PSI-PR-94-22
UCY-PHY-94/3
CERN-TH.7382/94

Beautiful Baryons from Lattice QCD [1]

C. Alexandrou[a], A. Borrelli[b], S. Güsken[c], F. Jegerlehner[b], K. Schilling[d,c], G. Siegert[c], R. Sommer[d]

[a] *Department of Natural Sciences, University of Cyprus, Nicosia, Cyprus*
[b] *Paul Scherrer Institute, CH-5232 Villigen PSI, Switzerland*
[c] *Physics Department, University of Wuppertal D-42097 Wuppertal, Germany*
[d] *CERN, Theory Division, CH-1211 Geneva-23, Switzerland*

Abstract

We perform a lattice study of heavy baryons, containing one (Λ_b) or two b-quarks (Ξ_b). Using the quenched approximation we obtain for the mass of Λ_b

$$M_{\Lambda_b} = 5.728 \pm 0.144 \pm 0.018\text{GeV}.$$

The mass splitting between the Λ_b and the B-meson is found to increase by about 20% if the light quark mass is varied from the chiral limit to the strange quark mass.

[1] work supported in part by DFG grant Schi 257/3-2.

1 Introduction

Considerable progress has been achieved in the computation of low lying hadronic states from lattice QCD (LQCD). It turned out that *quenched* LQCD reproduces the hadron spectrum in the light quark sector surprisingly well, once finite volume corrections are carefully taken into account [1]. This holds in particular for the ratio m_N/m_ρ, which has been a notorious problem for LQCD over quite some years.

Beauty physics is attracting much attention because the origin of CP violation in the B-system is still an open question. The investigation of such a system is a great challenge to LQCD. Considerable progress has been made in the lattice studies of heavy-light mesons like D- and B-mesons [2].

Heavy-light systems from the *baryonic* sector so far have been studied with lattice methods exploratively in the limit of infinite heavy quark mass [3, 4].

Throughout the current study the mass of the heavy quark has been kept finite. We will present results for the mass of Λ_b, a baryon composed of a b-quark and two light quarks, as well as for the mass of Ξ_b, a baryon composed of two b-quarks and one light quark.

As in the case of heavy-light mesons, the most dangerous source of systematic error stems from the fact that one is forced — in order to enter the region of heavy quark masses — to actually push the heavy mass close to the very limit of current lattice resolutions. We will attempt, however, to keep control on these effects of a finite lattice spacing a by a variety of precautions:

1. We avoid to compute the masses of the Λ_b and the Ξ_b directly, but rather calculate the mass splittings $\Delta_\Lambda = M_{\Lambda_b} - M_B$ and $\Delta_\Xi = M_{\Xi_b} - 2M_B$, with respect to the B-meson mass M_B. These splittings do not depend on the heavy quark mass in the infinite mass limit and are therefore less prone to contamination by finite a effects in the b and c quark mass regions.

2. We monitor the dependence of the splittings on the lattice spacing for our three β values, $\beta = 5.74, 6.0, 6.26$. This enables us (a) to check the assumption of weak a dependence and (b) to perform an $a \to 0$ extrapolation.

3. We will not calculate directly the mass splittings too close to the b quark mass. Instead we stop the calculation at approximately twice the charm quark mass and then extrapolate our data to the b mass.

Moreover, we investigate finite size effects at $\beta = 5.74$, on three lattices of spatial volumes $8^3, 10^3$ and 12^3, in lattice units. Our lattice parameters are detailed in table 1. To obtain a good signal for the ground state, we use smeared gauge invariant interpolating quark fields [5], defined for the standard Wilson action.

$\mathrm{N_S}$	$\mathrm{N_T}$	no. configs.	β	$a\, m_\rho$
8	24	175	5.74	0.542 ± 0.014
10	24	213	5.74	
12	24	113	5.74	
12	36	204	6.00	0.355 ± 0.016
18	48	67	6.26	0.260 ± 0.014

Table 1: Lattice parameters: space and time extents N_S and N_T, number of configurations, β and the ρ-mass in lattice units.

