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The α-Beauty Contest: Choosing Numbers, Thinking Intervals

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The α-Beauty Contest: Choosing Numbers, Thinking Intervals*

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Abstract

The 1-shot α-beauty contest is a strategic game under bounded rationality conditions, where equilibrium is approached if the game is played iteratively sufficiently many times. Experimental data of the 1-shot setting show a common pattern: The spectrum of announced numbers is a superposition of a skew background distribution and a regime of extra ordinarily often chosen numbers. Our model is capable of quantitatively reproducing this observation as well as the convergence towards equilibrium in the iterative setting. The approach is based on two basic assumptions: 1.) Players iteratively update their recent guesses in the sense of eductive reasoning and 2.) Players estimate intervals rather than exact numbers to cope with incomplete knowledge in non-equilibrium. The width of the interval is regarded as a measure for the confidence of the players' respective guess. It is shown analytically that the sequence of guessed numbers approaches a (finite) limit within only very few iterations. Moreover, if all players have infinite confidence in their respective guesses, the asymptotic Winning Number equals the rational Nash equilibrium 0, while if players have only finite confidence in their recent guess, the Winning Number in the 1-shot setting is strictly larger than 0. Our model is also capable of quantitatively describing the "path into equilibrium". Convergence is shown to be polynomial in the number of rounds played. The predictions of our model are in good quantitative agreement with real data for various α-beauty contest games.

Keywords: Experiments, α-Beauty Contest, Beliefs.

JEL Classification Numbers: C91, D84.

SSRN Classification: Behavioral Finance; Experimental Studies.

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1 Introduction and Overview of Results

Ever since Keynes 193 the α-beauty contest has served as an example for a strategic game under bounded rationality conditions to cover elementary features of price formation. The α-beauty contest game is described by Nagel, Bosch-Domenech, Satorra, and Garca-Montalvo as follows. "A certain number of players each chooses simultaneously a decimal number, let us say, from the interval [, 1]. The winner is the person whose number is closest to α times the mean of all chosen numbers, where $\alpha <$ 1 is a predetermined and known number. The winner gains a xed prize. If there is a tie, the prize is split amongst those who tie or a random draw decides the winner.
Being in an equilibrium would imply that each player s belief is consistent with what all the other players actually plan to choose. In fact, as Nagel 199 clearly demonstrated, the game theoretic Nash equilibrium is generally not observed in the 1-shot "α-beauty contest , rather than is approached after su ciently many rounds in the iterative setting, i.e. in the presence of communication.

Field experiments were conducted to estimate the behavior of probands when playing the Guessing Game in di erent settings Nagel 199 , Nagel and D y 199 . The number of probands in controlled laboratory experiments is naturally constrained to an order of magnitude of 1 only. Hence corresponding outcomes are strongly in uenced by individual behavior and may not cope for typical properties generated by the game. Since we are interested in such typical properties, we rst refer to studies with a much larger set of players such as the newspaper experiments conducted by Nagel, Selten, and Thaler. For an overview, see Nagel, Bosch-Domenech, Satorra, and Garca-Montalvo , where also a survey of 4 experiments on the 1-shot Guessing Games can be found. Figures 1 and are two samples obtained from an experiment done by the German newspaper "Die Zeit in and the Spanish newspaper Expansion, respectively Nagel, Bosch-Domenech, Satorra, and Garca-Montalvo .

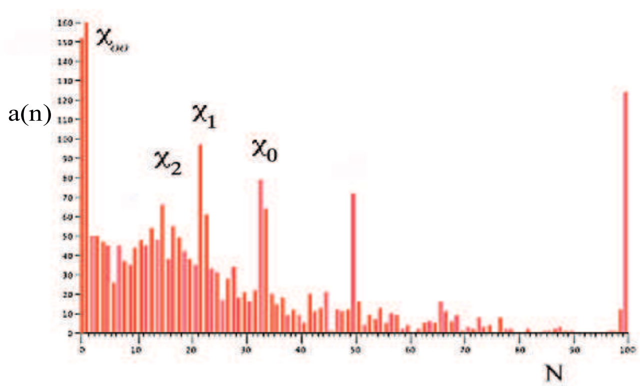

Figure 1 *The 1-shot Guessing Game played by approximately 3000 probands. Data are from the experiment done by the German newspaper "Die Zeit" in 2002.*

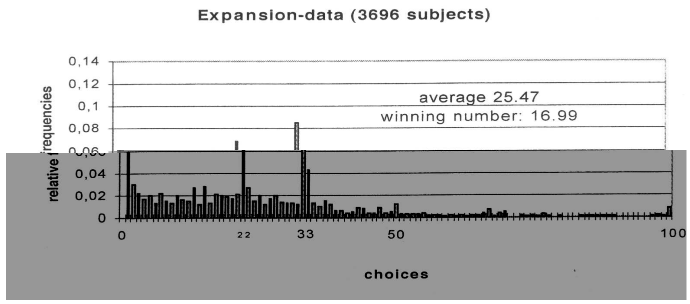

Figure *The 1-shot Guessing Game played by the Spanish newspaper Espansion. Data are taken from Nagel, Bosch-Domènech, Satorra, and Garca-Montalvo (2002).*

Game Theory predicts that if all players had identical beliefs and were perfectly rational, and if each player knows that all others were also perfectly rational, then would be the unique "stable" solution, in the sense of a game theoretic Nash equilibrium. Hence, under these conditions, it wold be rational to choose as the number to be announced. As experiments show, most players do not behave according to this. In fact, if only one player assumes that at least one player is irrational, then it is rational not to choose as the proposed Winning Number.

The rational regime in Figure 1, α /3 ϵ									
k		1		3	4		6		8..∞
χ_k	33		14	9	6	4		1	

Table 1 *The rational regime calculated according to our model, see next section.*

There are three main observations drawn from experimental data The *first observation* is that the Winning Number observed is approximately equal in different "comparable" experiments. The *second observation* is that the spectrum of numbers announced is a superposition of a broad and highly skewed distribution and a collection of numbers $\{\chi_1, \chi_2, ..\}$, which are extraordinarilly often chosen. We call the set of these numbers the "rational regime" (see Table 1). While the skew distribution reflects the players' uncertainty about other players' rationality, the rational regime represents different depth of thinking of players. The *third observation* is that equilibrium is established after a sufficient number of iterative rounds (see Figure 3). A reasonable model has to consistently explain these facts.

