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MATHEMATICS AND BEAUTY—VIII

TESSELATION AUTOMATA DERIVED FROM A SINGLE DEFECT

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Abstract—To help characterize complicated physical and mathematical structures and phenomena, computers with graphics can be used to produce visual representations with a spectrum of perspectives. In this paper, unusual "tesselation automata" (TA) are presented which grow according to certain symmetrical recursive rules. TA are a class of simple mathematical systems which exhibit complex behavior and which are becoming important as models for a variety of physical processes. This paper differs from others in that its focuses on symmetrical TA derived from a single defect, and reader involvement is encouraged by giving "recipes" for the various chaotic forms which represent a visually striking and intricate class of shapes.

INTRODUCTION

"Some people can read a musical score and in their minds hear the music.... Others can see, in their mind's eye, great beauty and structure in certain mathematical functions.... Lesser folk, like me, need to hear music played and see numbers rendered to appreciate their structures."

P. B. SCHROEDER

Today, there are several scientific fields devoted to the study of how complicated behavior can arise in systems from simple rules and how minute changes in the input of nonlinear systems can lead to large differences in the output; such fields include chaos and tesselation automata (TA) theory. In this paper, I briefly discuss some empirical results obtained by experimentation with a particular class of symmetrical TA. Some of the resulting patterns are reminiscent of the planar ornaments of a variety of cultures (ornaments with a repeating motif in at least two nonparallel directions).

"Tesselation automata" are a class of simple mathematical systems which are becoming important as models for a variety of physical processes. Referred to variously as "cellular automata", "homogeneous structures", "cellular structures" and "iterative arrays", they have been applied to and reintroduced for a wide variety of purposes [1–4]. The term "tesselation" is used in this paper for the following reasons: when a floor is covered with tiles, a symmetrical and repetitive pattern is often formed—straight edges being more common then curved ones. Such a division of a plane into polygons, regular or irregular, is called a "tesselation"—and I have chosen "tesselation" here to emphasize these geometric aspects often found in the figures in this paper.

Usually TA consist of a grid of cells which can exist in two states, occupied or unoccupied. The occupancy of one cell is determined from a simple mathematical analysis of the occupancy of neighbor cells. One popular set of rules is set forth in what has become known as the game of "LIFE" [2]. Though the rules governing the creation of TA are simple, the patterns they produce are very complicated and sometimes seem almost random, like a turbulent fluid flow or the output of a cryptographic system.

The term "chaos" is often used to describe the complicated behavior of nonlinear systems, and TA are useful in describing certain aspects of dynamical systems exhibiting irregular ("chaotic") behavior [5, 6]. Other simple algorithms studied by the author which produce interesting and complicated behavior are described in Ref. [7]. Apart from their curious mathematical properties, many nonlinear maps now have an immense attraction to physicists, because of the role they play in understanding certain phase transitions and other chaotic natural phenomenon [5].

The present paper is number eight in a "Mathematics and Beauty" series [7] which presents aesthetically appealing and mathematically interesting patterns derived from simple functions. The resulting pictures should be of interest to a range of scientists as well as home-computer artists.

MOTIVATION

One goal of this paper is to demonstrate and emphasize the role of recursive algorithms in generating complex forms and to show the reader how to create such shapes using a computer. Another goal is to demonstrate how research in simple mathematical formulas can reveal an inexhaustible new reservoir of magnificent shapes and images. Indeed, structures produced by these equations include shapes of startling intricacy. The graphics experiments presented, with the variety of accompanying parameters, are good ways to show the complexity of the behavior. This paper differs from others in that its focuses on TA derived from a single defect (explained below) using symmetrical rules, and that reader involvement is encouraged by giving "recipes" for the various chaotic forms which represent a visually striking and intricate class of shapes.

METHOD AND OBSERVATIONS

TA are mathematical idealizations of physical systems in which space and time are discrete [1]. Here I present unusual patterns exhibited by figures "growing" according to certain recursive rules. The growth occurs in a plane subdivided into regular square tiles. Note, in particular, that with the rules of growth in this paper, the figures will continue increasing in size indefinitely as time progresses. In each of my cases, the starting configuration is only 1 occupied square, which can be thought of a single defect (or perturbation) in a lattice of all 0s, represented by:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

TA Type 1

This is the simplest system to set up, yet the behavior is still interesting. Given the nth generation, I define the $(n + 1)$th as follows. A square of the next generation is formed if it is orthogonally contiguous to one and only one square of the current generation. Starting with the pattern in equation (1) for $n = 1$ pattern for $n = 2$ would be:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \tag{2}$$

Figure 1 indicates the results at $n = 200$. This TA is similar to that described in Refs [2, 4]. Note that no "death's" of squares occur (i.e. no $1 \rightarrow 0$ transitions can occur; deaths are employed in many CA experiments [2]). Note also that on the four perpendicular axes [which go through $(0, 0)$], all the squares will be present. These are the stems from which branching occurs.

TA Type 2. Time dependence of rules

A. "Mod 2" TA. Given the nth generation, I define the $(n + 1)$th as follows. A square of the next generation is formed if:

1. It is orthogonally contiguous to one and only one square of the current generation for even n (i.e. $n \bmod 2 = 0$).
2. It is contiguous to one and only one square of the current generation, where the local neighborhood is both orthogonal and diagonal, for odd n ($n \bmod 2 = 1$).

In other words, for (n mod $2 = 0$)

$$\text{if } \sum C_{\text{orth}} = 1 \rightarrow C_{ij} = 1, \tag{3}$$

where

$$C_{\text{orth}} = [C_{i,j+1}, C_{ij-1}, C_{i+1,j}, C_{i-1,j}]. \tag{4}$$

For (n mod $2 = 1$)

$$\text{if } \sum C_{\text{orth-diag}} = 1 \rightarrow C_{ij} = 1, \tag{5}$$

where

$$C_{\text{orth-diag}} = [C_{i,j+1}, C_{i,j-1}, C_{i+1,j}, C_{i-1,j}\, C_{I+1,j+1}, C_{i-1,j-1}, C_{i-1,j+1}, C_{i+1,j-1}]. \tag{6}$$

Notice the discrete symmetrical planes running through these TA. For example, see the planes in Figs 2 and 3.

We can use this observation to get a visual idea of resultant patterns, for large n, in a multi-defect system (see TA Type 3).

B. "Mod 6" TA. Given the nth generation, I define the $(n + 1)$th in a same manner as for Type 2A, except that $n \bmod 6 = 0$ vs $n \bmod 6 \neq 0$ determines the temporal evolution of the pattern (Fig. 4).

TA Type 3. Contests between defects

More than one initial defect can be placed on a large infinite lattice. We can let them each grow and finally merge (and compete) according to a set of rules. Figure 5 is a TA of Type 2A, and it shows three defects after just a few generations (this figure is magnified relative to others). Figure 6 shows the growth for large n.

To help see the numerous symmetry planes and to get an idea about the shape of the figure as it evolves, the reader can draw the primary radiating symmetrical discrete planes [see Type 2A] for example, see Fig. 6. For a recent fascinating article on competition of *TA rules*, see Ref. [8] which models biological phenomena of competition and selection by TA "subrule competition".

