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Beauty Quark Fragmentation Into Strange B Mesons

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Abstract

Using the recent measurement of the total production rate for B_s and B_s^* mesons in electron-positron annihilation to determine the strange quark mass parameter in the $\bar{b} \to B_s$, B_s^* fragmentation functions we calculate the momentum distributions of the B_s and B_s^* mesons.

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Recently the probability for a heavy quark to hadronize into a strange B or D meson has been measured at LEP by the DELPHI Collaboration [1]. These data can be used to determine the strange quark mass parameter, m_s, in the heavy quark fragmentation functions and, therefore, to predict the momentum distributions of the B_s and B_s^* mesons produced in the fragmentation process. Using the heavy quark fragmentation functions calculated in perturbative QCD at the scale of the heavy quark [2] for the initial condition we have numerically integrated the Altarelli-Parisi equation to obtain the momentum distributions of the B_s and B_s^* mesons at the scale of the Z mass. The resulting distributions are only moderately sensitive to the value of m_s, which we find to be about 300 MeV$/c^2$.

The fragmentation function for the process $\bar{b} \to B_s$ at the scale of the b mass is easily obtained from the $\bar{b} \to B_c$ fragmentation functions calculated using perturbative QCD by Braaten, Cheung, and Yuan [2]:

$$\begin{aligned} D_{\bar{b}\to B_s}(z,\mu_0) &= \frac{2\alpha_s(2m_s)^2|R(0)|^2}{81\pi m_s^3}\,\frac{rz(1-z)^2}{(1-(1-r)z)^6} \\ &\times\ [6-18(1-2r)z+(21-74r+68r^2)z^2 \\ &\quad -2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4]\,. \end{aligned} \tag{1}$$

Here z is the fraction of the $\bar{b}$ quark momentum carried by the B_s meson, $r = m_s/(m_b+m_s)$, and $R(0)$ is the B_s meson S-wave radial wavefunction at the origin. The corresponding fragmentation function for $\bar{b} \to B_s^*$ is

$$\begin{aligned} D_{\bar{b}\to B_s^*}(z,\mu_0) &= \frac{2\alpha_s(2m_s)^2|R(0)|^2}{27\pi m_s^3}\,\frac{rz(1-z)^2}{(1-(1-r)z)^6} \\ &\times\ [2-2(3-2r)z+3(3-2r+4r^2)z^2 \\ &\quad -2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4]\,, \end{aligned} \tag{2}$$

The wavefunction at the origin, $R(0)$, can be determined from the B_s meson decay constant, f_{B_s}, from the Van Royen-Weisskopf [3] relation, modified for color:

$$f_{B_s}^2 = \frac{3}{\pi}\frac{|R(0)|^2}{M_{B_s}}\,. \tag{3}$$

In Eqs. (1) and (2) the running coupling constant $\alpha_s(\mu) = \alpha_s(M_Z)/(1+b_0\alpha_s(M_Z)\log(\mu/M_Z))$, where $b_0 = (33 - 2n_f)/6\pi$, n_f is the number of flavors at the scale μ, $\alpha_s(M_Z) = 0.12$, and we have chosen the scale to be $2m_s$ [2]. The scale μ_0 of the fragmentation functions is of the order of m_b and we choose it to be $m_b + 2m_s$ [2]. Integrating the Altarelli-Parisi equation for the fragmentation functions from the initial scale μ_0 to the Z mass gives the momentum distributions of the B_s and B_s^* mesons produced by the fragmentation process in e^+e^- annihilation at the Z resonance. To determine the strange quark mass parameter, m_s, we use the recent measurement by the DELPHI collaboration [1] of the probability, f_s^w, that a weakly decaying strange heavy meson is produced during the hadronization of a heavy quark. In addition to the direct production $\bar{b} \to B_s$, the B_s meson can also result from the decays of excited states of B_s and B_c mesons. However, these have been excluded in the measurement of f_s^w, except for the radiative decay of the first excitation: $B_s^* \to B_s + \gamma$. Since the $B_s - B_s^*$ mass difference is so small we can determine m_s from the integrated fragmentation functions, which gives the measured probability f_s^w:

$$f_s^w = \int_0^1 dz \, \left[D_{\bar{b}\to B_s}(z,\mu_0) + D_{\bar{b}\to B_s^*}(z,\mu_0)\right] . \tag{4}$$