2 Smearing Techniques and Volume Effects

For the reasons given above, we will compute the baryonic masses with reference to the mass of the B-meson, by proper combinations that would eliminate the b-quark mass in the heavy quark limit. So we consider the splittings $\Delta_\Lambda = M_{\Lambda_b} - M_B$ and $\Delta_\Xi = M_{\Xi_b} - 2M_B$, respectively, in the single and double beauty sectors. In the first case, we need to compute the correlators, for the heavy-light pseudoscalar meson,

$$C_P(t) = \sum_{\vec{x}} \left\langle \left(\bar{Q}(x)\gamma^5 q^J(x) \right) \left(\bar{q}^J(0)\gamma^5 Q(0) \right) \right\rangle , \tag{1}$$

and for the Λ baryon [6]

$$C_\Lambda(t) = \sum_{\vec{x}} \Big\langle \left(\epsilon^{abc} Q_a(x) \left(q_b^J(x) \mathrm{C}\gamma_5 q_c^J(x) \right) \right) \left(\epsilon^{abc} Q_a(0) \left(q_b^J(0) \mathrm{C}\gamma_5 q_c^J(0) \right) \right)^\dagger \Big\rangle , \tag{2}$$

where $Q(x) = Q(\vec{x}, t)$ is the heavy quark field, and $q^J(x) = q^J(\vec{x}, t)$ is the light one, to which smearing of type J [5] has been applied[2]. C is the charge

[2]The correlator for the Ξ is obtained from the one for the Λ, by replacing the light quark q_b^J by a heavy one Q_b.

conjugation operator given by $C = i\gamma^4\gamma^2$.

Given the lattice results for these correlators, we perform a direct fit to their ratio

$$R_\Lambda(t) = \frac{C_\Lambda(t)}{C_P(t)} \to A e^{-\Delta_\Lambda t} \qquad (3)$$

in the large t limit.

It is by now well known that smearing [3, 7, 8] is crucial to obtain a decent overlap of the operators with the ground state. In this work, we make use of the experience acquired previously while computing properties of the heavy-light pseudoscalar states [3], where we found gauge-invariant 'Gaussian type' wave functions (of r.m.s radius 0.3 fm) to provide sufficient overlap. The smearing was applied to the light quark source in the mesonic case. In this study, we are using precisely this procedure for the heavy-light baryons as well, without any further optimization.

In order to establish ground state dominance we look for a plateau in the local mass of the ratio R_Λ. In fig. 1a we show as an example the local mass of R_Λ, for the two largest lattices: the solid line shows the fitted value for the plateau. Fig. 1b shows the corresponding local mass for the ratio $R_\Xi(t) = C_\Xi(t)/C_P^2(t)$ used to determine Δ_Ξ. These figures show the quality of the plateaus for representative κ values. Worse quality is found only in a few cases, and it resulted in larger statistical errors; for instance the two largest κ values at $\beta = 5.74$ given in table 3.

The pseudoscalar mass was extensively studied in ref. [9] and the values have been taken from there.

For checking the finite volume effects, we computed Δ_Λ on three lattices with $\mathrm{N_T} = 24$, $\beta = 5.74$ and sizes $\mathrm{N_S}$ =8, 10, 12, for $\kappa_l = 0.156$, and $\kappa_h =$ 0.125, 0.140, 0.150. We compare the results for the splitting in table 2. The values exhibit deviations of at most 4% . In fig. 2 we plot the dimensionless ratio Δ_Λ/m_ρ as a function of L, the lattice size in units of m_ρ; as can be seen, for these values of κ_h, the finite size effects of this ratio are smaller than our statistical errors.

In the following, we will fix the volume to about 1 fm (which corresponds to $N_S \simeq$8, 12 and 18 for $\beta = 5.74$, 6.0 and 6.26 respectively) and carry out a detailed study of the extrapolation to the continuum limit. As a tribute to possible finite size effects we will add the above maximal variation of 4% as an uncertainty to all results.

κ_l	κ_h	$\Delta_\Lambda, 8^3 \times 24$	$\Delta_\Lambda, 10^3 \times 24$	$\Delta_\Lambda, 12^3 \times 24$
0.156	0.125	0.564 ± 0.008	0.554 ± 0.005	0.569 ± 0.005
	0.140	0.592 ± 0.007	0.575 ± 0.005	0.588 ± 0.004
	0.150	0.621 ± 0.007	0.601 ± 0.005	0.612 ± 0.006

Table 2: Results for three different spatial lattice extents, $\mathrm{N_S}$ =8, 10, and 12, $\mathrm{N_T} = 24$, $\beta = 5.74$.

