NEWS & VIEWS

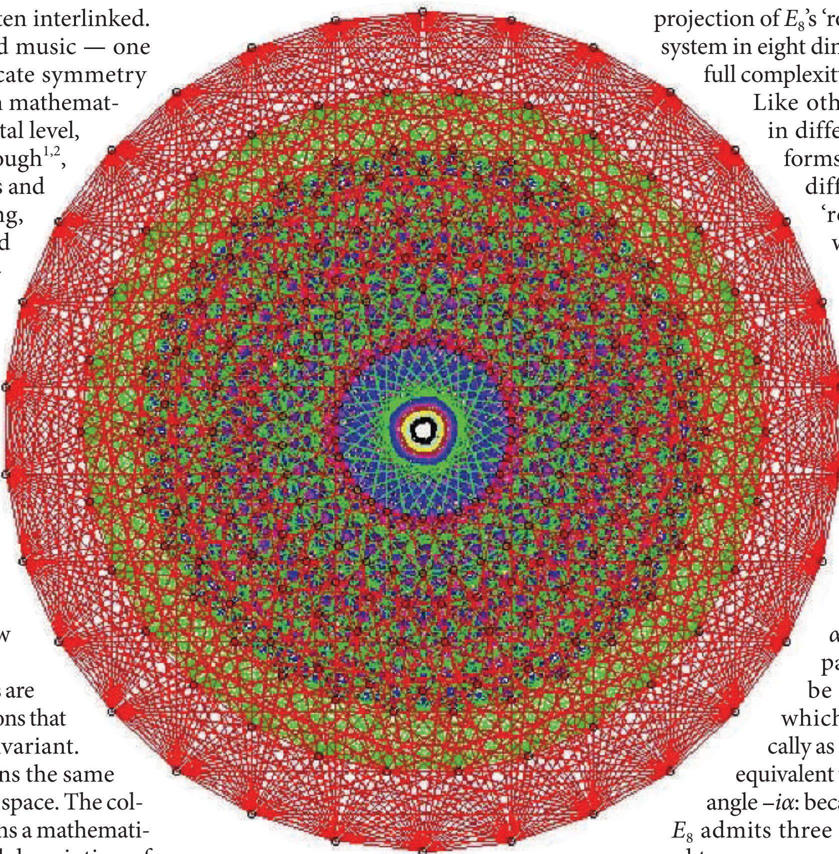
MATHEMATICS

A beauty and a beast

Hermann Nicolai

The mapping of the largest exceptional Lie group, E_8, is a milestone for enthusiasts for the aesthetics of mathematics. But this embodiment of complex symmetry could be of interest to fundamental physics, too.

Symmetry and beauty are often interlinked. This is so not only in art and music — one need only think of the intricate symmetry of a Bach fugue — but also in mathematics and, at its most fundamental level, physics. In a recent breakthrough[1,2], which involved 18 researchers and was four years in the making, mathematicians have mapped out one of the most mysterious and fascinating of all mathematical objects: the 'exceptional Lie group' E_8. In view of the magnitude of the computation and the sheer amount of data involved, the achievement has been likened to the mapping of the human genome[2]. But seeing the beauty in this complex beast can be hard: certainly more difficult than appreciating a Bach fugue without knowing the rules of counterpoint.



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In mathematics, symmetries are usually associated with operations that leave a geometrical object invariant. A sphere, for instance, remains the same under continuous rotations in space. The collection of such operations forms a mathematical 'group'. The mathematical description of continuous symmetries (as opposed to discrete symmetries, such as those that leave a crystal lattice invariant) is codified in the notion of a Lie group, named after the Norwegian mathematician Sophus Lie. Finite-dimensional Lie groups were classified more than a century ago, by Wilhelm Killing and Elie Cartan, by dint of considering only group elements infinitesimally close to identity: that is, to 'rotations' by arbitrarily small angles. Simply put[3], they identified four infinite series of such groups, labelled A_n, B_n, C_n and D_n for $n = 1, 2, 3\ldots$, which essentially correspond to linear transformations in spaces of arbitrary dimension that leave certain quadratic expressions invariant. There are also five exceptional groups that do not fit into these categories, designated G_2, F_4, E_6, E_7 and E_8.