Type 4. Defects in a centered rectangular lattice

In this type of TA, a single defect is placed in a lattice of the form:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}. \tag{7}$$

This is known as a "centered rectangular lattice" [9]. In some experiments, two different background lattices with adjacent boundaries are used, and the defect propagates from its beginning point in the centered rectangular lattice through the interface into the second lattice defined by:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{8}$$

(known simply as a "rectangular lattice"). Adding a defect to these two-phase systems bears some similarity to seeding supersaturated solutions and watching the crystallization process grow and "hit" the boundary of a solution with a different composition. In the examples in this paper, the two phases are also reminiscent of metal–metal interfaces—such a silicon 100 (centered rectangular) and chromium (rectangular). Note that with no defect present, the rules described have no effect on either lattice! Only when the defect is placed in the lattice does any growth occur.

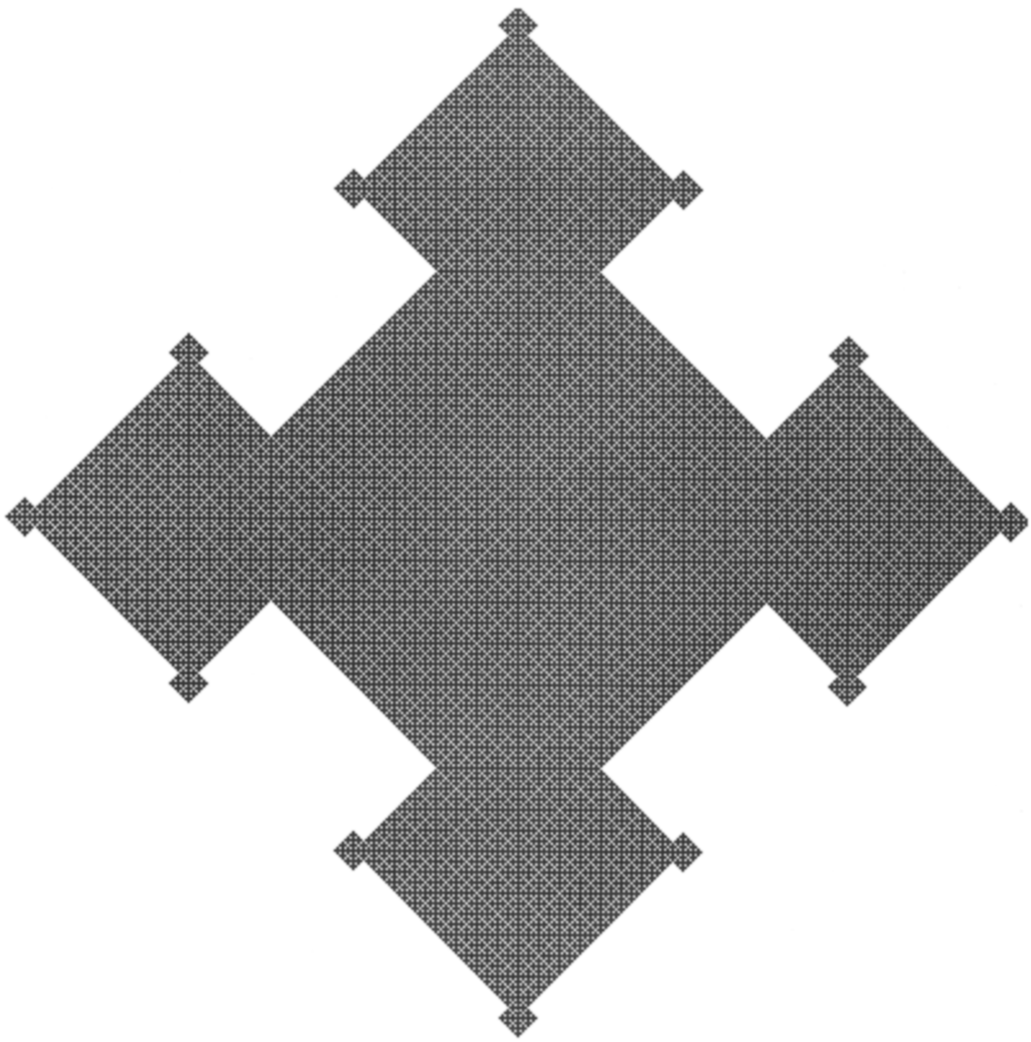


Fig. 1. TA Type 1 "growing" for 200 generations, starting with a single seen in the center of this figure.

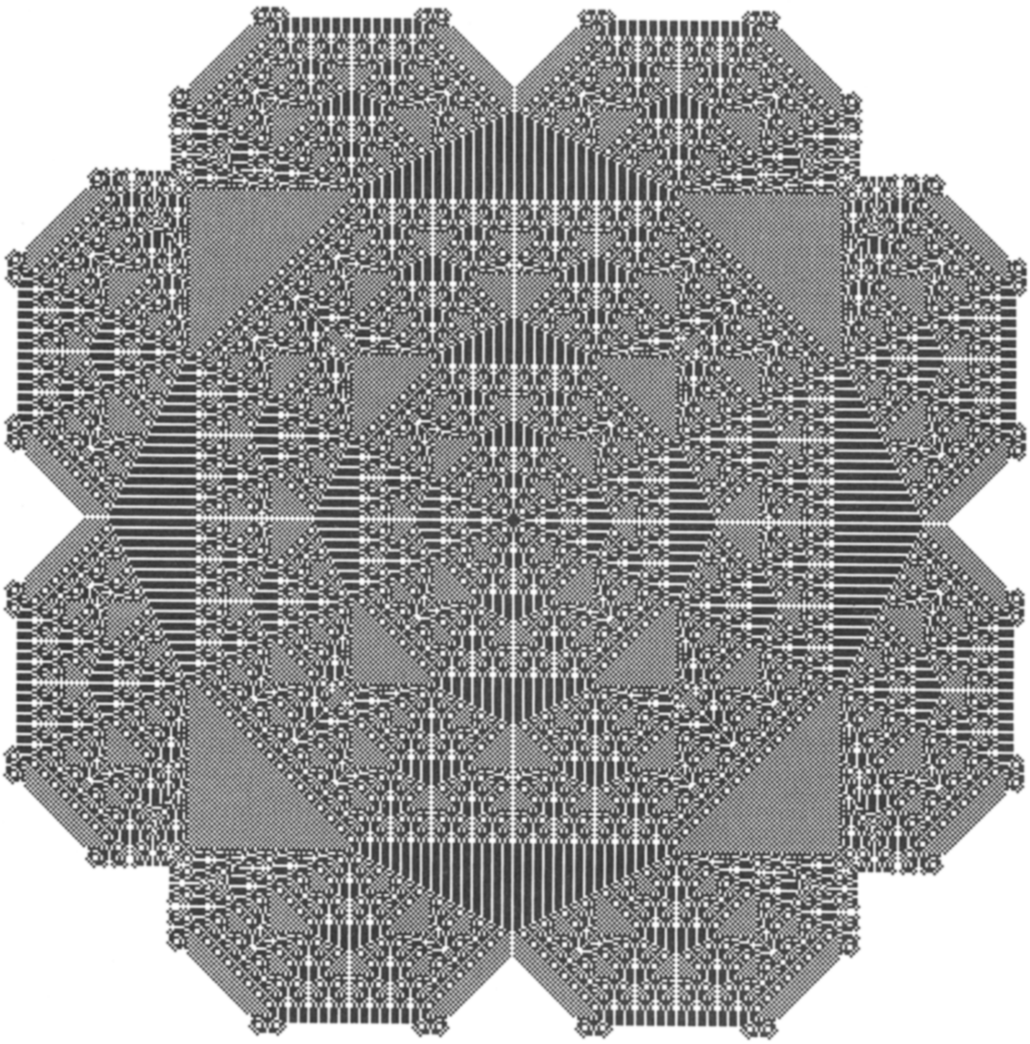


Fig. 2. TA Type 2A. The TA presented here has a time dependency to its rules of growth.