We start by considering the 1-shot game setting. The strategy chosen by one individual crucially depends on her guess about strategies of the others, e.g. on the distribution of strategies played at that time by others. Because of the lack of mutual consistency away from equilibrium, individual beliefs might differ from each others actual plans and are strongly based upon their individual recent hypothesis. Hence it is natural to assume that all players have minimal prejudice about other players guesses. In this situation, strategies and thus guesses are expected to be heterogenous over the players' population and context dependent. Authors including Nagel (1995), Stahl and Wilson (1995), Ho, Camerer, and Weigelt (1998), Costa-Gomes, Crawford, and Broseta (2001) proposed the idea that all k-step players think that all others are $k-1$ step players. This assumption was weakened by Camerer and coworkers (see for a review Camerer (2003)), claiming that all k-step players have an accurate guess about the relative proportion of players who are thinking $k' < k$ levels. Camerer proposed that this distribution is Poisson with intensity τ. This assumption also allows for taking into account that there are other players performing at most the same levels of thinking (Camerer, Ho, and Chong (2003)). It was found that τ varies between 0.1 and about 3.6 in non-equilibrium games, while most of players perform 1.6 thinking steps.

In most approaches numbers, let them be integers or reals, are considered as the fundamental entities for individual choice of strategies. It is well-known that the result of a calculation essentially depends on the choice of the set of elements, the corresponding operation is defined on. Of course it is not canonical that numbers have to be chosen for computation. Other entities might be considered. Intervals rather than numbers might be chosen to scope with the uncertain knowledge about others away from equilibrium. This fact invokes the key hypothesis of our model

"Strategy choices rely on estimates on INTERVALS rather than NUMBERS!"

A strategy thus is to choose an interval within the range of an admissible number according to which then the guess is calculated in the sense of a best reply. As an example If a player guesses that the Winning Number is about 30 then the interval chosen might be [20, 40]. Without any further knowledge, the assumption is that each agent has minimal prejudice in that all numbers in this interval are regarded equally probable, while also all admissible intervals are equally likely. This assumption represents the rational guess of a player that all others are bounded rational or even irrational (Plot 1993).

We will propose an elementary model for the 1-shot setting of the Guessing Game which is played by a large number of subjects performing k levels of thinking (for details see Section 2). Thereby we allow for infinite many levels of thinking, because a limit concerning depth of thinking is neither known nor a priori given. Fundamental assumptions are

(1) *Each individual estimates intervals rather than numbers*!
This assumption is actually an assumption about the choice of strategies, see above

ii) *Each individual successively updates his guessed number according to his recent believe and hence generates an infinite sequence of guessed numbers during k iterations!* This is actually to assume the mechanism of eductive reasoning.

Our model has explicit parameters M, N and α, where M is the number of players, N characterizes the set of alternatives and α is often is chosen to be $1/$ or $/3$ as well as an implicit one ϵ. This parameter ϵ can be regarded as a measure for the confidence of an agent in her recent guess in the sense that the larger ϵ is, the less confident is the player. As we will discuss in the next Section, the parameter ϵ is introduced to explain the skew distribution of guessed numbers observed in the experimental results. It is shown under very weak assumptions, i.e. $\leq \alpha$ and $\leq \epsilon \leq 1$ that the expected (asymptotic) Winning Number y^* in the 1-shot game yields

$$y^* \quad \frac{N}{\ } \; \frac{\epsilon\beta}{(\ -\beta)(\ -\epsilon)}, \qquad \leq \alpha, \epsilon \leq 1$$

where $\beta \quad \alpha \frac{M-1}{M-\alpha}$ is approximatively equals to α for M, the number of players, large enough. The asymptotic expected winning number $y^\star$ is therefore unequal to unless $\alpha \quad$ or $\epsilon \quad$, while $y^* \quad N/$ if $\alpha \quad 1$ and $\epsilon \quad$.

It is shown that the sequence of guessed numbers converges to a stationary value, see Proposition 1. This result was first obtained by Reimann (4) for a discrete set of alternative and unlimited many players. It is shown that the asymptotically expected guessing number is strictly larger than if and only if intervals have positive width. The individual width can be regarded as a measure for the confidence a player has in her guess, i.e. the smaller the interval width the more confident is the player that her guess is correct. Hence, if individuals have a finite degree of confidence in their individual belief, i.e. the intervals chosen have non-zero width, the system reaches a stationary state which is not the game-theoretic equilibrium. The Nash equilibrium is obtained in a 1-shot setting if and only if the width of the confidence interval is . In other words the Nash equilibrium would be played if and only if all players would assume that all other players are also rational. The convergence is rapid, i.e. within only 4 thinking steps the guessed number (approximately) equals the corresponding stationary value for usual choices of the parameter α and ϵ. This property of rapid convergence allows us to consider the iterative game setting using the model proposed for the 1-shot setting. We further deduce the distribution of numbers announced in an ensembles of players with heterogenous believes.

A reasonable model of the α-Beauty Contest must also explain the path towards equilibrium. In the second part of the paper we extend the previous model of the 1-shot game setting towards an iterative setting with t rounds. In the iterative setting, the game is played for several rounds - with the same rules - , while between rounds the Winning Number of the respective last round is made public, i.e. before playing the next round

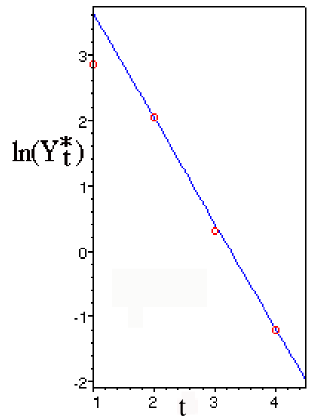

Figure 3: *The iterated Guessing Game: Experimental data (red circles) show a log-linear relationship (blue line) between the Winning Numbers announced after each round and the number of rounds. The data are from an experiment conducted at the University of Bergen, Norway, 2003.*

all players are informed about the resulting Winning Number of the previous round. If agents would be completely rational, communication would not have any impact. In the contrary, experiments show that communication does have a strong impact in that during only a very few rounds the Winning Number drops down to 0. This convergence should not be confused with the convergence within thinking steps in the 1-shot setting. In fact, the iterative game has two dynamical levels: the “fast” dynamics within one round and k levels of thinking and the “slow” dynamics over rounds. Since convergence in thinking levels steps is fast, it is a good approximation to assume that at the end of each round the corresponding stationary value is announced. Our model predicts that in an iterative setting the sequence of Winning Numbers drops to 0 rapidly according to

$$\ln y^*(t) = \ln y_0^* + t \ln\left(\frac{\epsilon}{1 - \beta(1 - \epsilon)}\right).$$

This linear relationship is in good agreement with real data, as shown in Figure 3 which presents results from the iterated Guessing Game conducted at the University of Bergen, Norway.