Since the integrated fragmentation functions are independent of the scale, we can choose the scale to be μ_0 in Eq. (4) and first determine m_s and then calculate the distributions $D_{\bar{b}\to B_s}(z, M_Z)$ and $D_{\bar{b}\to B_s^*}(z, M_Z)$ at the scale of the Z mass by numerically integrating the Altarelli-Parisi equation. As input we fix the heavy b quark mass at $m_b = 5.0$ GeV/c^2. To determine $R(0)$ we use Eq. (3) with the value $f_{B_s} = 207 \pm 34 \pm 22$ MeV from the lattice calculations of Bernard, Labrenz, and Soni [4]. Then from the measured value [1]

$$f_s^w = 0.19 \pm 0.06 \pm 0.08 \tag{5}$$

we find (adding the two errors in quadrature)

$$m_s = 318 \, ^{+47}_{-24} \quad MeV \, , \tag{6}$$

which is in a reasonable range [5]. The estimated error in m_s reflects only the experimental uncertainty in f_s^w.

Using this value of m_s we have calculated the momentum distributions $D_{\bar{b}\to B_s}(z)$ and $D_{\bar{b}\to B_s^*}(z)$, which are shown in Figs. 1. Figure 1(a) shows the fragmentation functions at the initial scale $\mu_0 = m_b + 2m_s$, while Fig. 1(b) shows the fragmentation functions at the scale $\mu = M_Z$. We have verified that the shapes of these distributions are only moderately sensitive to the value of m_s, although the peak does shift towards larger values of z if m_s is decreased, as one might expect. In fact, the maxima in both $D_{\bar{b}\to B_s}(z, M_Z)$ and $D_{\bar{b}\to B_s^*}(z, M_Z)$ occur at

$$z_{max}(B_s,\, B_s^*) = 0.92\,. \tag{7}$$

The average momentum fraction $\langle z\rangle$ at the Z mass scale is also not too different for the B_s and B_s^* contributions to the spectrum:

$$\langle z\rangle_{\bar{b}\to B_s} = 0.65 \quad and \quad \langle z\rangle_{\bar{b}\to B_s^*} = 0.68\ . \tag{8}$$

The spectrum also shows some evidence for approximate heavy quark spin symmetry at the level of about 50%. The ratio $\int dz D_{\bar{b}\to B_s^*}/\int dz D_{\bar{b}\to B_s}$ is about 2.1 compared to 3, as expected in the heavy quark limit from the spin independence. In terms of the ratio $P_V = B_s^*/(B_s + B_s^*)$, which heavy quark symmetry predicts to be 0.75, we find 0.68. To compare the $\bar{b}\to B_s$ and $\bar{b}\to B_s^*$ spectra individually with experiments, the two contributions $\bar{b}\to B_s$ and $\bar{b}\to B_s^*\to B_s\gamma$ have to be experimentally separated by observing the rather low energy photon, which has so far not been done.

To explore the sensitivity of our calculation to the assumptions we also considered the fragmentation of a charm quark into a D_s or D_s^* meson. Following ref. [1] we repeated the calculation assuming the probability that a D_s or D_s^* meson is produced in the hadronization of a c quark is the same as the probability that a B_s or B_s^* meson is produced in the hadronization of a b quark; that is, f_s^w is independent of the heavy quark flavor. Using the value $f_s^w = 0.19 \pm 0.06 \pm 0.08$ [1], the lattice calculation $f_{D_s} = 230 \pm 30 \pm 18$ MeV [4], and choosing $m_c = 1.5$ GeV$/c^2$ [5], we found