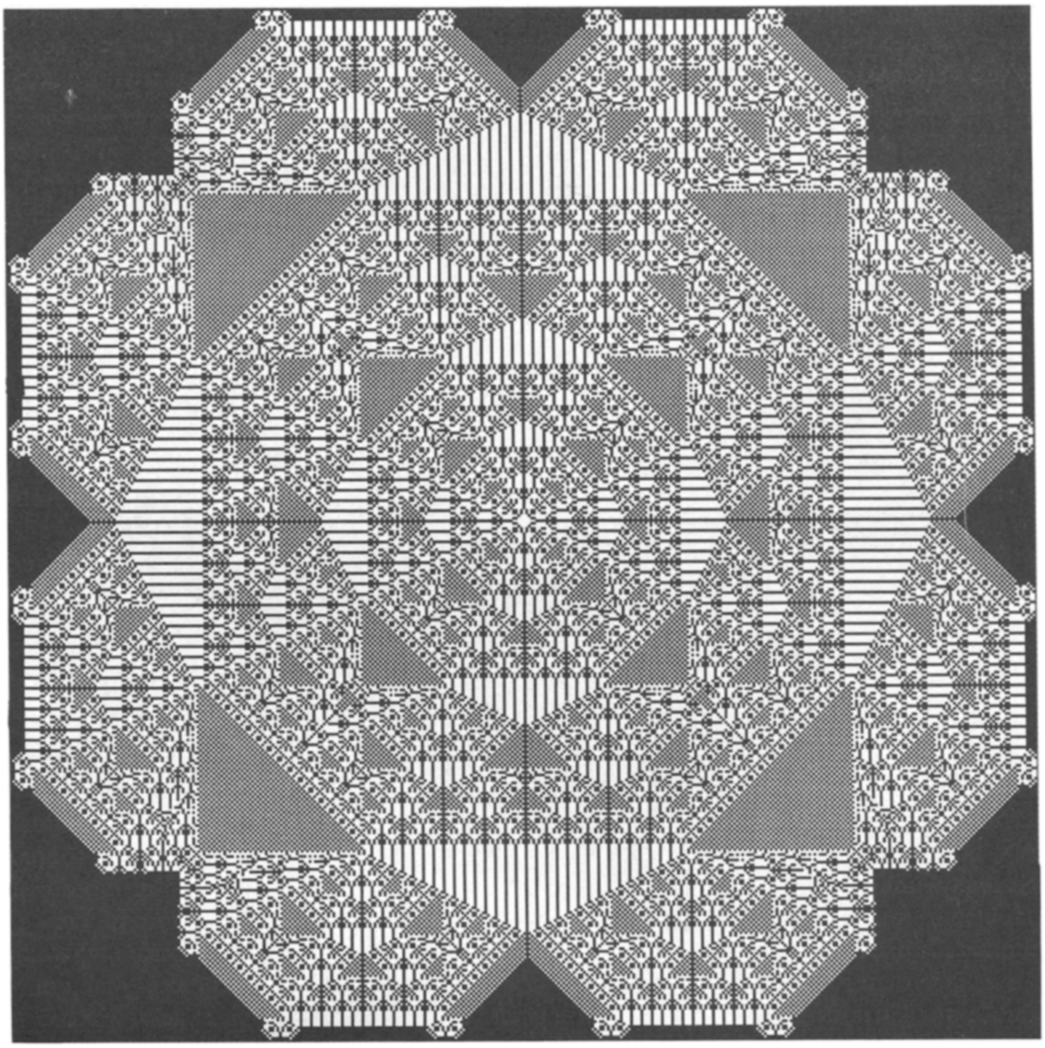


Fig. 3. Same as Fig. 2, but plotted as its negative.

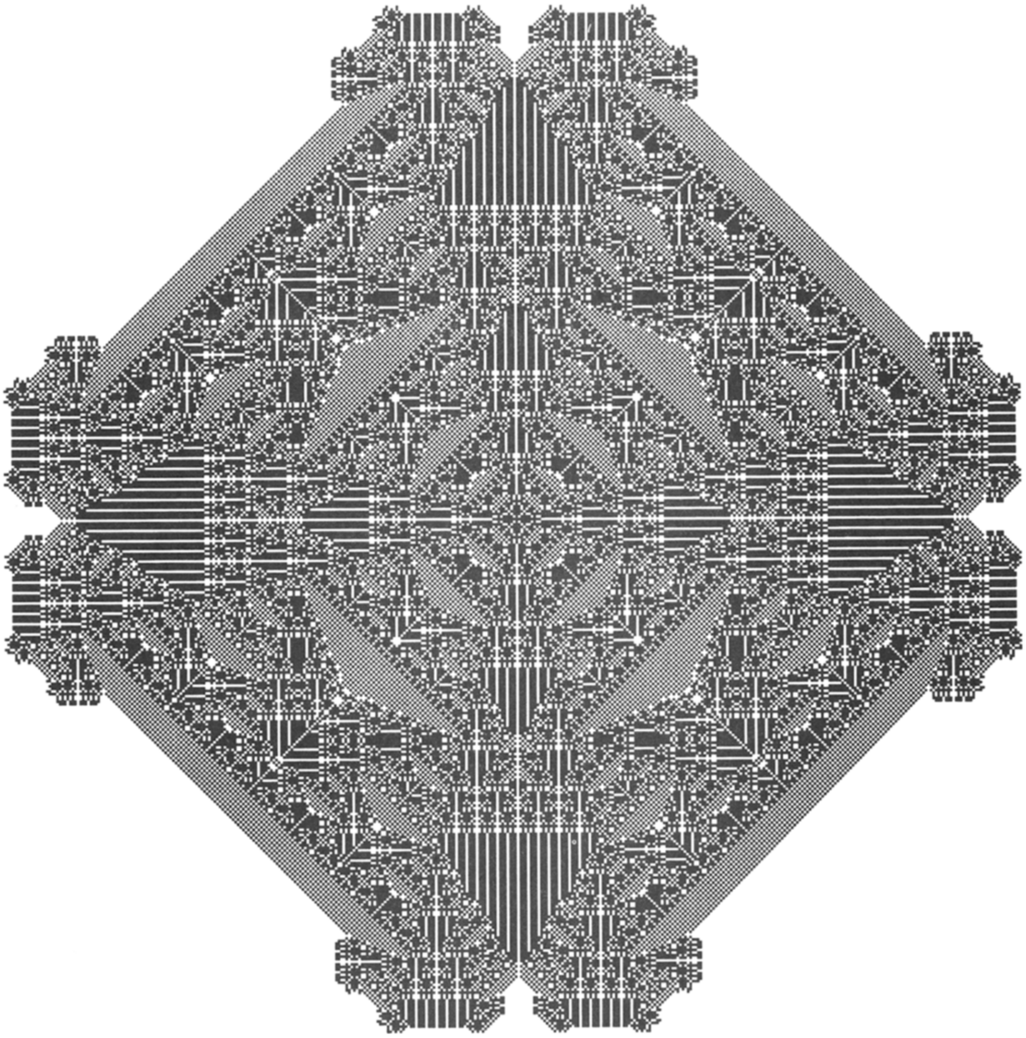


Fig. 4. TA Type 2B, with time dependent growth.