2 Outline of the basic model

The setting is as follows. There are $\mu = 1, \ldots, M < \infty$ agents or players, who choose a number $x^\mu \in \mathcal{A} = [0, N]$, where $N < \infty$, to guess the average number $y = \alpha \frac{1}{M} \sum_{\mu=1}^{M} x^\mu$ – called the Winning Number –, where $\alpha \in [0, 1]$ is fix and known to all players. We call $\mathcal{A}$ the set of alternatives. Let us consider a representative player μ. For given choices $x^1, \ldots, x^{\mu-1}, x^{\mu+1}, \ldots, x^M$ of players $\nu \neq \mu$, the optimal response of player μ is

$$x^\mu = \frac{\alpha}{M-\alpha} \sum_{\nu \neq \mu} x^\nu = \alpha \frac{M-1}{M-\alpha} \overline{x}^{(-\mu)},$$

where $\overline{x}^{(-\mu)} = \frac{1}{M-1} \sum_{\nu \neq \mu} x^\nu$, such that

$$x^\mu = y,$$

i.e. with her best response player μ announces exactly the Winning Number y.

Note that for M large enough, $\frac{M-1}{M-\alpha} \approx 1$ and thus $x^\mu \approx \alpha \overline{x}^{(-\mu)}$. Naturally, player μ does not have any information about other players choices, since she must announce her number simultaneously to the others. Therefore, she has to build beliefs about other players' choices. Let $(\Omega, \mathcal{F})$ be a measurable space. The random variable $X^\nu : \Omega \to \mathcal{A}$ for $\nu \in \{1, \ldots, M\}$ denotes players' μ choice in $\mathcal{A}$. The random variable $\overline{X} = \frac{1}{M} \sum_{\mu=1}^{M} X^\mu$ is the average over all announced numbers and $Y = \alpha \overline{X}$ is the Winning Number. Moreover, $\overline{X}^{(-\mu)} = \frac{1}{M-1} \sum_{\nu \neq \mu} X^\nu$ and $Y^{(-\mu)} = \alpha \overline{X}^{(-\mu)}$. We introduce the following definition.

Definition 1 (Belief). *The probability measure $\mathbb{P}^\mu$ on $(\Omega, \mathcal{F})$ is the belief of player $\mu \in \{1, \ldots, M\}$. $F^{\mu,\nu}$ denotes the cumulative distribution function of player ν's choice X^ν under player μ's belief $\mathbb{P}^\mu$, for all $\nu, \mu \in \{1, \ldots, M\}$.*

The probability measure $\mathbb{P}^\mu$ is an abstract way of characterizing players' believes concerning the state of nature driving players choices. Suppose for example the only two states of nature ω_1 and ω_2 exist, and that all other players choose the number if the state ω_1 occurs and the number N if the state ω_2 occurs. Then each player's believes is the probability associated to each state of nature. If one player believes the ω_2 occurs with probability 1, then she would announce the number αN. It is to note that players could have different beliefs on the state of nature. Nevertheless, to make the model tractable and to allow us to consider a representative player in the derivation of our results, we make the following simplifying assumptions.

Assumption 1. *Under $\mathbb{P}^\mu$, $X^1, \ldots, X^{\mu-1}, X^{\mu+1}, \ldots, X^M$ are independent and identically distributed for all $\mu \in \{1, \ldots, M\}$, i.e. other players' choices are independent under each player's belief and have the same distribution functions. Moreover,*

$$X^\mu = \alpha \frac{M-1}{M-\alpha} \overline{X}^{(-\mu)}.$$

We assume that players use an *iterative thinking process* to guess the numbers which will be announced by other players. We add an index k to our previous notation to indicate that we are considering the thinking process at step k, i.e. X_k^ν for $\nu \in \{1, \ldots, M\}$

is the number that player ν would announce at step k of her thinking process, $\overline{X}_k$ is the average over the X_k^νs and $\overline{X}_k^{(-\mu)}$ is the average over the X_k^νs for $\nu \neq \mu$. Moreover, F_k^μ denotes the cumulative distribution function of X_k^ν under $\mathbb{P}^\mu$ for $\mu, \nu = 1, \ldots, M$, where Assumption 1 is supposed to hold also for $X_k^1, \ldots, X_k^{\mu-1}, X_k^{\mu+1}, \ldots, X_k^M$. Let us consider player's μ thinking process.

STEP 0 First, player μ assumes that all alternatives in $\mathcal{A}$ are equally probable for all other players, i.e. F_0^μ corresponds to the uniform distribution on $[0, N]$. Under this assumption, the average number $\overline{X}_0^{(-\mu)}$ is the weighted sum of $M-1$ independent uniformly distributed random variables on $[0, N]$ and

$$y_0^\star = \mathbb{E}^\mu\left[X_0^\mu\right] = \alpha \frac{M-1}{M-\alpha} \frac{N}{2},$$

$$\left(\sigma_0^\star\right)^2 = \mathrm{Var}^\mu\left(X_0^\mu\right) = \alpha^2 \left(\frac{M-1}{M-\alpha}\right)^2 \frac{N^2}{12}.$$

$\mathbb{E}^\mu[\cdot]$ and Var^μ are the expectation and the variance respectively, under $\mathbb{P}^\mu$. Note that $y_0^\star$ and $\sigma_0^\star$ do not depend on μ.

STEP 1 In step 1, player μ takes into account the expected guessing number $y_0^\star$ she obtained from step 0. She considers an interval $\mathcal{I}_1^\mu$ around this number and assumes that the X_k^νs are independent distributed on $\mathcal{I}_1^\mu$ for all $\nu \neq \mu$. Here, the reference probability measure is $\mathbb{P}^\mu$. More precisely, player μ believes that for $\nu \neq \mu$ and conditioning on *unknown* realizations l_1^μ, u_1^μ of independent random variables $L_1^\mu \sim \mathrm{unif}\left(0, y_0^\star\right)$ and $U_1^\mu \sim \mathrm{unif}\left(0, N - y_0^\star\right)$ respectively, the interval is

$$\mathcal{I}_1^\mu\left(l_1^\mu, u_1^\mu, \epsilon\right) = [y_0^\star - \epsilon\, l_1^\mu, y_0^\star + \epsilon\, u_1^\mu] \subset [0, N],$$

where $\epsilon \in [0, 1]$ is fix and known to all players. Later we will provide a simulation result, where it is assumed that ϵ is stochastically in $[0, 1]$, with known distribution function. Moreover, here it is assumed that the X_1^νs are conditionally independent and uniformly distributed on $\mathcal{I}_1^\mu\left(l_1^\mu, u_1^\mu, \epsilon\right)$ given l_1^μ, u_1^μ. It follows that the conditional expectation of X_1^μ given l_1^μ, u_1^μ is

$$\mathbb{E}^\mu\left[X_1^\mu \mid \left(L_1^\mu, U_1^\mu\right) = \left(l_1^\mu, u_1^\mu\right)\right] = \alpha \frac{M-1}{M-\alpha} \left[y_0^\star + \frac{\epsilon}{2}\left(u_1^\mu - l_1^\mu\right)\right].$$