3 Continuum Limit

Λ **Splitting.** The results for Δ_Λ at the three β values 5.74, 6.00 and 6.26 at fixed volume are listed in table 3, all in lattice units. The extrapolations to u and s-type light quarks, — for given heavy quark κ_h — are performed as described in [3].

Since we are evaluating masses, it is natural to use m_ρ to set the lattice scale. The values of m_ρ for the various lattices have been listed in table 1; the experimental value used is: $m_\rho = 768$ MeV.

In fig. 3a we plot Δ_Λ in GeV extrapolated to the chiral limit as a function of $1/M_P$ in GeV^{-1}. Within the statistical precision achieved in this computation, the points show no dependence on a, although a is varied by about a factor two (cf. Table 1).

In the continuum, the $1/M_P$ expansion for Δ_Λ gives

$$\Delta_\Lambda^{\mathrm{cont.}}(M_P) = c_0 + \frac{c_1}{M_P} + O(M_P{}^{-2}) \; . \tag{4}$$

Assuming no dependence on the lattice spacing, this form can be fitted directly to the points in fig. 3a yielding $\Delta_{\Lambda_b} = 431(28)$ MeV at the mass of the B-meson. This result is included in the figure as the inverted triangle. The error bar of this point does not account for the fact that the simulation results exclude an a-dependence only within their precision.

A realistic error that includes the uncertainty of extrapolating the lattice data to the continuum is obtained by allowing for the leading a-dependence at each value of M_P [9]: We start out from a selected value of M_P (in physical units) and interpolate the lattice results from each (fixed) β-value to the value of M_P. This enables us to compare Δ_Λ at different values of a. A subsequent

$8^3 \times 24, \quad \beta = 5.74$

κ_h	$a\Delta_\Lambda$	$a\Delta_\Lambda(m_s)$	$a\Delta_\Xi$	$a\Delta_\Xi(m_s)$	$\Delta_\Lambda(m_s)/\Delta_\Lambda(m_u)$
0.06	0.25 ± 0.13	0.37 ± 0.08	-0.24 ± 0.02	-0.31 ± 0.02	1.46 ± 0.40
0.09	0.34 ± 0.10	0.42 ± 0.06	-0.17 ± 0.02	-0.23 ± 0.02	1.23 ± 0.18
0.125	0.38 ± 0.04	0.46 ± 0.02	-0.12 ± 0.02	-0.17 ± 0.01	1.20 ± 0.07
0.14	0.42 ± 0.02	0.50 ± 0.01	-0.01 ± 0.01	-0.009 ± 0.009	1.18 ± 0.04
0.15	0.47 ± 0.04	0.54 ± 0.02	0.09 ± 0.01	-0.080 ± 0.008	1.13 ± 0.05

$12^3 \times 36, \quad \beta = 6.00$

κ_h	$a\Delta_\Lambda$	$a\Delta_\Lambda(m_s)$	$a\Delta_\Xi$	$a\Delta_\Xi(m_s)$	$\Delta_\Lambda(m_s)/\Delta_\Lambda(m_u)$
0.1	0.22 ± 0.02	0.28 ± 0.01	-0.14 ± 0.02	-0.18 ± 0.01	1.23 ± 0.05
0.115	0.21 ± 0.02	0.28 ± 0.01	-0.10 ± 0.01	-0.147 ± 0.008	1.29 ± 0.08
0.125	0.25 ± 0.01	0.300 ± 0.008	-0.09 ± 0.01	-0.130 ± 0.007	1.19 ± 0.03
0.135	0.27 ± 0.01	0.310 ± 0.007	-0.06 ± 0.01	-0.101 ± 0.006	1.17 ± 0.03
0.145	0.31 ± 0.02	0.348 ± 0.007	0.01 ± 0.01	-0.049 ± 0.009	1.11 ± 0.04