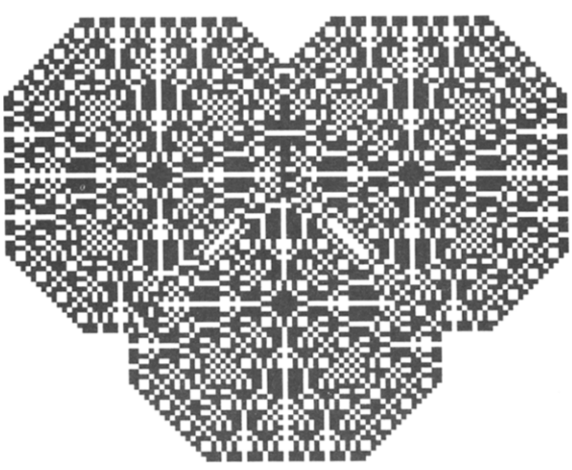


Fig. 5. Multi-defect system composed of three initial seeds of TA Type 2 (figure is magnified relative to Fig. 6).

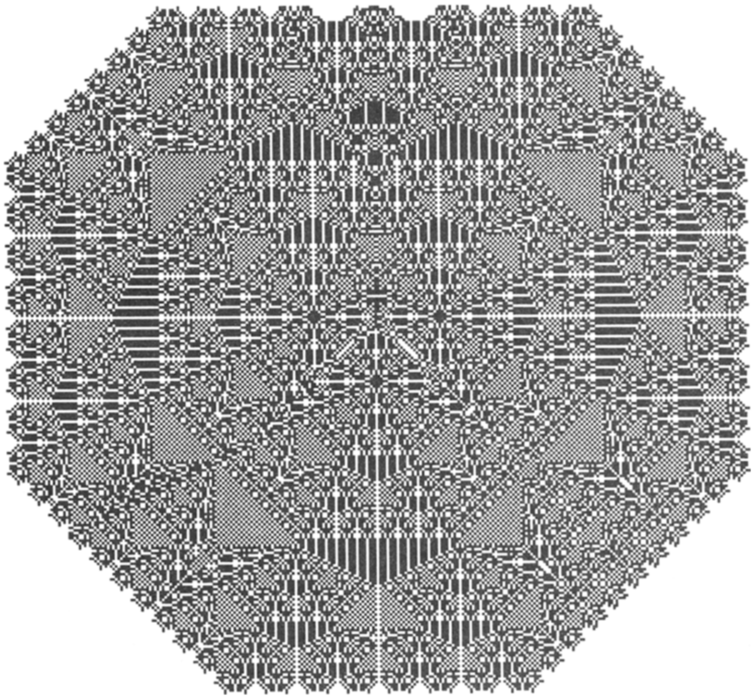


Fig. 6. Same as Fig. 5, except computed for longer time.

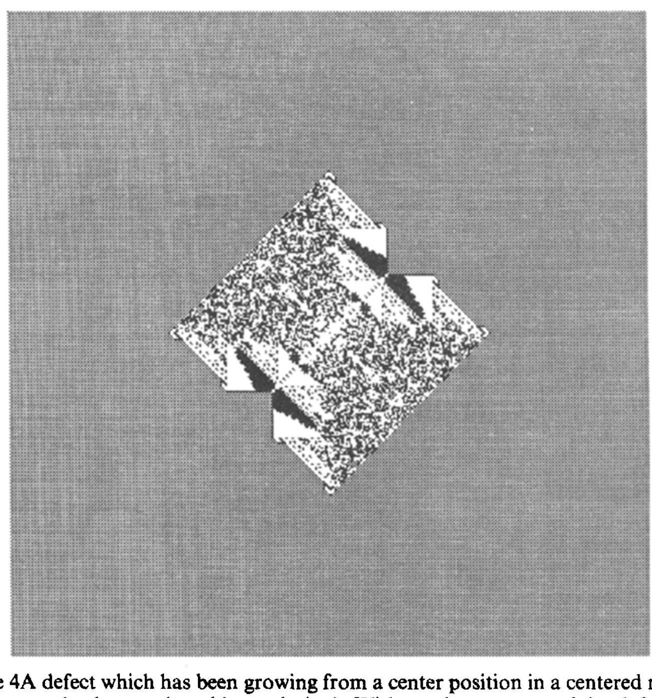


Fig. 7. TA Type 4A defect which has been growing from a center position in a centered rectangular lattice (seen as a diffuse grey background at this resolution). Without the presence of the defect, the rules have no effect on the lattice.

The rules for growth of the defects are as follows (note that deaths of cells can occur in these systems):

A. TA Type 4A.

$$\text{if } \sum C = 3 \wedge \text{ if } \; C_{i,j} = 0 \rightarrow C_{i,j} = 1, \tag{9}$$

$$\text{if } \sum C = 3 \wedge \text{ if } \; C_{i,j} = 1 \rightarrow C_{i,j} = 0, \tag{10}$$

where

$$C = [C_{i,j+1}, C_{i,j-1}, C_{i+1j}, C_{i-1j}, C_{i+1j+1}, C_{i-1j-1}]. \tag{11}$$

The symbol $\wedge$ denotes a logical "and". Figure 7 shows a defect which has been growing from a central position in a centered rectangular lattice (which is seen as a diffuse grey background at this resolution). Figures 8(a)–(d) show the propagation of the defect through a two-phase boundary. Note that the propagation behavior is visually different once in the second layer. For example, notice that the growth in the bottom layer appears to be constrained to planes 0° and 60° with respect to the lattice.

The introduction of "germ" cells appears to be useful in simulating real nucleation processes. An interesting paper in the literature describes solid–solid phase transformations of shape memory alloys, such as Cu–Zn–Al, using a 1-D cellular automata approach [10]. In this investigation, each cell represents several hundred atoms.

The search for multiphase systems, such as the ones in this paper, which are unaffected by a rule system *until* a defect is added, remains a provocative avenue of future research.