The conditional variance is

$$\mathrm{Var}^\mu\left[X_1^\mu \mid \left(L_1^\mu, U_1^\mu\right) = \left(l_1^\mu, u_1^\mu\right)\right] = \alpha^2 \left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{12}\left(u_1^\mu + l_1^\mu\right)^2.$$

Since l_1^μ and u_1^μ are unknown, player μ will base her decision on the unconditional expectation

$$\begin{aligned} y_1^\star &= \mathbb{E}^\mu\left[X_1^\mu\right] = \alpha\,\frac{M-1}{M-\alpha}\left[y_0^\star + \frac{\epsilon}{2}\left(\frac{N-y_0^\star}{2} - \frac{y_0^\star}{2}\right)\right] \\ &= \alpha\,\frac{M-1}{M-\alpha}\left[y_0^\star\left(1 - \frac{\epsilon}{2}\right) + \frac{\epsilon\,N}{4}\right]. \end{aligned}$$

Note that $y_1^\star$ does not depend on μ. The unconditional variance of X_1^μ can be easily obtained using that for a random variable X and a σ-algebra $\mathcal{B}$ we have

$$\operatorname{Var}\left[X\right] = \mathbb{E}\left[\operatorname{Var}\left[X \,|\, \mathcal{B}\right]\right] + \operatorname{Var}\left[\mathbb{E}\left[X \,|\, \mathcal{B}\right]\right] \qquad ([illegible])$$

(Feller 19[illegible]). Let $\mathcal{B} = \sigma(L_1^\mu, U_1^\mu)$, the σ-algebra generated by L_1^μ and U_1^μ, then it follows that

$$(\sigma_1^\star)^2 = \operatorname{Var}^\mu\left[X_1^\mu\right] = \alpha^2\left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{12}\left[\frac{7}{12}N^2 - \frac{2}{3}Ny_0^\star + \frac{2}{3}(y_0^\star)^2\right].$$

Proof. Using that

$$\mathbb{E}^\mu\left[X_1^\mu \,|\, L_1^\mu, U_1^\mu\right] = \alpha\frac{M-1}{M-\alpha}\left[y_0^\star + \frac{\epsilon}{2}(U_1^\mu - L_1^\mu)\right],$$

$$\operatorname{Var}^\mu\left[X_1^\mu \,|\, L_1^\mu, U_1^\mu\right] = \alpha^2\left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{12}(U_1^\mu + L_1^\mu)^2$$

and equation ([illegible]) we have

$$\begin{aligned} \operatorname{Var}^\mu\left[X_1^\mu\right] &= \mathbb{E}^\mu\left[\operatorname{Var}^\mu\left[X_1^\mu \,|\, L_1^\mu, U_1^\mu\right]\right] + \operatorname{Var}^\mu\left[\mathbb{E}^\mu\left[X_1^\mu \,|\, L_1^\mu, U_1^\mu\right]\right] \\ &= \mathbb{E}^\mu\left[\alpha^2\left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{12}(U_1^\mu + L_1^\mu)^2\right] \\ &\quad + \operatorname{Var}^\mu\left[\alpha\frac{M-1}{M-\alpha}\left[y_0^\star + \frac{\epsilon}{2}(U_1^\mu - L_1^\mu)\right]\right] \\ &= \alpha^2\left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{12}\left[\frac{7}{12}N^2 - \frac{2}{3}Ny_0^\star + \frac{2}{3}(y_0^\star)^2\right]. \end{aligned}$$

□

STEP k Given $y_{k-1}^\star$ from step $k-1$, player μ believes that all players build intervals $\mathcal{I}_k^\mu$ around $y_{k-1}^\star$ and the X_k^μ's are independent distributed on $\mathcal{I}_k^\mu$. Analogously to Step 1, conditioning on *unknown* realizations l_k^μ, u_k^μ of independent random variables $L_k^\mu \sim \operatorname{unif}[0, y_{k-1}^\star]$ and $U_k^\mu \sim \operatorname{unif}[0, N - y_{k-1}^\star]$ respectively, the interval is given by

$$\mathcal{I}_k^\mu(l_k^\mu, u_k^\mu, \epsilon) = [y_{k-1}^\star - \epsilon\, l_k^\mu, y_{k-1}^\star + \epsilon\, u_k^\mu] \subset [0, N],$$

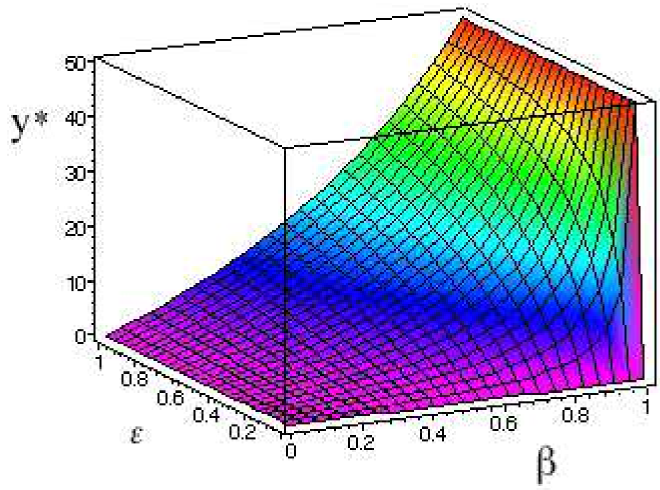

Figure 4. *Expected asymptotic Winning Number* $y^* = y^*(N,\beta,\epsilon)$ *for* $M \sim \infty$ *and* $N = 1$.

