirals in a Hindu temple[25]

Architecture in Hindu temples (see Fig.5) also reveal that the notion of self-similarity was used to create visually stunning forms. The English writer, Jonathan Swift (1773), best known for Gulliver's Travels wrote:

> So, Nat'ralists observe, a Flea
> Hath Smaller Fleas that on him prey,
> And these have smaller Fleas to bit'em,
> And so proceed as infinitum

And it was Mandelbrot (2001) who remarked Swift was merely repeating a saying by the German philosopher Leibniz, who in turn was repeating Aristotle!

POLYMATHY

The numerous examples of thinkers given thus far represent a unique sample of individuals who made remarkable contributions to the arts, sciences and mathematics, and who also happened to be philosophers. These individuals are best characterized as *polymaths*. The term *polymath* is in fact quite old and synonymous with the German term "Renaissance-mensch[26]." Although this term occurs abundantly in the literature in the humanities, very few (if any) attempts have been made to isolate the qualitative aspects of thinking that adequately describe this term. Most cognitive theorists believe that skills are domain specific and typically non-transferable across domains. This implicitly assumes that "skills" are that which

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[25] Retrieved from http://classes.yale.edu/fractals/IMA/FB/ArtFrac/ArtFractals.html

[26] The author prefers the use of the German term to avoid the sexist overtones of the English equivalent "Renaissance man"

one learns as a student within a particular discipline. However such an assumption begs the question as to why *polymathy* occurs in the first place. Although the numerous historic and contemporary examples presented are of eminent individuals, it has been found that *polymathy* as a thinking trait occurs frequently in non-eminent samples (such as high school students) when presented with the opportunities to engage in trans-disciplinary behavior. In particular the use of unsolved classical problems and mathematics literature has been found to be particularly effective in fostering trans-disciplinary thinking (see Sriraman, 2003a,2003b, 2004b, 2004c).

Thinking traits of polymaths

Root-Bernstein (2003) has been instrumental in rekindling an interest in mainstream psychology in a systematic investigation of *polymathy*. That is the study of individuals, both historical and contemporary, and their trans-disciplinary thinking traits which enabled them to contribute to a variety of disciplines. His analysis of the works and biographies of numerous innovators both historical and contemporary reveals that arts advance the sciences and scientists are inspired by the arts.

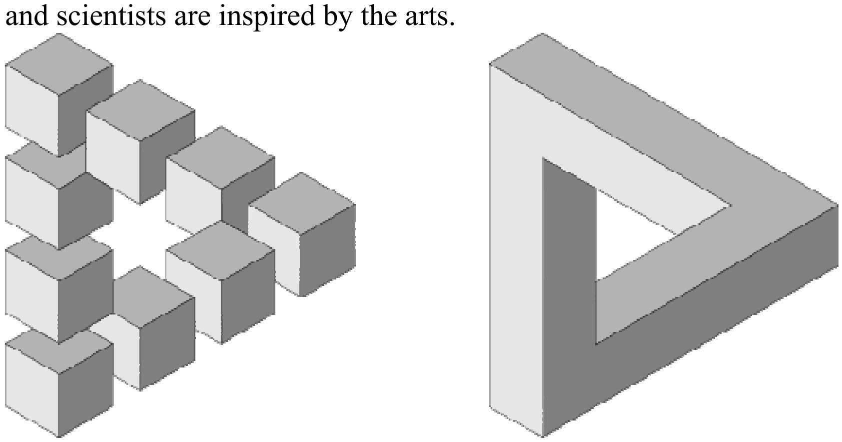

Fig.6 The impossible Penrose Tribar[27]

One recent example provided by Root- Bernstein (2003) is the effect of Escher's drawings on a young Roger Penrose, the mathematical physicist, who visited one of Escher's exhibitions in 1954. Stimulated by the seemingly impossible perspectives conveyed by Escher in 2-dimensions, Penrose began creating his own impossible objects such as the famous Penrose "impossible" tribar which shows a 3-dimensional triangle that twists both forwards and backwards in 2-dimensions. Root- Bernstein writes:

> *"*Roger Penrose showed his tribar to his father L.S. Penrose, a biologist who dabbled in art… [who] invented the impossible staircase in which stairs appear to spiral both up and down simultaneously…[and] sent Escher a copy…[who] then developed artistic possibilities of the impossible staircase in ways that have since become famous*"* (p.274)

Another well known consequence of Escher's artistic influence on mathematicians is the investigation of tiling problems (both periodic and aperiodic) popularized by both Roger Penrose and Martin Gardner, which helped cystallographers understand the structure of many metal alloys which are aperiodic (Peterson, 1985 as quoted by Root-Berstein, 2003, p.274).

[27] Retrieved from http://mathworld.wolfram.com/PenroseTriangle.html

Common thinking traits of the polymaths described in this paper in conjunction with the thousands of polymaths (historical and contemporary) as analyzed by Root-Bernstein, Dean Simonton and many others are: (1) Visual geometric thinking and/or thinking in terms of geometric principles, (2) Frequent shifts in perspective, (3) thinking in analogies, (4) Nepistemological awareness (that is, an awareness of domain limitations), (5) Interest in investigating paradoxes (which often reveal interplay between language, mathematics and science), (6) Belief in *Occam's Razor* [Simple ideas are preferable to complicated ones], (7) Acknowledgment of Serendipity and the role of chance, and (8) the drive to influence the Agenda of the times.

IMPLICATIONS

The tension between the disciplines that came out of the Renaissance, namely natural philosophy–art – alchemy (metallurgy/chemistry)- theology during the post Renaissance continues today in the modern day antipathy between the ever increasing sub-disciplines within arts, science, mathematics and philosophy. Many of the thinking processes of polymaths who unified disciplines are commonly invoked by artists, scientists, mathematicians and philosophers in their craft albeit the end products are invariably different. These disciplines explore our world for new knowledge. Literature is an excellent medium to create frequent shifts in perspective. Paradoxes can be easily investigated by exploring geometry motivated by Art. After all Art suggests new possibilities and pushes the limits of our imagination, whereas science verifies the actual limitations of these possibilities using mathematics. Both are driven by the need to understand reality with philosophy (and theology) often serving as the underlying framework linking the three. Models and Theory building lie at the intersection of art-science-mathematics. The history of model building in science conveys nepistemological awareness of domain limitations. Arts imagine possibilities, science attempts to generate models to test possibilities, mathematics serves as the tool. The Implications for education today is to move away from the post Renaissance snobbery rampant within individual disciplines at the school and university levels. By building bridges today between disciplines, the greatest benefactors are the potential innovators of tomorrow. Our Symposium is a worthy start towards this endeavor.

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