B. TA Type 4B. This case (see Fig. 9) is the same as the subset 4A, except that

$$\text{if } \sum C = 3 \wedge C_{i,j} = 1 \rightarrow C_{i,j} = 0 \tag{12}$$

$$\text{if } \sum C = 3 \wedge C_{i,j} = 0 \rightarrow C_{i,j} = 1 \tag{13}$$

$$\text{if } \sum C \neq 3 \wedge C_{i,j} = 1 \rightarrow C_{i,j} = 1 \tag{14}$$

$$\text{if } \sum C \neq 3 \wedge C_{i,j} = 0 \rightarrow C_{i,j} = 0. \tag{15}$$

Type 5. Larger local neighborhood

In TA Types 1–4, the neighborhood was defined as being within one cell of the center cell under consideration. In this system, the local neighborhood is larger. The rule is as follows:

$$\text{if } \sum C = 0(\text{mod } 2) \rightarrow C_{i,j} = 0, \tag{16}$$

$$\text{if } \sum C \neq 0(\text{mod } 2) \rightarrow C_{i,j} = 1, \tag{17}$$

where

$$C = [C_{i-2,j+2}, C_{i+2,j+2}, C_{i,j+1}, C_{i-1,j}, C_{i+1,j}. C_{i,j-1}, C_{i-2,j-2}, C_{i+2,j-2}]. \tag{18}$$

Figures 10(a)–(e) show the evolution of a two-state background defined by the lattices in equations (7) and (8) for several different snapshots in time. Unlike Type 4, the background without a defect is disturbed by this rule-set. Notice the visually unusual behavior of this system with both symmetry and stochasticity present. Also note the interesting growth of the two defects which have been placed next to each other in the top layer.

SUMMARY AND CONCLUSIONS

> "Blindness to the aesthetic element in mathematics is widespread and can account for a feeling that mathematics is dry as dust, as exciting as a telephone took.... On the contrary, appreciation of this element makes the subject live in a wonderful manner and burn as no other creation of the human mind seems to do."
>
> P. J. Davis and R. Hersch

Among the methods available for the characterization of complicated artistic, mathematical and natural phenomena, computers with graphics are emerging as an important tool (for several papers by this author, see Ref. [11]). In natural phenomena, there are examples of complicated and ordered

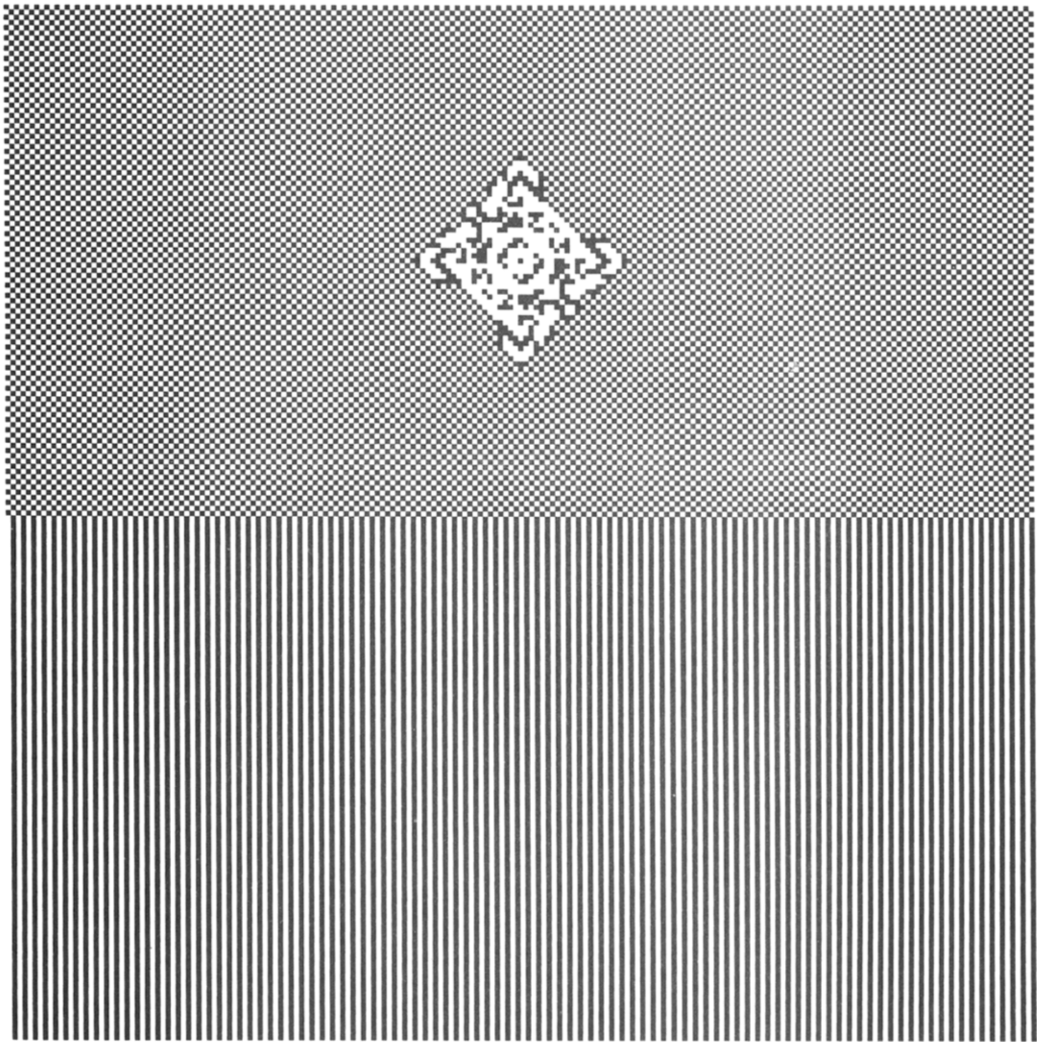


Fig. 8(a). Magnified picture of the beginning of propagation of a Type 4A defect through a two-phase system. The top phase is a centered rectangular lattice, while the bottom phase is a rectangular lattice.

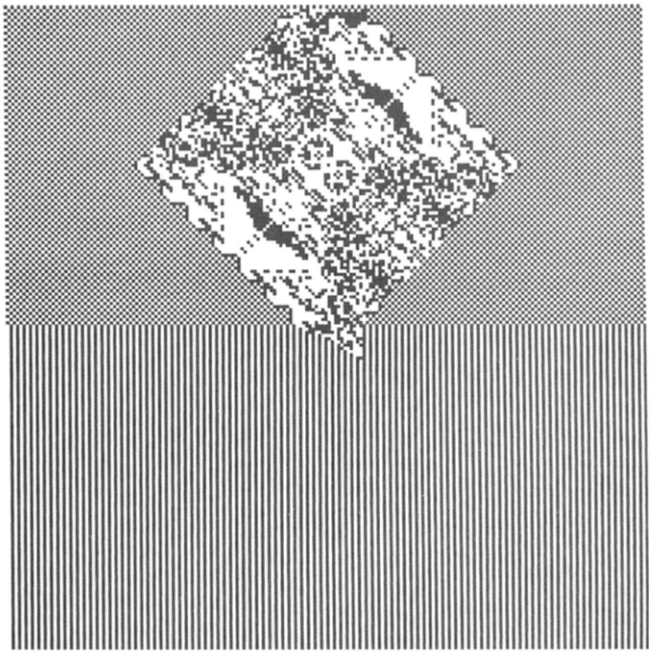


Fig. 8(b). Same as Fig. 8(a) except less magnified and computed for 60 generations. The defect has just "broken through" the boundary.