and the X_k^μs are conditionally uniformly distributed on $\mathcal{I}_k^\mu(l_k^\mu, u_k^\mu, \epsilon)$. The conditional expectation of X_k^μ given l_k^μ, u_k^μ is

$$\mathbb{E}^\mu\left[X_k^\mu \mid (L_k^\mu, U_k^\mu) = (l_k^\mu, u_k^\mu)\right] = \alpha \frac{M-1}{M-\alpha}\left[y^\star_{k-1} + \frac{\epsilon}{}(u_i^\mu - l_i^\mu)\right],$$

and the conditional variance is

$$\mathrm{Var}^\mu\left[X_k^\mu \mid (L_k^\mu, U_k^\mu) = (l_k^\mu, u_k^\mu)\right] = \alpha^2\left(\frac{M-1}{M-\alpha}\right)^2 \frac{\epsilon^2}{1}(u_k^\mu - l_k^\mu)^2.$$

Following the same approach as by Step 1, the unconditional expectation and variance are respectively

$$y_k^\star = \mathbb{E}^\mu\left[X_k^\mu\right] = \alpha\frac{M-1}{M-\alpha}\left[y^\star_{k-1}(1 - \frac{\epsilon}{}) + \frac{\epsilon}{}\frac{N}{}\right],$$

$$(\sigma_k^\star)^2 = \mathrm{Var}^\mu\left[X_k^\mu\right] = \alpha^2\left(\frac{M-1}{M-\alpha}\right)^2\frac{\epsilon^2}{1}\left[\frac{}{1}N^2 - \frac{}{3}N y^\star_{k-1} + \frac{}{3}(y^\star_{k-1})^2\right].$$

$y_k^\star$ and $\sigma_k^\star$ do not depend on μ.

The sequence of expected Winning Numbers $(y_k^\star)_{i\geq 0}$ converges to a limit $y^\star$ as stated in the following Proposition. Figure 4 gives the asymptotic expected Winning Number $y^\star$ as function of α and ϵ.

Proposition 1. *Let* $M > 0$, $\mathcal{A} = [0, N]$ *and* $(y_k^\star, k \geq 0)$ *the sequence of expected guessing numbers defined above. Then*

$$y_k^\star = \beta^{k+1}\left(1 - \frac{\epsilon}{2}\right)^k \frac{N}{2} + \beta\frac{\epsilon N}{2}\frac{1 - \beta^k\left(1 - \frac{\epsilon}{2}\right)^k}{1 - \beta\left(1 - \frac{\epsilon}{2}\right)}. \qquad (2)$$

where $\beta = \alpha\frac{M-1}{M-\alpha}$. *Thus* $(y_k^\star)_{i\geq 0}$ *is decreasing and* $y_k^\star \to y^\star$ *for* $k \to \infty$, *where*

$$y^\star = y^\star(N, M, \alpha, \epsilon) = \begin{cases} \frac{N}{2}\frac{\epsilon\beta}{2-\beta(2-\epsilon)} & \text{for } (\alpha, \epsilon) \neq (1, 1), \\ \frac{N}{2} & \text{for } (\alpha, \epsilon) = (1, 1). \end{cases} \qquad (3)$$

Consequently, the sequence $(\sigma_k^\star, k \geq 1)$ *is increasing and*

$$\sigma^\star = \lim_{k\to\infty}\sigma_k^\star = \alpha\frac{M-1}{M-\alpha}\frac{\epsilon}{\sqrt{12}}\sqrt{\frac{7}{12}N^2 - \frac{2}{3}Ny^\star + \frac{2}{3}{y^\star}^2}. \qquad (4)$$

Proof. From the computation above it follows that

$$y_0^\star = \beta\frac{N}{2} \quad \text{and} \quad y_k^\star = \beta\left[y_{k-1}^\star\left(1 - \frac{\epsilon}{2}\right) + \frac{\epsilon N}{2}\right].$$

By applying iteratively the last equation we obtain

$$\begin{aligned} y_k^\star &= \beta\left\{\beta\left(y_{k-2}^\star\left(1 - \frac{\epsilon}{2}\right) + \frac{\epsilon N}{2}\right)\left(1 - \frac{\epsilon}{2}\right) + \frac{\epsilon N}{2}\right\} \\ &= \beta^2\left(1 - \frac{\epsilon}{2}\right)^2 y_{k-2}^\star + \alpha^2\frac{\epsilon N}{2}\left(1 - \frac{\epsilon}{2}\right) + \beta\frac{\epsilon N}{2} \\ &= \dots = \beta^k\left(1 - \frac{\epsilon}{2}\right)^k y_0^\star + \beta\frac{\epsilon N}{2}\sum_{l=0}^{k-1}\beta^l\left(1 - \frac{\epsilon}{2}\right)^l \\ &= \beta^{k+1}\left(1 - \frac{\epsilon}{2}\right)^k\frac{N}{2} + \beta\frac{\epsilon N}{2}\frac{1 - \beta^k\left(1 - \frac{\epsilon}{2}\right)^k}{1 - \beta\left(1 - \frac{\epsilon}{2}\right)}. \end{aligned}$$

For $k \to \infty$ and $(\beta, \epsilon) \neq (1, 1)$ we obtain

$$y_k^\star \to \frac{N}{2}\frac{\epsilon\beta}{2 - \beta(2 - \epsilon)}.$$

For $(\beta, \epsilon) = (1, 1)$, it follows directly from the iteration that $y_k^\star = y_0^\star$ for all $k \geq 0$. Finally, note that $\beta = 1$ if and only if $\alpha = 1$.
To prove that $(\sigma_k^\star, k \geq 1)$ is increasing, note that $x \mapsto \frac{7}{12}N^2 - \frac{2}{3}Nx + \frac{2}{3}x^2$ is decreasing on $[0, \frac{N}{2}]$. Thus, $\sigma_k^\star = \alpha\frac{M-1}{M-\alpha}\frac{\epsilon}{\sqrt{12}}\sqrt{\frac{7}{12}N^2 - \frac{2}{3}Ny_{k-1}^\star + \frac{2}{3}(y_{k-1}^\star)^2}$ increases as $y_{k-1}^\star$ decreases to $y^\star$. The continuity of $f(x)$ implies that $\sigma_k^\star \nearrow \sigma^\star$. $\square$