$18^3 \times 48, \quad \beta = 6.26$

κ_h	$a\Delta_\Lambda$	$a\Delta_\Lambda(m_s)$	$a\Delta_\Xi$	$a\Delta_\Xi(m_s)$	$\Delta_\Lambda(m_s)/\Delta_\Lambda(m_u)$
0.09	0.15 ± 0.01	0.188 ± 0.009	-0.07 ± 0.01	-0.107 ± 0.008	1.25 ± 0.07
0.10	0.15 ± 0.02	0.19 ± 0.01	-0.073 ± 0.008	-0.110 ± 0.006	1.29 ± 0.07
0.12	0.16 ± 0.01	0.201 ± 0.007	-0.062 ± 0.06	-0.010 ± 0.004	1.25 ± 0.04
0.135	0.18 ± 0.01	0.216 ± 0.008	-0.032 ± 0.005	-0.071 ± 0.004	1.21 ± 0.04
0.145	0.21 ± 0.01	0.243 ± 0.009	0.022 ± 0.006	-0.024 ± 0.005	1.14 ± 0.04

Table 3: The mass splittings Δ_Λ and Δ_Ξ at the chiral limit and at the strange quark mass in lattice units for $\beta = 5.74, 6.00, 6.26$. The last column gives the ratio $\Delta_\Lambda(m_s)/\Delta_\Lambda(m_u)$ for the same β values.

linear extrapolation in a will then yield the continuum estimate for Δ_Λ, at the chosen physical value of M_P.

Reiteration of the procedure on a set of masses M_P determines the numerical dependence of $\Delta_\Lambda^{\rm cont.}$ on M_P. In order to remain in compliance with the assumption of linearity in a, we used only a subset of the data listed in table 3, by excluding the data for the two heaviest masses at any value of β.

A final fit of the continuum limit values $\Delta_\Lambda^{\rm cont.}$ to eq.(4) yields

$$\Delta_{\Lambda_b} = 458 \pm 144 \pm 18 \,\mathrm{MeV}\ . \tag{5}$$

Here the first error is purely statistical and stems from a full jacknife analysis of our data. The second error represents the 4% uncertainty due to finite volume effects. Together with the known experimental value $M_B = 5.27$ GeV, we obtain an estimate for the mass of Λ_b :

$$M_{\Lambda_b} = 5.728 \pm 0.144 \pm 0.018 \,\mathrm{GeV}. \tag{6}$$

We note in passing, that the parameter c_0 is in agreement with the estimates obtained directly in the static approximation [3], albeit within the larger uncertainties of the latter.

In the same way we obtain the mass splitting $\Delta_{\Lambda_b}(m_s)$ where the light quark mass is extrapolated to the strange quark mass. The data fitted are shown in fig. 3b, and the result of the extrapolations described above is

$$\Delta_{\Lambda_b}(m_s) = 597 \pm 91 \pm 24 \,\mathrm{MeV}\ , \tag{7}$$

with the same meaning of errors. Instead of looking directly at the splitting at the strange quark mass, one may consider the ratio

$$r = \frac{\Delta_\Lambda(m_s)}{\Delta_\Lambda(m_u)}, \tag{8}$$

which is expected to have an even weaker a-dependence. Indeed the ratio for the different β values shown in fig. 3c follows a "universal curve" with quite small statistical errors. A linear fit to the (weak) mass dependence yields

$$r = 1.25 \pm 0.03 \pm 0.05. \tag{9}$$

This ratio represents the change in the Λ_b mass when replacing the u quark by an s quark.

Ξ **Splitting.** The mass splitting for the Ξ baryon is investigated using the same techniques as for the Λ. Table 3 displays the results after extrapolation of the light quark to the d and s mass respectively. We then convert all data to physical units, and plot Δ_Ξ vs. $1/M_P$ at the chiral limit in fig. 4. In this case the data show a statistically significant a-dependence, especially for heavy masses. We extrapolate to the continuum limit as above for pseudoscalar masses M_P below the charm mass. We use the same ansatz as in the case of Δ_Λ to fit the $1/M_P$ dependence of Δ_Ξ and obtain:

$$\Delta_{\Xi_b} = -90 \pm 170 \text{ MeV} \ , \tag{10}$$

This mass shift together with the experimental value for M_B determines

$$M_{\Xi_b} = 10.45 \pm 0.17 \text{ GeV} \quad , \tag{11}$$

for the physical Ξ_b–mass.