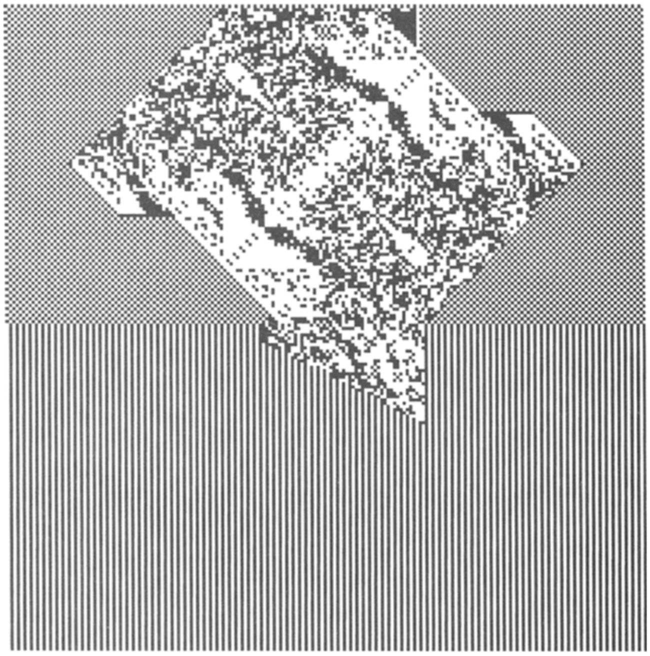


Fig. 8(c). Same as Fig. 8(b), for 80 generations.

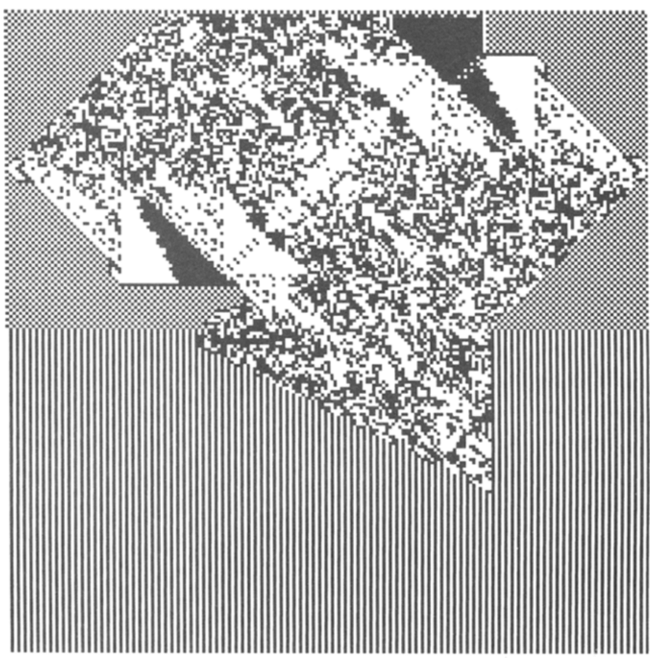


Fig. 8(d). Same as Fig. 8(c), for 100 generations.

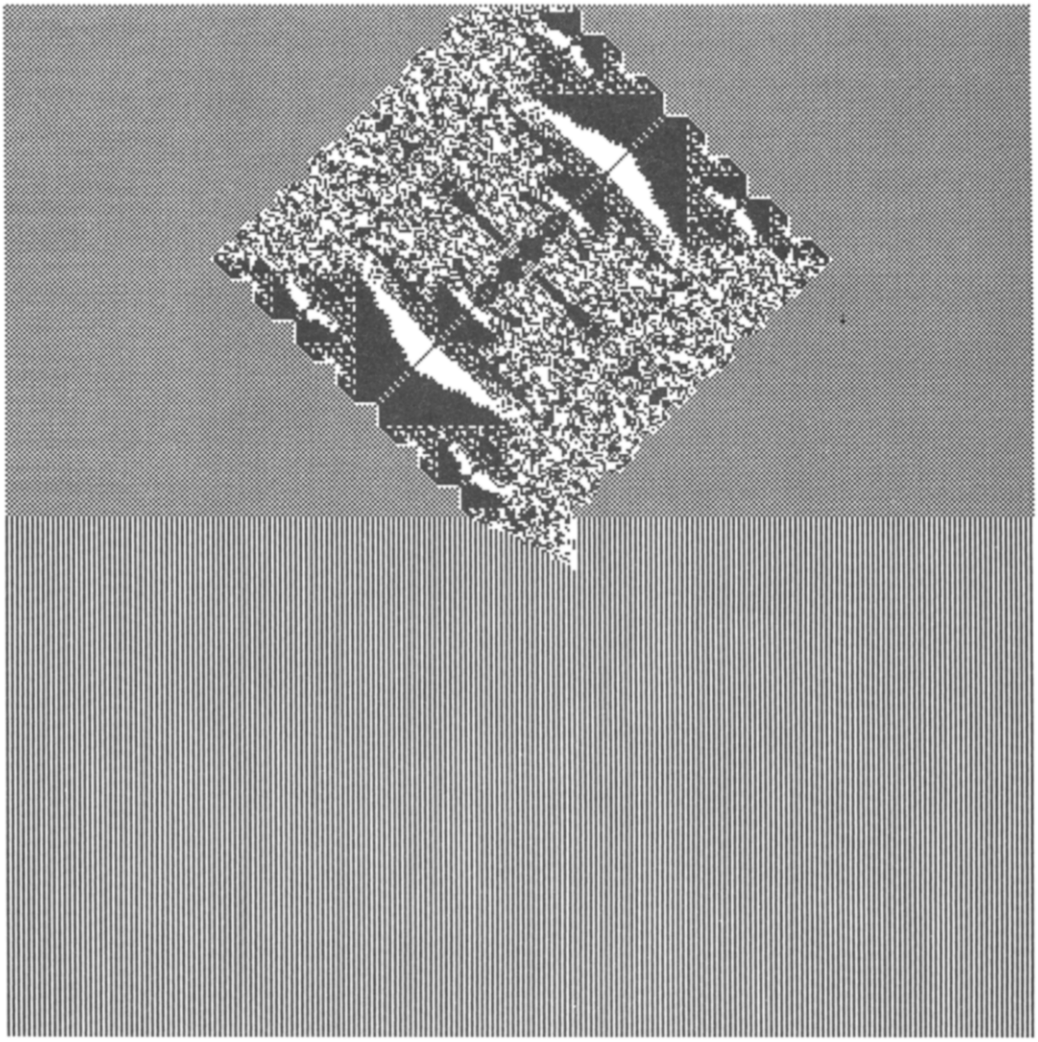


Fig. 9(a). Propagation for TA. Type 4B in a two-phase system ($n = 120$).

structures arising spontaneously from "disordered" states and examples include: snowflakes, patterns of flow in turbulent fluids, and biological systems. As Wolfram points out [1], TA are sufficiently simple to allow detailed mathematical analysis, yet sufficiently complex to exhibit a wide variety of complicated phenomena, and they can perhaps serve as models for some real processes in nature.

In contrast to previous systems where mathematical and aesthetic beauty relies on the use of imaginary numbers [12], there calculations use integers—which also facilitates their study with programming languages having no complex data types on small personal computers. The forms in this paper contain both symmetry and stochasticity, and the richness of resultant forms contrasts with the simplicity of the generating formula. Running TA at high speeds on a computer lets observers actually see the process of growth.

TA portraits contain a beauty and complexity which corresponds to behavior which mathematicians were not able to fully appreciate before the age of computer graphics. This complexity makes it difficult to objectively characterize structures such as these, and, therefore, it is useful to develop graphics systems which allow the maps to be followed in a qualitative and quantitative way. The TA graphics program allows the researcher to display patterns for a specified length of time and for different rule systems.