The Proposition states that the thinking process generates a sequence of expected Winning Numbers that converges quickly to an asymptotic expected Winning Number $y^\star$, that depends on α, ϵ, N and the number of players M. The convergence is very fast, so that $y^\star$ well approximates the expected Winning Numbers obtained after only a few levels of thinking. This is shown in Figure , which gives the numbers of thinking levels k that should be performed in order that $y_k^\star - y^\star \le \delta$ (note that by Proposition 1, $y_k^\star \ge y^\star$), depending on the parameters β and ϵ. The curves in Figure (a) correspond to fixed numbers of thinking levels k, such that the expected Wining Number after k levels of thinking $y_k^\star$ and the asymptotic Winning Number $y^\star$ differ of at most of δ. The Figure (b) gives instead the number of thinking levels as function of ϵ for $\beta = \frac{2}{3}$ and $\delta = 1$ and . Here convergence is within only 3-4 steps. The vertical lines correspond to the usual range for the parameter ϵ, as discussed in the Introduction. From the Proposition 1 we also obtain that $y^\star$ is strictly positive, unless $\alpha = 0$ or $\epsilon = 0$, i.e. the asymptotic expected Winning Number differs from the game-theoretic equilibrium if the confidence parameter is strictly positive. Finally, the variance of the k-th level of thinking Winning Number Y_k is strictly increasing as k increases to ∞. This is due to the asymmetry of the intervals $\mathcal{I}_k$ generated during the thinking process.
The asymptotic Winning Number $y^\star$ can be also written as

$$y^\star = y_0^\star \frac{\epsilon}{(1-\beta)(1-\epsilon)},$$

i.e. it corresponds to the initial guess $y_0^\star$ multiplied by a factor $c(\epsilon) < 1$ depending only on the confidence parameter ϵ (here β and M are given by the game setup). Therefore, if players were informed about the initial expected Winning Number number $y_0^\star$, then their thinking process would generated the asymptotic expected Winning Number $y^\star = y_0^\star c(\epsilon)$. This observation is the starting point of our discussion on the iterated Guessing Game of Section 4.

Next, we derive the density probability function of the Winning Number at each level of thinking and asymptotically, i.e. for $k \to \infty$. For sake of simplicity, let now consider a countable infinite group of players, i.e. $M = \infty$. Then by the Central Limit Theorem, under $\mathbb{P}^\mu$ and conditionally on the upper and lower bound l_k^μ, u_k^μ for the interval $\mathcal{I}_k^\mu(l_k^\mu, u_k^\mu, \epsilon)$, the random variable $X_k^\mu = \alpha\,\overline{X}_k^{(\mu)}$ is normally distributed with mean $m_k^\mu = \alpha\,\left(y_{k-1}^\star + \frac{1}{2}\,\epsilon\,(u_k^\mu - l_k^\mu)\right)$ and variance $(s_k^\mu)^2 = \frac{1}{12}\,\alpha^2\,\epsilon^2\,(u_k^\mu + l_k^\mu)^2$. The unconditional density function f_k of X_k^μ under $\mathbb{P}^\mu$ is then given by

$$f_k(x) = \frac{1}{y_k^\star (N - y_k^\star)} \frac{\sqrt{6}}{\sqrt{\pi}\,\epsilon\,\alpha} \int_0^{y_k^\star} \int_0^{N-y_k^\star} g(u, l, y_{k-1}^\star)\, du\, dl,$$

where $g(u, l, y) = \frac{1}{u+l} \exp\left(-\frac{1}{2}\left(\frac{\sqrt{12}\,(x-\alpha\,y-\frac{1}{2}\,\epsilon\,\alpha\,(u-l))}{\epsilon\,\alpha\,(u+l)}\right)^2\right)$.

(a)

(b)

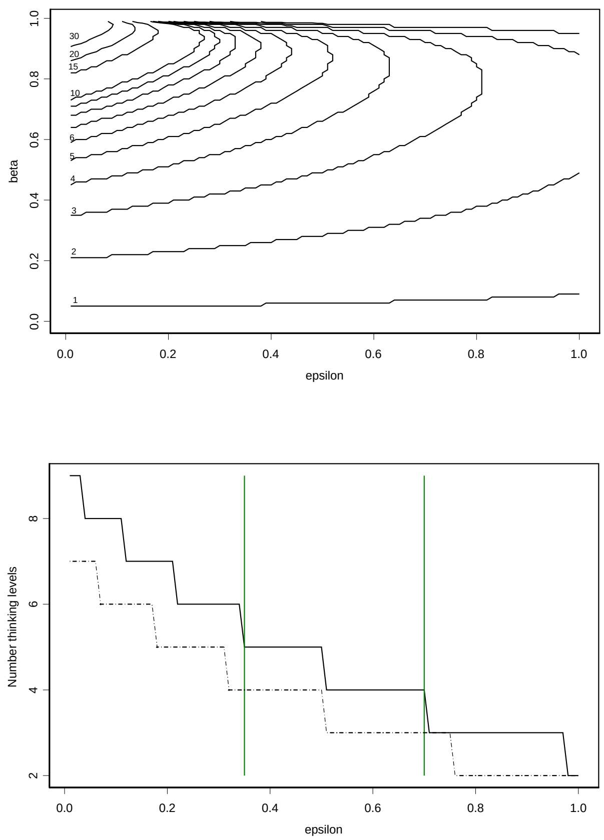

Figure *(a) Number of thinking levels such that $y_k^\star - y^\star \leq$, depending on β and ϵ. The curves correspond to fixed numbers of thinking levels k. (b) Number for thinking levels such that $y_k^\star - y^\star \leq \delta$ as function of ϵ, for β $\frac{2}{3}$ and δ 1 (full line) and δ (dotted line).*

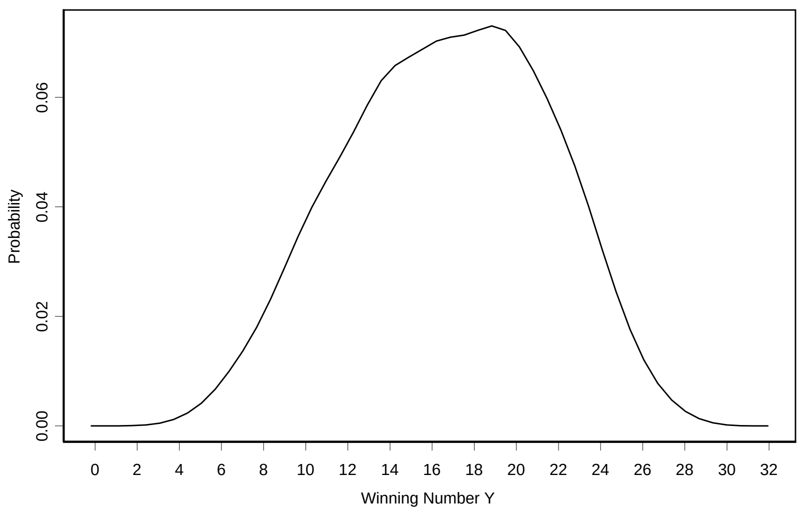

Figure 6 *Asymptotic distribution of the Winning Number Y, with α $\frac{2}{3}$, N 1 , ϵ . (simulation with M 1).*

Obviously, each player will announce her guessing number only once. In the limit $k \to \infty$ we obtain the asymptotic unconditional density f under $\mathbb{P}^{\mu}$

$$f(x) \quad \frac{1}{y^*(N - y^*)} \frac{\sqrt{6}}{\sqrt{\pi}\,\epsilon\,\alpha} \int_0^{y^*} \int_0^{N-y^*} g(u, l(y^*))\, du\, dl.$$

The density function f is illustrated in Figure 6.