Visualizing rotations in three-dimensional space is straightforward (as it is, with some training in mathematics, in higher dimensions!), but the 'visualization' of exceptional symmetries and their action on geometrical objects is much harder. The results of such attempts are often collectively (and jokingly) referred to as the 'botany' of these Lie groups. For instance G_2, by far the 'easiest' of the exceptional groups, can be defined as the group that leaves invariant the multiplication table of a system of hypercomplex numbers known as octonions. E_8 stands out as the largest and most difficult of the exceptional Lie groups. It has 248 dimensions, and its smallest non-trivial realization requires a space of 57 dimensions[1,2] (see ref. 4 for a physicist's description of this object). In short, E_8 is as intricate as symmetry can get. Pictured here is a two-dimensional projection of E_8's 'root system' — a lattice-like system in eight dimensions that embodies its full complexity.

Like other Lie groups, E_8 comes in different versions, called real forms. Roughly speaking, these differ according to whether 'rotations' are performed with a real-number angle or an imaginary-number angle. More specifically, if it is possible to return to the starting point after a finite rotation, one speaks of a compact realization. A simple example is rotation in space by 360°, which can be represented mathematically through multiplication by $e^{i\alpha}$ with the (real) angle $\alpha = 2\pi$. A simple non-compact transformation would be translation along a line, which is realized mathematically as multiplication by e^{α}. This is equivalent to rotation by an imaginary angle $-i\alpha$: because $i^2 = -1$, then $e^{i(-i\alpha)} = e^{\alpha}$.

E_8 admits three real forms, one compact and two non-compact. Quite generally, the non-compact forms are much more tricky to deal with. This makes the main advance just reported[1] so impressive: it concerns the most subtle of all non-compact forms in Lie-group theory, the 'split-real form' of E_8, sometimes denoted $E_{8(8)}$.

Aside from pure mathematics, what is the wider significance of this achievement? One answer lies in fundamental physics. Symmetry concepts played a central role in the establishment of the two most successful theories of modern physics: general relativity, and quantum-field theory as embodied in particle physics' standard model. In general relativity, symmetry enters through the principle of general covariance: that the laws of physics should not depend on the coordinate system in which they are formulated. This principle enabled Albert Einstein to formulate in one stroke the equations of the gravitational field governing the evolution of the Universe, as well as many other phenomena that would otherwise be intractable (the interaction of light with gravity, for instance).

In the standard model, symmetry is embodied by the principle of gauge invariance, which determines the way in which elementary particles can interact. Given this principle, and the apparatus of modern quantum-field theory, all that is needed to properly formulate the standard model is the specification of the symmetry group, the matter-particle content, and the transformation properties of these matter fields (quarks and leptons) under the chosen symmetry group. Gauge invariance automatically ensures the mathematical consistency ('renormalizability') of the theory, allowing us to extract definite predictions from seemingly infinite expressions, and thus making the standard model one of the best-tested theories of physics.

Yet in spite of their success, neither general relativity nor the standard model can be final theories of physics[5]. This is first of all because of a basic incompatibility between the two theories, reflected in the appearance of 'non-renormalizable' infinities when Einstein's theory is quantized following the standard rules of quantum mechanics. Equally importantly, neither theory is able to answer some obvious questions. For instance, what sets the pattern of elementary particles found in nature apart from other possible such patterns? Similarly, what is so special about the standard model's symmetry group, denoted $SU(3) \times SU(2) \times U(1)$, which seems mathematically undistinguished? And, connected to those questions, how did the Universe, and with it space-time and matter, come into being at the moment of the Big Bang?

To avoid the existing mathematical discrepancies, the yet-to-be-constructed unified theory (sometimes dubbed 'M theory') must be tightly constrained, and possibly even uniquely determined, by symmetry principles. One important difference between Einstein's theory of gravity and the standard model concerns the way in which symmetries are realized. In general relativity, symmetries act in physical space and time, whereas the gauge transformations of particle physics act in an abstract internal space (in which one can, for example, 'rotate' a proton into a neutron and vice versa).