Some of these figures contain what is known as nonstandard scaling symmetry, also called dilation symmetry, i.e. invariance under changes of scale (for a classification of the various forms of self-similarity symmetries, see the second reference in Ref [12]). For example, if we look at any

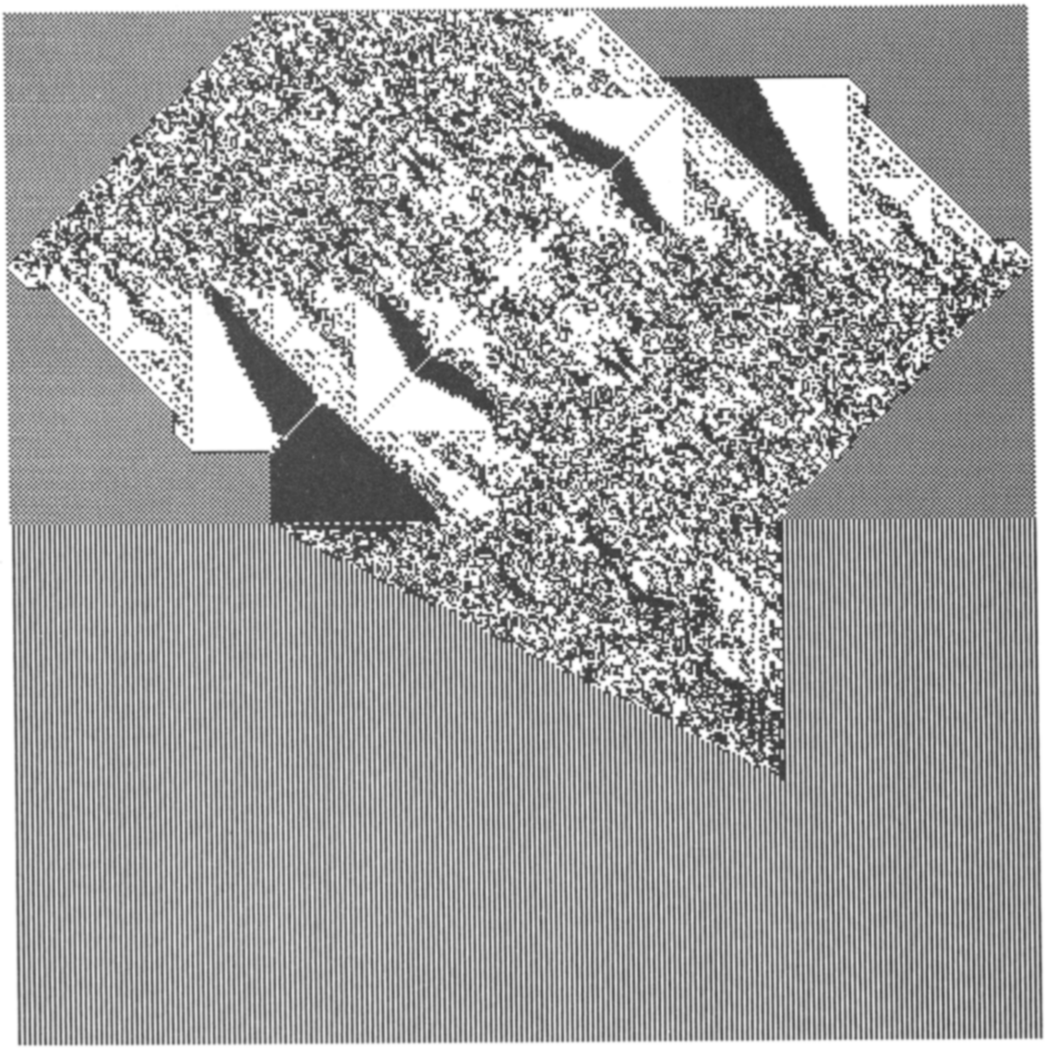


Fig. 9(b). Propagation for TA Type 4B in a two-phase system ($n = 200$).

one of the geometric motifs we notice that the same basic shape is found at another place in another size. Dilation symmetry is sometimes expressed by the formula ($\mathbf{r} \rightarrow a\mathbf{r}$). Thus, an expanded piece of some TA can be moved in such a way as to make it coincide with the entire TA, and this operation can be performed in an infinite number of ways. Other more trivial symmetries in the figures include the bilaterial symmetries and the various rotation axes and other mirror planes in the TA. Note the dilation symmetry has been discovered and applied in different kinds of phenomenon in condensed matter physics, diffusion, polymer growth and percolation clusters. One example given by Kadanoff [13] is petroleum-bearing rock layers. These typically contain fluid-filled pores of many sizes, which, as Kadanoff points out, might be effectively understood as 2-D fractal networks known as gaskets [13], and I would add that TA may also serve as visual and physical models for these types of structures.

These figures may also have a practical importance in that they can provide models for materials scientists to build entirely new structures with entirely new properties [14]. For example, Gordon *et al.* [14] have created wire networks on the micron size scale similar to some of these figures with repeating triangles. The area of their smallest triangle was $1.38 \pm 0.01\ \mu m^2$, and they have investigated many unusual properties of their superconducting network in a magnetic field (see their paper for details).

From an artistic standpoint, TA provide a vast and deep reservoir from which artists can draw. The computer is a machine which, when guided by an artist, can render images of captivating power and beauty. New "recipes", such as those outlined here, interact with such traditional elements as

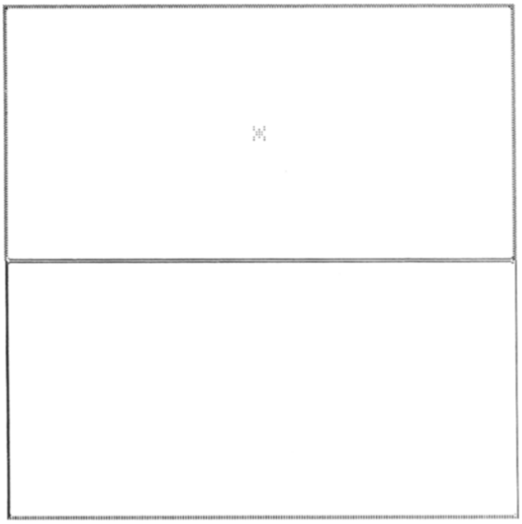


Fig. 10(a). Evolution of a two-state background defined by the lattices in equations (7) and (8). Two adjacent defects have been placed in the top layer, and the result for two generations ($n = 2$) is shown.

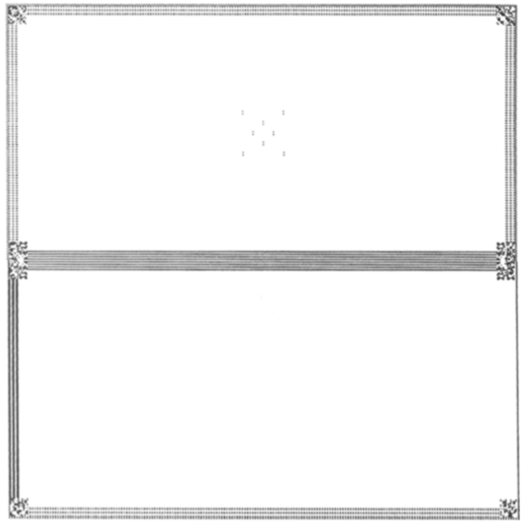


Fig. 10(b). Figure after 8 generations ($n = 8$).