3 The distribution in the case of a heterogenous population

Our results from Section were obtained under the assumption that all players are homogeneous, i.e. having the same confidence parameter ϵ and also performing the same number of levels of thinking (see Proposition 1). The advantage of these assumptions is that we were able to derive a closed form solution for the expected Winning Number at each level of thinking. The experimental results (see Figures 1 and) do not support the assumption of an homogeneous population of players. For this reason, facing real data, we take an heterogenous ensemble, by making both parameters – the confidence parameter and the number of thinking levels – random variables. Following Camerer, Ho, and Chong (3) we will simulate the Winning Number Y under the assumption that the number of thinking levels is Poisson distributed with some intensity τ. Figure shows

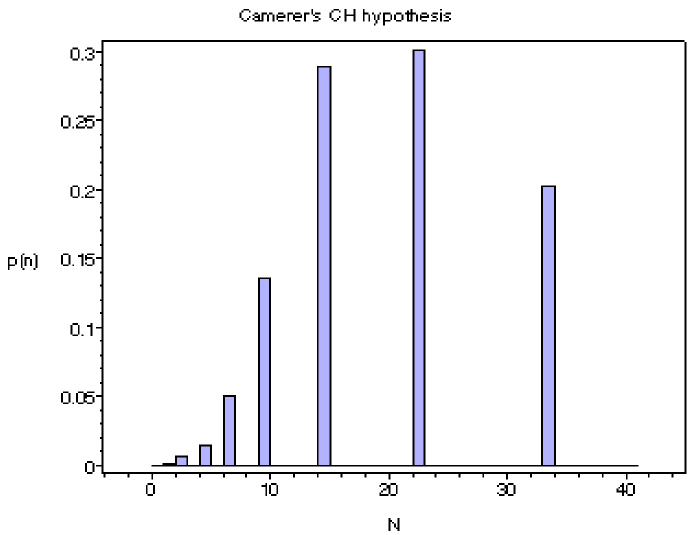

Figure *Relative frequency of guessed numbers due to the model of "Cognitive Hierarchy" by Camerer, Ho, and Chong (2003). The spectrum of guessed numbers is discrete.*

the results obtained by Camerer, Ho, and Chong (3), where no confidence parameter ϵ enters in the model (or equivalently, ϵ). The Figure 8(b) shows instead our simulation result, where the number of thinking level of each player is the outcome of a Poisson distributed random variable with τ 4, while the confidence parameter is randomly chosen in $\mathcal{E}$ $\{ \,^{-i} \; i \; , \ldots, \nu\}$ and ν . For a comparison with our simulation, Figure 8(a) reports the results from the German newspaper "Die Zeit" already shown in Figure 1. We obtain a distribution of the Winning Number, that is now a superposition of a background distribution which is driven by the confidence parameter ϵ for the several level of thinking, and a series of peaks which correspond to the realizations of ϵ near to zero.

4 Convergence towards equilibrium

We finally consider the case that the Guessing Game is played for a number of rounds. The rule is the same for any round and corresponds to the 1-shot game discussed previously, while between any two rounds the Winning Number determined in the preceding round is made public. Within each round any player performs k levels of thinking. We can assume that all players perform the same number of iterations, since the convergence in each round is rapid, as shown in Proposition 1. Therefore, we assume that the Winning Number announced at the end of each period is the respective asymptotic Winning Number of this round, denoted by $y^\star(t)$. While in round 1 all players start with the initial guess $y^\star(\;) \quad \beta \frac{N}{2}$ the Winning Number announced at the end of round 1 is $y^\star(1) \quad c(\epsilon_1)\, y^\star(\;)$, where ϵ_1 is the confidence parameter of the first round and $c(\cdot)$ is defined as in Section .

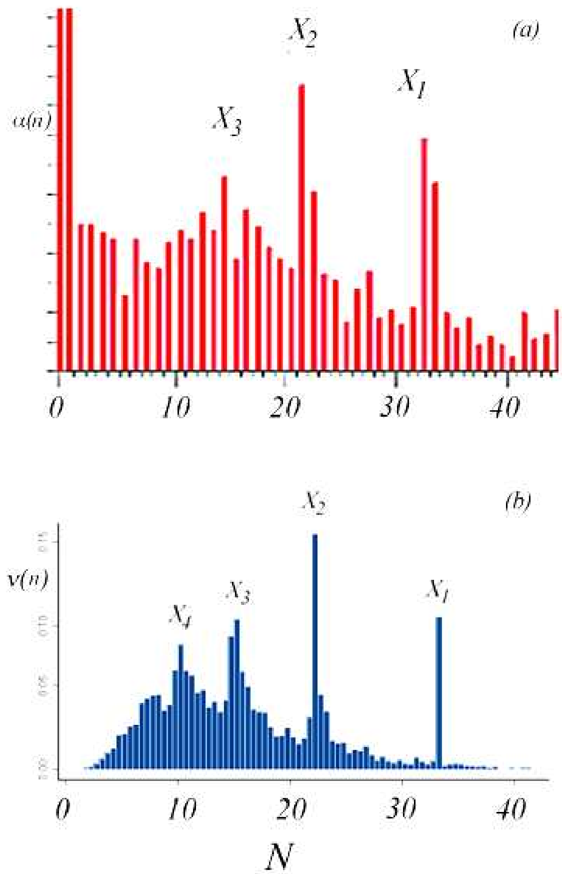

Figure 8 *Distribution of the Winning Number Y, under the assumptions that the number of thinking levels is Poisson distributed with intensity τ 4 and the confidence parameter ϵ is uniformly distributed on $\mathcal{E}$ $\{\ ^{-i}\ i\ \ ,\ldots,\nu\}$ with ν . The other parameters are α $\frac{2}{3}$, N 1 and M 1 .*