An important step on the long road to a unified theory was the development of supersymmetry, a new kind of symmetry relating the particle groups known as bosons and fermions[6]. This led to supergravity, an extension of Einstein's theory, and superstring theory[7], which is considered by many to be the leading contender to unify physics. Surprisingly, it turned out that the 'most supersymmetric' extension of Einstein's theory — supergravity in 11 space-time dimensions[8,9] — has the split-real forms of E_6, E_7 and E_8 automatically built into it[10], albeit in a rather hidden form.

This seminal discovery was all the more remarkable because it revealed completely unsuspected connections. Who could have anticipated what is, in effect, a link between the esoterics of exceptional Lie groups and the absence of long-range (tensor) forces other than gravity in nature? More recent studies of gauged maximal supergravity theories in three dimensions[11] have confirmed the intimate links between supergravity and the split-real form $E_{8(8)}$.

As yet, we have no idea what the true extent of E_8's involvement in the scheme of things will be. If proponents of superstring theory are right, the compact form of E_8 could be realized as a gauge symmetry in the framework of 'grand unification'. But it is equally possible that E_8 will be realized in a different and more subtle way, intertwining space-time and matter, and possibly involving the split-real form, rather than the compact form.

The ambitious search for a fundamental symmetry of nature might even force us to venture into the unknown territory of infinite-dimensional exceptional symmetry groups, of which the finite-dimensional E_8 is just a subset. The prime candidate is E_{10}, about which we know next to nothing, other than that it exists. Physicists should not let themselves get carried away by these intriguing possibilities, as experiment remains the final arbiter. But they would be well advised to take note of the exciting developments[1] in deciphering the E_8 group. ■

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CHEMICAL BIOLOGY

Ignore the nonsense

Anton Schmitz and Michael Famulok

A small molecule forces the protein-translation machinery to overlook the signals that would otherwise result in its premature termination. Genuine stop signs are, however, read and obeyed.

Several inherited diseases are caused by mutations in single nucleotides within genes. These mutations can transform the products of messenger RNA codons, the sets of three nucleotides that determine which amino acid is incorporated into the growing protein chain. When such 'nonsense' mutations are transcribed into a 'stop' codon, the cellular machinery that translates mRNA into protein misinterprets the codon as a signal to terminate protein synthesis. These false stop codons are known as premature termination codons (PTCs) and result in the formation of truncated proteins that cannot function properly and may even damage the cell, eventually leading to disease. Depending on the disorder, nonsense mutations account for 5–70% of cases of genetic disorders, including cystic fibrosis, muscular dystrophy and several types of cancer. On page 87 of this issue, Welch *et al.*[1] report that a small organic molecule known as PTC124 can force the translation machinery to ignore PTCs, without preventing it from reading the real stop signals*.

It has been known for the past 10 years that the antibiotic gentamycin can prompt ribosomes — the core component of the cellular protein-synthesis machinery — to read through PTCs, thereby generating full-length proteins[2]. Nevertheless, the clinical benefit of gentamycin is limited, because to be effective it has to be used at very high concentrations, which are associated with severe side effects. There is now hope that PTC124, which, like gentamycin, ignores PTCs but lacks its adverse side effects, could be more beneficial in the clinic. Indeed, interim results of phase II clinical trials[3] indicate that patients with PTC-induced forms of cystic fibrosis and Duchenne muscular dystrophy might benefit from treatment with PTC124 — a promising result that has been commented on for some time[4,5].

Welch *et al.*[1] describe an astonishing feature of PTC124 — its selectivity for PTCs. Why is this so striking? All organisms with membrane-bound cell nuclei (eukaryotes) have evolved mechanisms to protect themselves from the harmful products of nonsense mutations. There are two lines of defence. The first relies on the fast and efficient degradation of the truncated proteins after the translation of PTC-containing mRNAs. The second acts before these proteins are synthesized. This quality-control mechanism, known as

*This article and the paper concerned[1] were published online on 22 April 2007.