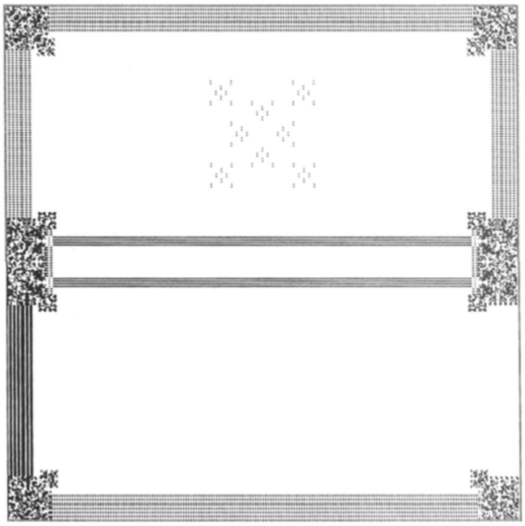


Fig. 10(c). $n = 20$.

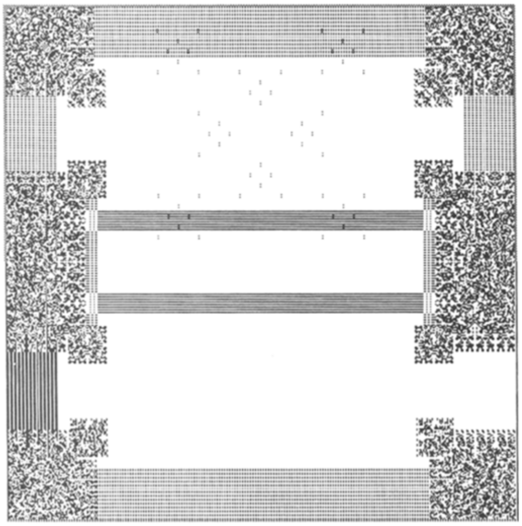


Fig. 10(d). $n = 40$.

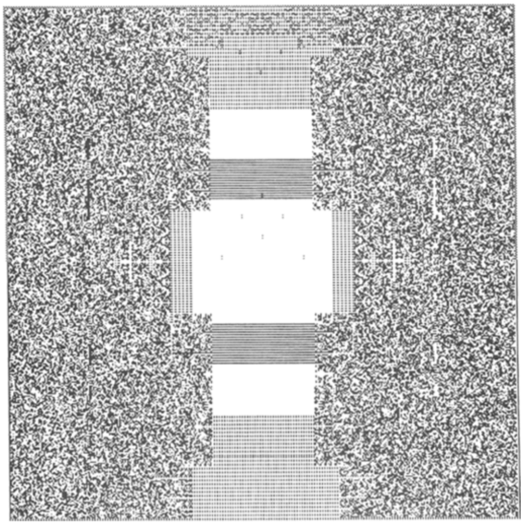
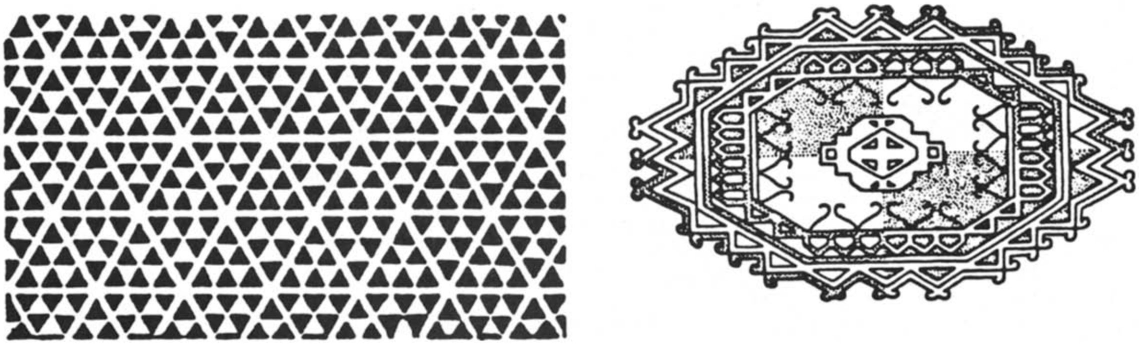


Fig. 10(e). $n = 80$. Note that the patterns, previously well ordered, appears to be on the route to "chaos".

form, shading and color to produce futuristic images and effects. The recipes function as the artist's helper, quickly taking care of much of the repetitive and sometimes tedious detail. By creating an environment of advanced computer graphics, artists with access to computers will gradually change our perception of art.

Also from a purely artistic standpoint, some of the figures in this paper are reminiscent of Persian carpet designs [15], ceramic tile mosaics [15], Peruvian striped fabrics [15], brick patterns from certain Mosques [16], and the symmetry in Moorish ornamental patterns [17]:



Scheme 1

The idea of investigating the ornaments and decorations of various cultures by consideration of their symmetry groups appears to have originated with Polya [15, 18]. This artistic resemblance is due to the complicated symmetries produced by the algorithm, and it is suggested that the reader explore the various parameters to achieve artistic control of the visual effect most desired.

In summary, all the TA shown here have an infinite variety of shapes, and although the equations seem to display what might be called "bizarre" behavior, there nevertheless seems to be a limited repertory of recurrent patterns. A report such as this can only be viewed as introductory. However, it is hoped that the techniques, equations, and systems will provide a useful tool and stimulate future studies in the graphic characterization of the morphologically rich structures produced by relatively simple generating formula.

Acknowledgement—I owe a special debt of gratitude to Charles Bennett for introducing me to TA Type 4B.

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APPENDIX

Recipe for Picture Computation

In order to encourage reader involvement, the following pseudocode is given. Typical parameter constants are given within the code. Readers are encouraged to modify the equations to create a variety of patterns of their own design. Initially, the C array is 0 for all its elements, except for a value of 1 placed in its center. For the program below, a temporary array, Ctemp, is used to save the new results of each generation. The routine below would be called $n = 200$ times in a typical simulation.

```
ALGORITHM:  TA GENERATION (TYPE 2A )
INPUT:      1 DEFECT, Centered in the C array
OUTPUT:     TA PATTERN
TYPICAL PARAMETER VALUES:
            Size = 400
            N is the generation counter - goes from 1 to 200

do i = 2 to size-1;                     (* X - direction          *)
  do j = 2 to size-1;                   (* Y - direction          *)
    if C(i,j) = 0 then do;              (* Test for vacancy       *)
      if mod(n,2) = 0 then              (* Test for even number *)
        sum = C(i,j+1)+C(i,j-1)+C(i+1,j)+C(i-1,j);
      else
        sum = C(i,j+1)+C(i,j-1)+C(i+1,j)+C(i-1,j) +
          C(I+1,j+1) + C(i-1,j-1)+ C(I-1,j+1)+C(i+1,j-1);
      if sum = 1 then Ctemp(i,j) = 1;
    end;                                 (* End j loop            *)
end;                                     (* End i loop            *)
```

Program 1