The Beauty of Everyday Mathematics

by Norbert Herrmann

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REVIEWED BY PAMELA GORKIN

First-semester calculus used to be one of my favorite courses to teach. After seeing the definition of derivative and using it to calculate lots of derivatives, my students were thrilled by the ease with which they could take derivatives using the product rule. After struggling to compute areas under a curve using Riemann sums, my students were amazed by the simplicity the Fundamental Theorem of Calculus provides. I could be misremembering that, of course. But what is true now is that our students arrive on campus knowing the product rule, but unable to recall the definition of derivative. They know rules for integration by heart, but don't know what is "fundamental" about the Fundamental Theorem of Calculus. It's much more difficult for these students to see the beauty of calculus, but my hope is that if we find the right examples, we'll still be able to grab our students' attention. The appearance of a new book aimed at teachers and students of mathematics, *The Beauty of Everyday Mathematics* by Norbert Herrmann, provides such examples.

Translated from the German *Mathematik ist überall*, Herrmann's book contains unusual examples of mathematics in everyday life. Here's one that I can relate to: I have two identical and perfectly round beer coasters that I borrowed, permanently, from a bar I go to (occasionally, of course). I put them next to each other and slide the leftmost coaster over the top of the coaster on the right, keeping the centers of the coasters on the line segment joining the old centers. When the coasters are next to each other, there is no overlapping area. When they are on top of each other, the entire area of the coaster on the bottom is covered. So here's a natural question that two beer-drinking mathematically inclined people might ask: When is half the area of the bottom coaster covered? Now most mathematicians will be able to find an equation that, when solved, will provide the solution. If you spend a few minutes, use some familiar formulas, and apply a double-angle formula, you'll probably come up with the following:

$$\alpha - \sin(\alpha) = \pi/2,$$

where α is the angle between the two radii that are drawn from the center to the two points of intersection of the coasters. There are, however, a few things that you should note here: First, this is pretty sophisticated stuff. If you are thinking of purchasing this book for someone else, your recipient needs to be pretty savvy, mathematically speaking. The second thing you should notice is that it's pretty unlikely that a nonmathematician would look at $\alpha - \sin(\alpha) = \pi/2$ and say, "Hey, I know how to show α is pretty darn close to 2.309881 radians." But it's a great way to introduce Newton's method, which is precisely what Herrmann does. Not only that, he presents a second method to solve this equation, which he calls "The Fixed Point Procedure." It's a clever example that leads to some beautiful mathematics.

A second example, reproduced as it appears in the text, sounds similar, but the solution is quite different.

> "A group of young people has decided to picnic in the great outdoors and is now sitting on the grass somewhere, battling flies and, well, with this specific can of soda. Because this stupid container doesn't want to stay upright on the grass; instead, it just wants to tip over and spill its delicious contents so that the ants can enjoy it. This is exactly the kind of task which makes physicists and mathematicians roll up their sleeves together."

The English translation leaves much to be desired, but it's possible to overlook that and appreciate the author's creativity and humor. Let's focus, then, on the problem of interest. What is it? Well, when the can is empty, the center of gravity is obviously in the middle of the can. And when the can is full the center of gravity is also obviously in the middle of the can. Apparently, the center of gravity travelled down and then back up again, but it pretty clearly never hit the bottom. So, "how much soda do you have to drink so that the center of gravity reaches its lowest point?" You may have noticed a few things here. The author has an entertaining style, and the problem is again, I think, very interesting. The solution begins by describing the important variables as well as recalling how one computes the center of gravity. It's nontrivial, but presented at the right level for the intended audience.

Some chapters focus on the challenges of parking your car: How can you model, mathematically, the act of parallel parking? What's the right position to enter a parking space? Other chapters present problems of a visual nature: How far should one person walk behind another in order to have the best possible visual angle of that person's legs? There's also a chapter consisting of things we find amusing, such as Hilbert's Hotel Infinity or the fact that "from a mathematical point of view, all numbers are interesting." Hermann even manages to introduce induction, using the toasting problem as his motivation: if *N* people are in a room and everyone toasts everyone else, how many times would the glasses clink?

Hermann must be a remarkable teacher. He chooses the topics carefully, the problems are well motivated, and the presentation is clever, entertaining, and clear. The English translation, as well as some curious misprints, may interfere with your enjoyment, but overall *The Beauty of Mathematics* is a charming little book.

Department of Mathematics
Bucknell University
Lewisburg, PA 17837
USA
e-mail: pgorkin@bucknell.edu

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