$$m_s = 267\ ^{+36}_{-17}\quad MeV\,, \tag{9}$$

which is in reasonable agreement with the value of m_s obtained above in the B_s case. With this value of m_s we also calculated the momentum spectra for $c \to D_s$, for $c \to D_s^*$, and their sum at the initial scale $\mu_0 = m_c + 2m_s$ as well as at the scale $\mu = M_Z$. They are shown, respectively, in Figs. 2(a) and 2(b). As expected, the D_s spectra are softer than the B_s spectra shown in Figs. 1(a) and 1(b) because of the smaller charm quark mass. The peaks of the spectra occur at 0.68 for D_s and at 0.75 for D_s^* and the mean momentum fractions are $\langle z \rangle = 0.49$ for D_s and $\langle z \rangle = 0.52$ for D_s^* at the scale $\mu = M_Z$ [Fig. 2(b)]. The ratio of the two fragmentation probabilities for the D_s case also differs significantly more from the heavy quark spin symmetry prediction of 3, being about 1.6, than in the B_s case, as one expects. Also, the ratio $P_V = D_s^*/(D_s + D_s^*)$ is 0.62, rather than 0.75.

To further test the sensitivity of our results to the input parameters assumed we have repeated the calculation of the D_s and D_s^* spectra using the measurement of the decay constant f_{D_s} recently reported by the CLEO collaboration [6]. Although this measured value $f_{D_s} = 344 \pm 37 \pm 52 \pm 42$ MeV [6] differs substantially from the lattice result $f_{D_s} = 230 \pm 30 \pm 18$ MeV [4], we found

$$m_s = 306 \ ^{+45}_{-22} \quad MeV \,, \tag{10}$$

which is not significantly different from the value above [Eq. (9)], and the changes in the fragmentation spectra shown in Fig. 2 are negligible.

To summarize, we have calculated the momentum distributions of B_s and B_s^* mesons produced by $\bar{b}$ quark fragmentation. The measured total production probability was used to determine the strange quark mass parameter. The result for m_s was quite reasonable, being well above the current quark mass value, but lower than values often used for the constituent quark mass. We note that at the scale $2m_s$ that $\alpha_s(2m_s) \approx 0.8$ is uncomfortably large, which could mean there are significant corrections, which have not been calculated, to the fragmentation function at the heavy quark scale, which was calculated using perturbative QCD. But this only affects the normalization and we determined the parameter m_s from the experimental value of the total production probability. Indeed, we repeated the calculation for D_s and D_s^* meson production by fragmentation and found substantially the same value for m_s and verified that the momentum distributions were only moderately sensitive to the value of m_s. The results were also not sensitive to the values assumed for the heavy quark masses or the meson decay constants. Essentially, the experimental production probability fixes the normalization, the perturbative QCD result determines the shape of the momentum spectrum at the heavy quark mass scale, and the Altarelli-Parisi equation governs the evolution up to the scale M_Z; so perhaps it is not so surprising that the results are not too sensitive to the particular values of the parameters.

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The first two errors are the statistical and systematic errors, respectively, while the last error reflects the uncertainty in the absolute $D_s^+ \to \phi\pi^+$ branching ratio.

Figure Captions

1. (a) Momentum distributions for the fragmentation processes $\bar{b} \to B_s$, $\bar{b} \to B_s^*$, and their sum at the scale $\mu_0 = m_b + 2m_s$.

   (b) Momentum distributions for the fragmentation processes $\bar{b} \to B_s$, $\bar{b} \to B_s^*$, and their sum at the scale $\mu = M_Z$.

2. (a) Momentum distributions for the fragmentation processes $c \to D_s$, $c \to D_s^*$, and their sum at the scale $\mu_0 = m_c + 2m_s$.

   (b) Momentum distributions for the fragmentation processes $c \to D_s$, $c \to D_s^*$, and their sum at the scale $\mu = M_Z$.