Estimates of ϵ in various iterated Guessing Games								
$session_\alpha$	$a_{\frac{2}{3}}$	$b_{\frac{2}{3}}$	$c_{\frac{2}{3}}$	$d_{\frac{2}{3}}$	$e_{\frac{2}{3}}$	$f_{\frac{2}{3}}$	$g_{\frac{1}{2}}$	$h_{\frac{1}{2}}$
α	/3	/3	/3	/3	/3	/3	1/	1/
ϵ	.99	.4	.6	.1	.3	.	.33	.
R^2	.99	.98	.96	.98	.9	.99	.98	.9

Table *Estimation of the confidence parameter ϵ for several experimental results. $a_{\frac{2}{3}}, b_{\frac{2}{3}}, c_{\frac{2}{3}}$, and $d_{\frac{2}{3}}$ are Nagel's sessions 4 – from Nagel (1995) in which α /3 while $e_{\frac{2}{3}}$ is from Weber (http://www.andrew.cmu.edu/user/rweber/) and $f_{\frac{2}{3}}$ is from the experiments at the University of Bergen, while $g_{\frac{2}{3}}, h_{\frac{1}{2}}$ are Nagel's sessions 1, 3 for α 1/ taken also from Nagel (1995).*

The number $y^\star(1)$ is made public to all players at the beginning of round , so that all players start their new thinking process of round with $y^\star(1)$ and they arrive at $y^\star()$ $y^\star(1)c(\epsilon_2)$. Note that since parameters β, N are given by the game rule, $y^\star(t)$ essentially only depends on the sequence $\epsilon_1, \ldots, \epsilon_t$ of confidence parameters up to time t. It might be expected that due to some adaptive mechanisms, ϵ_t might change over rounds. In fact, as a first order approximation we assume that the confidence parameter is constant over rounds, i.e. ϵ_t ϵ $\mathcal{O}(t)$ for all t. Under these assumptions the iterative setting is governed by the following recurrence equation

$$y^\star(t) \approx y^\star(t-1)\,\frac{\epsilon}{-\beta(-\epsilon)}, \qquad (6)$$

where $y^*()$ $\beta\frac{N}{2}$. Hence

$$y^\star(t) \approx y^\star()\left(\frac{\epsilon}{-\beta(-\epsilon)}\right)^t,$$

or

$$\ln y^\star(t) \approx \ln y^\star() \; t \ln\left(\frac{\epsilon}{-\beta(-\epsilon)}\right).$$

Therefore, since $\frac{\epsilon}{2-\beta(2-\epsilon)} < 1$, as $t \to \infty$ the expected Winning Number $y^*(t)$ converges to . The following Figure 9 show real data from experimental studies. In Figure 9 three sessions are shown (data are from Nagel (199)). The agreement with the prediction of our model with constant confidence parameter ϵ is surprising well. On the other hand it is not surprising that ϵ might be different in different experiments. Table summarizes the numerical estimates.

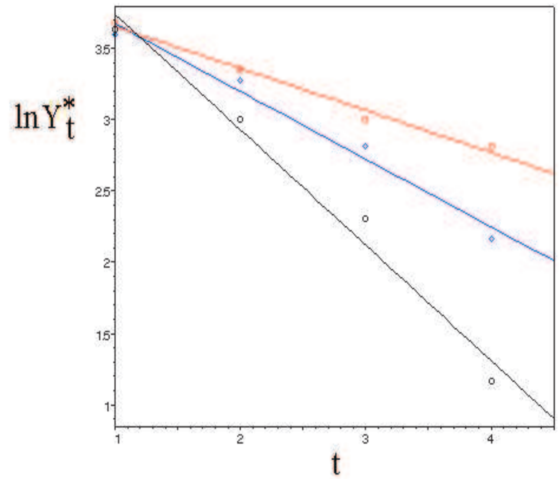

Figure 9 *Data of the iterated Guessing Game due to Nagel (sessions 4, 5, 7) with* α /3 *played with 15 - 18 subjects with* $\epsilon_s \approx 1$, .4, . *(Nagel 1995).*

5 Conclusion

Ever since Keynes, the α-Beauty Contest has served as a paradigmatic framework for strategic price formation processes on markets. It was shown experimentally that the 1-shot α-Beauty Contest is a non-equilibrium game under bounded rationality conditions, while equilibrium is established after a sufficient number of iterative rounds. Unfortunately today non-equilibrium systems are only poorly understood, i.e. there does not exist a general theory of non-equilibrium systems describing their typical properties and their routes into equilibrium. Hence without any general theory, one is left with studies of exemplary systems.

In this note, we studied the α-Beauty Contest with arbitrary parameters and in different settings, such as the 1-shot setting and the iterative setting with communication between rounds. Our studies started from considerations of real data. Experimental data of the 1-shot setting of the -equilibrium game exhibit a common pattern The spectrum of announced numbers is a superposition of a skew background distribution and a regime of extra ordinarily often chosen numbers. Our basic model proposed is able to explain this pattern. It reproduces as well as generalizes classical results. It is based on the assumption that players successively update their recent beliefs according to estimating intervals rather than numbers. Note that our model has only one free parameter ϵ, which is the a measure for the confidence of the player in her guess. It was shown analytically that if players have only finite confidence in their recent belief, the expected Winning Number is strictly positive. The Nash equilibrium is obtained if the players' confidence in their guesses is infinite. The rate of convergence is shown to be high so that the expected asymptotic Winning Number is obtained after only very few thinking steps. This result of the model is consistent with the observation by Nagel and others that about 4 levels of thinking are sufficient to generate this expected Winning Number. If the players' population is heterogenous with respect to the level of confidence, i.e. if the confidence parameter varies over the player's population, numbers announced will have a highly skewed distribution in agreement with real data. Hence our model explains the existence of a skew distribution of announced numbers observed in real data. The combination of Camerer's Cognitive Hierarchy Theory with our model was shown to be able to reproduce the typical pattern observed in real data. A model of the α-Beauty Contest must also describe the route into equilibrium in the iterative setting as is observed in experiments. It is a straight forward consequence of the fast convergence over thinking steps, that convergence towards equilibrium should be polynomial in the number of rounds played. This prediction from our model is in good quantitative agreement with various experimental data.

On a market, there is partial information exchange among traders. Thus it would be interesting to consider the case of partial information among players between rounds. While our model makes clear predictions about the rate of convergence to maximum information gain and equilibrium, concrete data are needed for further investigation.

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