Another possibility to compute the Ξ_b mass, is to consider the ratio $R\,'_{\Xi}(t) = C_\Xi(t)/C_P^{\bar{h}h}(t)$, which should yield the splitting between the Ξ_b mass and the Υ-meson mass. We evaluated this quantity but the resulting plateaus turned out to be of worse quality than those for the other channel studied; this is due to the fact that smearing was applied only to light quarks, and so the heavy-heavy channels suffer from relevant contamination by higher-mass states.

4 Discussion

The mass splitting technique is a viable method to compute the Λ_b mass on the lattice. Both the lattice spacing dependence of the mass splitting and its dependence on the heavy quark mass are weak. Thus an extrapolation to the continuum and to the b–quark mass is possible. Our actual value of $M_{\Lambda_b} = 5728 \pm 144 \pm 18$ MeV can be compared to the value 5630 MeV suggested by Martin et al. within the naive quark model approach [4].

Experimentally, the determination of the Λ_b mass has a somewhat controversial history ever since 1981 [12]. Recently, the UA1 collaboration has measured the mass from 16 events in the decay channel $J/\Psi\Lambda$[3]. Their value

[3]It is an open question, why this decay channel has not been observed in the CDF experiment.

is [13] $M_{\Lambda_b} = 5640 \pm 50 \pm 30$MeV. The DELPHI collaboration is presently quoting [14] a preliminary mass value, $M_{\Lambda_b} = 5635^{+38}_{-29} \pm 4$MeV, which is based on one candidate event in the $\Lambda_c^+\pi^-$ and in the $D^0 p\pi-$ decay modes. These numbers are in agreement with our result.

For M_{Λ_c} we obtain $2433 \pm 88 \pm 18$ MeV, which is in rough accord with the experimental value of 2285 MeV [12]. Looking at the light quark dependence we found a 20% increase as we lift the quark mass from the chiral limit to the strange quark mass. Finally, an estimate for the Ξ_b mass is given in eq.(11).

Our errors do include – as the dominant part – the uncertainty induced by an extrapolation to the continuum limit. Nevertheless, it is desirable to further check these extrapolations through simulations with higher lattice resolutions and/or different lattice actions.

Acknowledgements

We thank the Personal at the Computer Centers CSCS in Manno Switzerland and Jülich, Germany for their support. KS thanks A. Martin for a valuable discussion.

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Figure captions

Figure 1a
The Δ_Λ local masses, given by $\mu_\Lambda^{\rm loc}(t) = \log[\frac{R_\Lambda(t-a)}{R_\Lambda(t)}]$ for $\kappa_l = 0.1525$, $\kappa_h = 0.125$ at $\beta = 6.0$, for a $12^3 \times 36$ lattice, and for $\kappa_l = 0.1492$, $\kappa_h = 0.12$ at $\beta = 6.26$, for a $18^3 \times 48$ lattice.

Figure 1b
The same as figure 1a, but for $\mu_\Xi^{\rm loc}(t) = \log[\frac{R_\Xi(t-a)}{R_\Xi(t)}]$.

Figure 2
Δ_Λ/m_ρ is shown vs L for three heavy quark masses at $\beta = 5.74$. The light quark mass was fixed to about twice the strange quark mass.

Figure 3a
The Λ mass splitting is shown vs $1/M_P$ in GeV^{-1} at the chiral limit. The solid line is a global fit assuming no a effects. It gives the value

shown with the inverted triangle at the B-meson mass. The value obtained after extrapolation to the continuum limit and to the B-meson mass, is shown by the open circle. This extrapolation is discussed in the text.

Figure 3b
As for figure 3a but at the strange quark mass.

Figure 3c
The ratio $\Delta_\Lambda(m_s)/\Delta_\Lambda(m_u)$ is shown vs $1/M_P$. The notation is the same as for figure 3a.

Figure 4
The Ξ mass splitting, $\Delta_\Xi = M_\Xi - 2M_P$ is shown vs $1/M_P$ in GeV^{-1} at the chiral limit. The notation is the same as for figure 3a.